THE GROUND
OF
ARTES

R. RECORD
1586

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The Ground of

Artes : teaching the perfect worke and practise of Arithmetike, both in whole numbers and Fractions,

after a more easie and exact sort, than hitherto hath beene set forth.

Made by M. ROBERT RECORD, *D. in Physicke.*

And nowe lately diligently corrected and beautified with sundrie newe Rules and necessarie Additions: And further endowed with a third part, of Rules of Practise, abridged into a briefer method than hitherto hath beene published: with diuerse such necessarie Rules, as are incident to the trade of Merchandise.

Whereunto are also added diuerse Tables and Instructions that will bring great profite and delight vnto Merchants, Gentlemen, and others, as by the Contents of this Treatise shall appeare.

By *Iohn Mellis* of short *Southwarke*, Schoolemaster.

AT LONDON, *Printed by Henrie Midleton for* Iohn Harison. 1586.

To the Reader.

That which my friend hath well begun
For verie loue to common weale,
Of good will hath beene ouer runne
Correcting each part euerie deale.
And beautified for thy behoose
With briefer wayes for practise lore,
As by the triall and the proose
Was neuer yet in print before.
Of numbers vse the endlesse might
No wit nor language can expresse,
Applie, and trie, both day and night
And then this truth thou wilt cõfesse.

The Bookes Verdict.

To please or displease sure I am,
But not of one sort to euerie man.
To please the best sort would I faine,
The froward displease shal I certaine.
Yet wish I wil, though not with hope,
All eares & mouthes to please or stop.

To the Right worshipfull M. Robert Forth Doctor of Lawe, and one of the Masters of the Queenes Maiesties high Court of Chauncerie.

Hereas heeretofore (Right woorshipfull Master) I traueled to the inlarging of this Author, so it is that the former Impression being worne out, I haue now taken in hand the like, wherin I haue abbreuiated some Rules to a more plainer method, and in other places inlarged both the Author and my Additions, with sundrie rules and questions for the better vnderstanding of such as delight in this excellent Arte. And for as much as my first labour being dedicate to your Worship, was right courteously accepted of you, I am nowe the more incouraged to present you with these my second labours, which of like affection (as one the greatest Iewel that God hath bestowed vpon me) I dedicate againe to your Worship as a testimonie of the duetifull zeale and good will I beare you, not doubting but that you will accept the giuers minde more than the gift, which I confesse is smal. But being throughly acquainted with the great fauor your worship beareth to such as delight in any good exercise, it hath imboldened me to put foorth this simple Addition vnder your Woorships defence. The entyre loue and exercise of this excellent Arte, with *Drawing of Proportions, Mappes, Cards, Buildings, Plattes, &c.* were the onely studies whereunto I euermore

more haue beene enclined. Touching Drawing it was onely *Dei beneficio*, naturally giuen me from my youth without instruction of any man, more than *Loue* thereof, delectation, desire, and practise. In this Arte also hauing great delight, I had no other instruction at my first beginning, but onely this good Authors booke, but afterwards I greatly increased the same during the time I serued your Worship in *Cambridge*, in going to the *Arithmetike* Lecture at the common Schoole: And more furthered since the time that I left your worships seruice, which is about 14 yeres past, by continuall exercise therein (the mother and nurse of Science) during which time my onely vocation hath beene, thinking it a meete exercise for a common wealth) in training vp of youth to write and drawe, with teaching of them the infallible principles and briefe practises of this worthie Science, hauing (I praise God for it) brought vp a number to become faithfull & seruiceable to their masters in great affaires, and many of them good members of a common wealth, which is no small comfort to me in Christ. Amongest which number, a Countreyman of mine hath oftentimes beene verie importunate with me, to doe a deede of Charitie vpon the ground of Arts, vz. to peruse and amende it of the imperfections and faultes, that haue crept into it through negligéce of often printing. Which earnest request of his, bred two strifes in mee: The one was, I was loath to doe it, knowing my selfe inferior to a great number that might better doe it than I. And the other considering it is a Booke hath doone many a thousand good, which when a young beginner commeth to a cónfused or mistaken figure, it bringeth him into a wonderful discouragement and maze: which thing considered, for meere loue to a common wealth, and to the booke, being my first Author, I willingly graunted to do my good will. And passing vnder the file of correction, I here and there increast it with such necessarie Additions as I knewe might incourage a young learner, and more would haue doone, but for feare the booke would

rise

rise too thicke or growe to deere. And being thus entred into the vaine thereof, and knowing that this Author was the only light & the chiefe Lodestone vnto the vulgar sort of English men in this worthie science, that euer writ in our naturall tongue, I haue (according to my simple knowledge) yeelded againe some part of my receiued talent with aduantage: and endowed him to the further increase of his memorie, with Rules of breuitie and practise, abridged into a briefer method than hitherto hath bin published in our English tongue, with other right necessarie Additions, Rules & Tables, which I trust will doe my Countrie good, and be right commodious to all sorts of men: all which I commit to the fauourable censure of your Worship, and all such as loue knowledge, desiring their fauourable correction heerein if ought be amisse. The which with greater affection than I am able to vtter, I dedicate to your Woorship, as a meete Patrone, both for learning, godlinesse, and loue of the same, which comming from your Worship into the hands of many shall (I doubt not) doe many good, as heretofore it hath alreadie doone. So shall you (as the best benefactor of these labors) be partaker of al their prayers that shal reape profit or knowledge by this worthie Arte, in commendation whereof if I should write, I should rather blemish than adorne it. For the Authors Epistle vnto that famous Prince of woorthie memorie King *Edward* the sixt, & his Preface to the Reader, are sufficient.

Thus crauing fauorable acceptation of this my homely and duetifull present, I humblie leaue you to the conduction of the Almightie, whom I beseech long to preserue in continuall health, with daily increase of Worship, to the glorie of his name, and to the ioy of all such as vnfeignedly loue you.

At my house in Southwarke this 24 of August,
Your Worships most bounden to command,

IOHN MELLIS.

To the moſt mightie Prince EDVVARDE the ſixt, by the grace of God king of Englande, Fraunce, and Irelande, &c.

THE excellencie of mans nature, being ſuch, as it is by Gods diuine fauour (moſt mighty Prince) not onely created in highnes of degree farre aboue all other corporall things, but by perfectiō of reaſon and ſearch of wit, much approching toward the image of God, as not onely the holy Scriptures doe teſtifie, but alſo thoſe natural Philoſophers, which exactly did conſider the nature of man, and namely the farre reach and infinite compaſſe of the workes of the minde, were inforced to confeſſe, that man ſcarcely was able to knowe himſelfe. And if he would duely ponder the nature of himſelfe, he ſhould finde it ſo ſtrange, that it might ſeeme vnto him a verie miracle. And thereof ſprang that ſaying: *Magnum miraculum eſt homo, maximum miraculum ſapiens homo.* For vndoubtedly as man is one of the greateſt miracles that euer God wrought, ſo a wiſe man is plainely the greateſt.

And therefore was it that ſome did account the head of a man the greateſt miracle in the worlde, becauſe not onely of the ſtrange workemanſhip that is in it, but much more of the efficacie of reaſon, wit, memorie, imagination, and ſuch other powers and

woꝛkes of the minde, which can moꝛe easilie conceiue any thing in a maner, than vnderstande it selfe. And amongest al the creatures of God it findeth none moꝛe difficult to be perceiued than these same powers of it selfe, whereby it doth conceiue and iudge: as it may be well coniectured by the diuersitie of opinions, that the wisest Philosophers did vtter touching the spirit of man and the substance of it: whereof at this present I intende to make no rehearsall, but who so listeth to reade thereof, may finde it largely set foꝛth not onely in Aristotle his bookes *de Anima*, but also in Galene his booke called *Historia Philosophica*, and againe in Plutarch his woꝛke, *de philosophorum placitis*, whose woꝛds are also repeated of Eusebius in the xv. Booke, τῆς εὐαγγελικῆς προπαρασκευῆς, vnto whom I remit them that haue desired to vnderstand the intricate difficulty of knowing our owne selues, as touching our best part, and that part whereby we deserue to beare the name of men.

This matter seemed so obscure and difficult, in knowledge, that Galene, who foꝛ his excellent wisedome and iudgement in naturall woꝛkes, is called of many men a Miracle in nature, yet in searching the nature and substance of the spirite of man, he not onely confesseth himselfe ignoꝛant, but counteth it plaine temeritie to attempt to find it: so farre aboue the hope of mans knowledge is that part, whereby man doeth knowe and iudge of things. And although the ignoꝛant soꝛt (which hate all things that they knowe not) doe litle esteeme the profoundnesse of mans spirite and of reason, the chiefe power and facultie of it, yet as there is a kinde of feare and obedience of vnreasonable beasts vnto man by the woꝛking power of God, so is there in those small reasoned persons a certaine kinde of reuerence toward wisedome and reasons, which they doe shewe oftentimes, and by power of perswasion are inforced to obey reason, will they will they.

And

And hereby came it to passe, that the rudenesse of the first age of man was brought vnto some more ciuill trade, as it is well declared by Cicero in the beginning of his first booke *De inuentione Rhetorica*, where he saith thus.

Nam fuit quoddam tempus quum in agris homines passim bestiarum more vagabantur, & sibi victu ferino vitam propagabant, nec ratione animi quicquam, sed pleraq; viribus corporis administrabant. Nondum diuinae religionis, non humani officij ratio colebatur: Nemo legitimas viderat nuptias, non certos quisquam inspexerat liberos, non ius æquabile quid vtilitatis haberet, acceperat: ita propter errorem atque insciti am, cæca ac temeraria dominatrix animi cupiditas, ad se explendam viribus corporis abutebatur, pernicios issimis satellitibus.

Quo tempore quidam, magnus videlicet vir & sapiens, cognouit quæ materia esset, & quanta ad maximas res oportunitas in animis inesset hominum, si quis eam posset elicere, & præcipiendo meliorem reddere: Qui dispersos homines in agris, & in tectis syluestribus abditos ratione quadam compulit in vnum locum, & congregauit: & eos in vnamquamque rem inducens vtilem atque honestam primo propter insolentiam reclamantes, deinde propter rationem atque orationem studiosius audientes, ex feris & immanibus mites reddidit, & mansuetos.

This long repetition of Tullies words will seeme tedious to them which loue but litle, and care much lesse for the knowledge of reason, but vnto your Maiestie (I dare say) it is a delectable remembraunce, and vnto me it seemed so pleasant, that I could scarce stay my penne from writing all that mine eyes did so greedily reade.

This sentence of Cicero am I loath to translate into English, partly for that vnto your Maiestie it needeth no translation, but especially knowing how farre that grace of Tullies eloquence doeth excell any English mans tongue, and much more exceedeth the baseneſse of my barbarous style, yet for the fruit of the sentence, I had rather vnto my meere English Countreymen vtter the rudenesse of my translation, than to de-

defraude them of the benefite of so good a lesson, trusting they will also gladly and greedily imbrace all good sciences that may helpe to the iust furniture of the same, when they consider that informed reason was the onely instrument, or at least the chiefe meane to bring men vnto Ciuile regiment, from barbarous maners and beastly conditions.

For the time was (saith Tullie) that men wandered abroade in the fields vp and downe like beastes, and vsed no better order in feeding than they, so that by reasons rule they wrought nothing, but most of their doing did they atchieue by force of strength. At this time there was no iust regard of religion toward God, nor of duetie toward man. No men had seene right vse of mariage, neither did any man know their owne children from other, nor no man had felt the commoditie of iust Lawes: so that through errour and ignorance, wilfull lust, like a blinde and headie ruler, abused bodily strength as a most mortall minister for the satisfying of his desire. At that time was there one, which not onely in power, but also in wisedome was great: and he considered, howe that in the mindes of men was both apt instruments, and great occasion to the due accomplishment of most waightie affaires, if a man could applie them to vse, and by teaching of rules frame them to better trade. This man with perswasion of reason gathered into one place the people that were wandring about the fieldes, and lay lurking in wilde cottages and woods: And bringing them into one common societie, did trade them to all such things, as either were profitable or honest, although not without repyning at the first, by reason that they had not beene so accustomed before. Yet at length through reason and perswasion of wordes they obeyed him more diligently, and so of a wilde and cruell people, he made them courteous and gentle.

Thus hath Tullie set forth the efficacie of reason and

and perswasion, how it was able to conuert wild people to a mildenesse, and to change their furious cruelnesse into gentle courtesie. Were it not nowe a great reproch in this our time (when knowledge raigneth so large) that men should shewe themselues lesse obsequious to reason: vnlesse it may be thought, that nowe euerie man hauing sufficient knowledge of him selfe, needeth not to hearken to the perswasion of others.

Indeede he that thinketh himselfe wise, will not esteeme the reason of any other, be he neuer so wise, so that of such one it may be well saide, He that thinketh himselfe wiser than he is, may iustly be counted a double fole: wherefore such men are not to be permitted in open audience to talke, but must be put to silence, and made to giue eare to reason, which reason consisteth not in a multitude of wordes, heaped rashly together, and applyed for one purpose, but reason is the expressing of a iust matter with wittie perswasions, furnished with learned knowledge. Such knowledge had Moses being expert in all learning of the Ægyptians, as the Scriptures declare, and therefore was able to perswade the stubborne people of the Iewes, although not without great pain. Such knowledge and such reasons did Druys shewe, which was the first Lawe maker of all the West parts of Europe. Like reason and wisedome did Xamolxis vse amongest the Gothes: Lycurgus vnto the Lacedemonians, Zeleucus to the Locrians, Solon to the Athenienses, & Donwallo Molmitius two thousand yeres past, amongest the olde Britanes of this Realme. And thereby it came to passe, that their Lawes continued long, till more perfect reason altered many of them, and wilfull power oppressed most of them.

Druys was sonne to K. Sarron, & succeeded him in his kingdome.

At the beginning when these wise men perceiued howe harde it was to bring the rude people to vnderstand reason, they iudged the best meanes to attaine this honest purpose to depende of learning in euerie kinde:

kinde: for by learning, as Ouid saith: *Pectora mollescunt, asperitas q; fugit*: Stoute stomacks doe waxe mild, and sharpe fiercensse is exiled. Therefore as Berosus doeth testifie, Sarron that was the third king ouer all this West part of Europe, for to bring the people from beastly rage to manly reason, did erect Schooles of liberall Artes, which toke so good successe, that his name continued in that sort famous aboue two thousand yeres after: for Diodorus Siculus which was in the time of Iulius Cæsar maketh mention of the learned men of the Gothes, and named them Sarronides, that is to say, Sarron his scholers and followers.

Among these Artes that then were taught, some did informe the tongue, and make men able both to vtter aptly their minde, and also to perswade, as Grammer, Logike, and Rhetorike, although not so curiously as in this time: some other did appertaine to the iust order of partition of Landes, the true vsing of waights, measures, and reckenings in all sorts of bargaines, and for order of building and sundrie other vses, those were Arithmetike and Geometrie. Againe, to incourage men to the honour of God, they taught Astronomie, whereby the wonderfull workes of God were so manifestly set forth, that no mans tongue nor pen can in like sort expresse his infinite power, his vnspeakable wisedome, and his exceeding goodnesse toward man, whereby he doeth bountifully prouide for man all necessaries, not onely to liue, but also to liue pleasantly. And so was their confidence in Gods prouidence strongly stayed, knowing his goodnesse to be such, that he would helpe man as he could, and his power to be so great, that he could doe what he would: and thirdly his wisedome to be so pure, that he would doe nothing, but that was best. Beside these Sciences, they taught also Musicke, which most commonly they did applie partly to religious Sciences, to drawe men to delight therein, and partly to songs made of the maners of men in praise

of

of Vertue and discommendation of Vice, whereby it came to passe, that no man would displease them, nor doe any thing euill that might come to their hearing: for their Songs did make euill men more abhorred in that time, than any excommunication doeth in this time. The posteritie of these Musitians continue yet both in Wales and Irelande called Bardes vnto this day, by the auncient name of Bardus, their first founder. This Bardus Druidus the v. king of the Celtes, raigned 69 yeres, and died 1832 yeres before Christ

And as these Sciences did increase, so did vertue increase thereby. Againe, as these Sciences did decay, so vertue lost her estimation, and consequently was litle in vse: whereof to make a full declaration, were a thing meete for a Prince to heare, but it would require a peculiar treatise. Wherefore at this present I count it sufficient lightly to haue touched this matter in generall wordes, and to say no more of the particularitie thereof, but onely touching one of those sciences, that is, Arithmetike, by which not onely iust partition of landes was made, but also touching buying and selling, all assises, weights, and measures were deuised, and all reckenings and accounts driuen pea by proportion of it, were the true orders of Iustice limited, (as Aristotle in his Ethikes doeth declare) & the degrees of estates in the common wealth established. Although that proportion be called Geometricall, and not Arithmeticall, yet doeth that proportion appertaine to the Arte of Arithmetike: and in Arithmetike is taught the Progression of such proportions, and al things thereto belonging. Wherefore I may well say, that seeing Arithmetike is so many wayes needefull vnto the first planting of a common wealth, it must needes be as much required to the preseruation of it also: for by the same meanes is any common wealth continued, by which it was erected and established. And if I shall in small matters in appearance, but in deede verie waighty, put one example or two, what shal we say for the statutes

of

of this Realme, which be the onely stay of good order in maner nowe: As touching the measuring of grou̇nd by length and breadth, there is a good and an auncient statute made by Arte of Arithmetike, and nowe it shall be to litle vse, if by the same Art it be not practised and tryed. For the assise of bread and drinke, the two most common and most necessarie things for the sustentation of man, there was a goodly ordinance in the Lawe made, which by ignorance hath so growen out of knowledge and vse, that fewe men doe vnderstande it, and therefore the statute bookes wonderfully corrupted, & the Commons cruelly oppressed: notwithstanding some men haue written, that it is too doubtfull a matter to execute those Assises by those statutes, by reason they depende of the standerde of the coine, which is much chaunged from the state of that time when those statutes were made. Thus shall euerie man reade that listeth in the Abridgement of Statutes in the title of Waights and Measures, in the seuenth number of the English Booke, where he should haue translated a good ordinance, which is set forth in the French booke: but no maruel if the Abridgement doth omit it, seeing the great booke of statutes doeth omit the same statute, as it hath done diuerse other verie good Lawes. And this is the fruite of ignorance, to reiect all that it vnderstandeth not, although they vse some clokes for it: but such clokes, as being allowed, might serue to repel al good lawes, which God forbid.

Againe there is an auncient order for assise of fyre wood, and coales, which was renued not many yeres past, and nowe howe Auarice and Ignorance doeth canuas the statute, it is too pitifull to talke of, and more miserable to feele.

Furthermore, for the statute of Coinage, and the standerd thereof, if the people vnderstoode rightly the statute, they should not, nor would not (as they often doe) gather an excuse for their follie thereby, but as I saide,

saide, these statutes by wisedome and good knowledge of Arithmetike were made, and by the same must they be continued. And let Ignorance no more medle with the vse of them, than it did with the making of them. Oh in howe miserable case is that Realme, where the Ministers and Interpreters of the Lawes are destitute of all good sciences, which be the keyes of the Lawes? How can they either make good lawes or maintaine them, that lacke that true knowledge wherby to iudge them? And happie may that realme be accounted where the Prince himselfe is studious of learning, and desireth to vnderstand equitie in all Lawes. Therefore most happie are we the louing subiectes of your Maiestie, which may see in your highnesse not onely such towardnesse, but also such knowledge of diuerse Artes as seldome hath beene seene in any Prince of such yeres, whereby we are inforced to conceiue this hope: Certainely, that he which in those yeres seeketh knowledge when knowledge is least esteemed, and of such an age can discerne them, to be enemies both to his royall person, and to his realme, which labour to withdrawe him from knowledge to excessiue pastime, and from reasonable studie to idle or noysome pleasures, he must needes, when he commeth to more mature yeres, be a most prudent Prince, a most iust Gouernour, and a right Iudge, not onely of his subiects commonly, but also of the ministers of his Lawes, yea, and of the Lawes themselues, and to be able to conceiue the true equitie and exact vnderstanding of all his Lawes and statutes, to the comfort of his good subiects, and the confusion and reproch of them which labour to obscure or peruert the equitie of the same Lawes and statutes. Howe some of those statutes may be applyed to vse, as well in our time, as in any other time, I haue particularly declared in this booke, and some other I haue omitted for iust considerations, till I may offer them first vnto your Maiestie, to weigh them, as to

your

your Highnesse shall seeme good: for many thinges in them are not to be published without your Highnes knowledge and approbation, namely because in them is declared all the rates of alloyes for all standerdes from one ounce vpwarde, with other mysteries of Mynte matters, and also most part of the varieties of coynes, that haue beene currant in this your Maiesties realme by the space of sixe hundred yeares last past, and many of them that were currant in the time that the Romanes ruled here.

All which, with the auncient description of Englande and Irelande and my simple censure of the same, I haue almost compleated to be exhibited to your Highnesse.

In the meane season most humbly beseeching your Maiestie, to accept this simple treatise, not worthie to be presented to so high a Prince, but that my lowly request to your maiestie is, that this amongest other of my bookes may passe vnder the protection of your highnesse, whom I beseech God most earnestly and daily, according to my duetie, to aduance in all honour and princely regalitie, and to increase in all knowledge, iustice, and godlie policie.

AMEN.

Your Maiesties most obedient
Subiect and Seruant

ROBERT RECORDE.

To

TO THE LOVING Reader.

The Preface of Master R. RECORDE.

ORE oftetimes haue I lamented with my selfe the infortunate condition of *England*, seeing so many great Clerkes to arise in sundrie other partes of the world, and so fewe to appeare in this our nation: whereas for pregnancie of naturall wit (I think) fewe Nations doe exceede Englishmen: But I cannot impute the cause to any other thing than to the contempt and misregarde of learning. For as Englishmen are inferior to no men in mother wit, so they passe all men in vaine plesures, to which they may attaine without great paine or labour: & are as slacke to any, neuer so great commoditie, if there hang of it any painefull studie or trauelsome labour.

Howbeit, yet all men are not of that sort, though the most part be, the more pitie it is: but of them that are so glad, not onely with painefull studie and studious paine to attaine learning, but also with as great studie and paine to communicate their learning to others, and make all *England*, if it might be, partakers of the same, the most part are such, that vnneth they can supporte their owne necessarie charges, so that they are not able

to beare any charges in doing of that good, that elſe they deſire to doe.

But a greater cauſe of lamentation is this: that when learned men haue taken paines to do things for the aide of the vnlearned, ſcarce they ſhall be allowed for their well doing, but derided and ſcorned, and ſo vtterly diſcouraged to take in hande any like enterprice againe. So that if any be found (as there are ſome) that doe fauour learning and learned wits, and can be content to further knowledge, yea onely with their worde, ſuch perſons, though they bee rare, yet ſhall they incourage learned men to enterpriſe ſome things, at the leaſt, that *England* may reioyce of. And I haue good hope that *England* wil (after ſhe hath taken ſure taſt of learning) not onely bring forth more fauourers of it, but alſo ſuch learned men, that ſhe ſhall be able to compare with any Realme in the world. But in the meane ſeaſon, where ſo fewe regarders of learning are, how greatly they are to be eſteemed that doe fauour and further it, my penne will not ſuffice at full to declare.

Therefore gentle reader, whereas I doe vpon moſt iuſt occaſion iudge, yea and knowe aſſuredly, that there be ſome men in this Realme, which both loue, and alſo much deſire to further good learning, and yet am not well able to write their condigne praiſe for the ſame, I thinke it better with ſilence to ouerpaſſe it, than eyther to ſay too litle of it, or to prouoke againſt them the malice of ſuch other, which doe nothing themſelues that is praiſe woorthie, and therefore cannot abide to heare the praiſe of any other mans good deede.

And conſidering their great fauour vnto learning, though I my ſelfe be not worthie to be reckoned in the number of great learned men, yet am I bolde to put my ſelfe in preace with ſuch abilitie, as God hath lent mee, though not with ſo great cunning as many men, yet with as great affection as any man, to help my Countreymen, and will not ceaſſe daily (as much as my ſmall abilitie wil

ſuffer

ſuffer me) to indite ſome ſuch thing, that ſhall be to the inſtruction, though not of learned men, yet at the leaſt of the vulgare ſort, whoſe argument alwayes ſhalbe ſuch, that it ſhall delight all learned wits, though they do not learne any great things out of it.

But to ſpeake of this preſent booke of Arithmetike, I dare not nor will not ſet it foorth with any woords, but remit it to the iudgement of all gentle readers, & namely ſuch as loue good learning, beſeeching them ſo to eſteeme it, as it doeth ſeeme worthie. And ſo either to accept the thing for it ſelfe, either at the leaſt to allowe my good indeuour. But I perceiue I neede not to vſe any perſwaſions vnto them, whoſe gentle nature and fauourable minde is readie to receiue thankefully, and interprete to the beſt, of all ſuch enterpriſes attempted for ſo good an ende, though the thing doe not alwayes ſatisfie mens expectation.

This conſidered, did bolden me to publiſh abroad this litle booke of the Arte of numbring, which if you ſhall receiue fauourably, you ſhall incourage mee to gratifie you hereafter with ſome greater thing.

And as I iudge ſome men of ſo louing a minde to their natiue Countrey that they would much reioyce to ſee it to proſper in good learning and wittie Artes, ſo I hope well of all the reſt of Engliſh men, that they will not be vnmindefull of his due praiſe, by whoſe meanes they are helped and furthered in any thing. Neither ought to eſteeme this thing of ſo litle value, as many men of litle diſcretion oftentimes doe: For who ſo ſetteth ſmall price by the wittie deuiſe and knowledge of numbring, he litle conſidereth it to be the chiefe point (in maner) wherby men differ from all bruite beaſtes: for as in all other things, (almoſt) beaſts are partakers with vs, ſo in numbring we differ cleane from them, and in maner peculiarly, ſith that in many things they excell vs againe.

The Foxe in craftie wit exceedeth moſt men,
A Dogge in ſmelling hath no man his peere,
To foreſight of weather if you looke then,
Many beaſts excell man, this is cleere.
The wittineſſe of Elephants doth letters attaine,
But what cunning doth there in the Bee remaine?
The Emmet foreſeeing the hardneſſe of winter,
Prouideth victuals in the time of Sommer.
The Nightingale, the Linet, the Thruſh, the Lark,
In Muſicall harmonie paſſe many a Clarke.
The Hedgehog of Aſtronomie ſeemeth to knowe,
And ſtoppeth his caue where the winde doth blowe.
The Spider in weauing ſuch arte doth ſhowe,
No man can him mende, nor followe I trowe.
When a houſe will fall, the Miſe right quicke,
Flee thence before. Can man doe the like?

Many things elſe of the wittineſſe of beaſts and birds might I here ſay, ſaue that another time I intende to write wherein they excell in maner all men, as it is daily ſeene: but in number was there neuer beaſt found ſo cunning, that could knowe or diſcerne one thing from many, as by daily experience you may well conſider, when a Bitch hath many whelpes, or a Hen many Chickens: and likewiſe of other whatſoeuer they be, take from them all their young, ſauing onely one, and yee ſhall perceiue plainely that they miſſe none, though they will reſiſt you in taking them away, and will ſeeke them againe if they may knowe where they be, but elſe they will neuer miſſe them truely but take away that one that is left, and then will they crie and complaine: and reſtore to them that one, then are they pleaſed againe: ſo that of number this may I iuſtly ſay, It is the onely thing (almoſt) that ſepa-

separateth man from beastes. He therefore that shall contemne number, he declareth himselfe as brutish as a beast, & vnworthie to be counted in felowship of men. But I trust there is no man so fowlie ouerseene, though many right smally doe it regard.

Therefore will I nowe stay to write against such, and returne againe to this booke, which I haue written in the forme of a Dialogue, because I iudge that to be the easiest way of instruction, when the Scholer may aske euery doubt orderly, and the Master may aunswere to his question plainely.

Howbeit I thinke not the contrarie, but as it is easier to blame an other mans worke than to make the like, so there will be some that will finde fault, because I write in a Dialogue: but as I coniecture, those shall be such, as doe not, cannot, either will not perceiue the reason of right teaching, and therefore are vnmeete to bee aunswered vnto, for such men with no reason will bee satisfied.

And if any man obiect, that other Bookes haue beene written of Arithmetike alreadie so sufficiently, that I needed not nowe to put penne to the booke, except I will condemne other mens writings: to them I aunswere. That as I condemne no mans diligence, so I knowe that no one man can satisfie euerie man, and therfore like as many doe esteeme greatly other bookes, so I doubt not but some will like this my booke aboue any other English Arithmetike hitherto written, and namely such as shall lacke instructers, for whose sake I haue so plainely set forth the examples, as no Booke (that I haue seene) hath doone hitherto: which thing shal be great ease to the rude readers.

Therefore gentle Reader, though this booke can be small ayde to the learned sorte, yet vnto the simple ignorant (which needeth most helpe) it may bee a good furtherance and meane vnto knowledge. And though vnto the king his Maiestie priuately I doe it dedicate, yet

I doubt not (such is his clemencie) but that he can bee content, yea and much desirous, that all his louing subiects shall take the vse of it, & employ the same to their most profite. Which thing if I perceiue that they thankfully doe, and receiue with as good will as it was written, then will I shortly with no lesse kindenesse set forth such Introductions into Geometrie and Cosmographie, as I haue at other times promised, and as hitherto in English hath not beene enterprised, wherwith I dare say all honest hearts will be pleased, and all studious wits greatly delighted.

I will say no more, but let euerie man iudge as he shall see cause. And thus for this time I will stay my penne, committing you al to that true fountain of perfect number, which wrought the whole worlde by number and measure: he is Trinitie in Vnitie, and Vnitie in Trinitie: To whom be all praise, honour and glorie.

AMEN.

Here followeth a Table of all the Contents of this Booke.

The Contents of the first Dialogue containeth the Declaration of the profite of Arithmetike.

Numeration with an easie & large Table.
Addition.
Subtraction.
Multiplicatiō.
Diuision.
} with diuers examples, and all their proues, and some newe formes of workings, &c.

Reduction with diuerse declarations of Coines, Waights and Measures of sundry formes newely added, with a new Table, containing most part of the golde Coynes throughout Christendome, with the true waight and valuation of them in currant money English, &c.

Progression both Arithmeticall and Geometricall, with diuerse sundrie questions touching the same.

The Golden Rule of three: and the Backer Rule of three: with diuers questions thereunto belonging, newely added and augmented.

The double rule of Proportion.

The Rule of three compoſed of 5 numbers.

The Rule of Felowship, both with time, and without time.

Unto all these are added their prooves.

The second Dialogue containeth

The first 5 kindes of Arithmetike wrought by Counters.

The common kindes of casting of accounts after the Merchants fashion, & Auditors also.

The Contents of the second part, touching Fractions.

What a Fraction is.

Numeration in Fractiõs.

The order of working fractions.

Multiplication.

Division.

with diuers familiar questions proponed for the perfit vnderstanding, & proofe of eche of thē.

Reduction of

diuers fractions into one denomination in 3 varieties.

Fractions of Fractions.

Improper Fractions.

Fractions to the smallest denomination, with easy rules how to cōuert thē thereunto.

Fractions in other partes of things, with a table demonstratiue of their proportiõs.

Fraction, and howe it may be turned into any other Fraction, or into what denomination you list.

Againe of { Multiplication, Duplation, Diuision, Mediation, Addition, Subtraction } in Fractions.

The Golden Rule with diuers questions, and their proofes.

The Backer Rule.

A question of Loane.

The statute of Assise of Bread and Ale recognised and applied to this time, with newe tables thereunto annexed.

The statute of measuring of ground, with a Table thereof faithfully calculated and corrected.

Questions of societie, with the reason of the Rules and proofes of their workes.

To finde three numbers in any proportion.

The Rule of Alligation, with diuerse questions and the proofes of their workes, with many varieties of such solutions.

The rule of Falshode, or falce Position, with diuers questions and their proofes.

The Contents of the third Addition to this Booke.

The first Chapter entreateth of Rules of Breuitie and Practise, after a briefer Methode than hitherto hath beene published in the

the English tongue.
The second Chapter treateth of briefe Reduction of diuers Measures, as Elles, Yards, Braces, &c. by Rules of Practise.
The third Chapter intreateth of the Rule of three in broken numbers after the trade of Merchants, something differing from Master Records order, which is comprehended in 3 Rules.
The fourth Chapter intreateth of Losse and Gaine in the trade of Merchandise.
The fifth Chapter intreateth of Losse and Gaine in the trade of merchandise vpon time, &c. with necessarie questions therein wrought by the double Rule of three, or the Rule of 3 composed.
The sixth Chapter intreateth of rules of payment, and one of the necessariest rules that appertaineth to buying and selling, &c.
The seuenth Chapter intreateth of buying and selling in the Trade of Merchandise, wherein is taken part readie money, and diuerse dayes of payment giuen for the rest, and what is won or lost in the 100 pound forbearance for 12 moneths, &c.
The eight Chapter intreateth of Tares and allowances in the trade of merchandise, solde by waight, and of their Losses and gaines therein, &c.
The ninth Chapter intreateth of Lengths & breadths

breadths of Arras, and other Clothes, with diuers questions incident thereunto.

The tenth Chapter entreath of reducing of Paunes of Geanes into English yardes.

The eleuenth Chapter entreateth of Rules of Loane and Interest with diuers questions incident thereunto.

The twelft Chapter entreateth of the making of Factors.

The thirteenth Chapter entreateth of Rules of Barter or Exchange of merchandize, wherein is taken parte ware, and parte readie moneye, with their proffes, and diuers other necessarie questions thereunto belonging.

The fourteenth Chapter entreateth of exchanging of money from one place to an other, with diuerse necessarie questions incident thereunto.

The fifteenth Chapter intreateth of sixe sundrie formes of practise for the Reduction of English, Flemish, and French money, and howe each of them may easily be brought to others money sterling.

The sixteenth Chapter containeth a briefe note of the ordinarie Coynes of most places of Christendome for traffike, and the maner of their exchaunging from one Citie or Towne to an other, which knowen the Italians call Pary: whereby they finde the Gaine or Losse vpon the Exchange.

change.

The seuentéenth Chapter containeth also a declaration of the diuersitie of the waights and measures of most places of Christendome for traffike, proportionated in equalitie one to another, as also vnto our English measure and waight, whereby the ingenious practitioner may easilie reduce the waight and measure of each Countrey into other.

The eightéenth Chapter intreateth of diuerse Sportes and Pastimes, done by Number.

FINIS.

A Collection of such Tables as are contained in this Treatise.

A large Table of Numeration.
A Table of Multiplication.
A Table of all the golde and siluer Coynes in this Realme, with the most vsuall golde Coynes throughout Christendome with their seuerall waights of pence & graines, and what they are worth in currant money English.
Certaine Tables or Notes of the Contents Ale, Beere, wine, Butter, Sope, Salmon, Eeles, &c. both what such vessels ought to containe by the statute, and what those vessels emptie ought to wey.
A Table of the quantitie of drie measures, as Peckes, Bushels, Quarters, Weyes, &c.
A Table of the proportion of measure, touching Lengths or breadths: to wit, from the ynch to the foote, and so to the yarde, the Elle with their partes, the perch, the rod, the furlong: the myle, &c.
A Table made by Progression Arithmetical, which containeth a double table of Multiplication.
A Table or demonstration of a figure or measure for the perfect vnderstanding of Fractions

ons of Fractions.

A Table of the contents of the Statute for the Assise of the waight of bread. From 1 shilling the quarter to 20 shillings, faithfully corrected.

A necessarie Table of the Statute of measuring of ground, vpon the breadth giuen, what length it ought to containe: faithfully corrected according to the equitie of the statute, wherein the Author declareth how necessarie this worthie Arte of Arithmetike is vnto Gentlemen students of the Lawe, and such other as are desirous of infallible trueth.

Briefe Tables of the readie reducing of English, French, and Flemish money, each into others common currant moneyes.

A briefe Table or collection of the common and vsuall moneyes of most places of Christendome for traffike, the maner of their paymentes or exchanging from one Citie or towne to an other: right necessarie for Merchants, and other Occuppyers, Trauellers, &c.

Tables of the Waights, Measures, and Customes of most places of Europe for traffike, whereby through the aide of the Rule of 3, the ingenious may easilie reduce our measure to the perfect valuation of other Countries measure or waight, & likewise theirs to ours.

Last-

Lastly, a Table demonstrating the true solution of 3 diuers things hidden of 3 seuerall persons in pastime.

Note that this △ Caracter representeth the Crowne by exchange.

Item whereas I haue augmented diuers necessarie questions and rules vnto the Author. The beginning of which Additions are marked with this * note, And where they ende, with this ‡ note.

FINIS.

Before the Introduction of Arithmetike, it were very good to haue some vnderstanding and knowledge of these figures and Notes.

j.	1	one.	xx.	20	twentie.
ij.	2	two.	xl.	40	fourtie.
iij.	3	three.	l.	50	fiftie.
iiij.	4	foure.	lx.	60	sixtie.
v.	5	fiue.	lxx.	70	seuentie.
vj.	6	sixe.	xc.	90	ninetie.
vij.	7	seuen.	C.	100	a hundred.
viij.	8	eight.	CC.	200	two hundred.
ix.	9	nine.	D.	500	fiue hundred.
x.	10	ten.	DC.	600	sixe hundred.
xj.	11	eleuen.	M.	1000	a thousand.
xij.	12	twelue.	M. D.	1500	a thousand fiue hund.
&c.	&c.	&c.	&c.	&c.	&c.

112 ℔ make a C. waight.
56 halfe a C. waight.
28 a quarter of a hundred waight.
16 ounces make a pound.
A poũd Troy which is the standard of Englande, wherby is only wayed Gold, Siluer & Bread, is 12 ounces.

12 ynches make a foote.
3 foote make a yarde.
The yarde is parted into 4 quarters.
And ech quarter into 4 nailes
5 quarters of a yarde make an Elle. Which Elle is also parted in 4 quarters.
And each quarter into foure nayles.

A Dialogue betweene the Master and the Scholer, teaching the Art and vse of Arithmetike with penne.

The Scholer speaketh.

SYR, such is your authoritie in mine estimation, that I am content to consent to your saying, & to receiue it as trueth, thogh I see none other reason that doeth leade mee thereunto: whereas else in mine owne conceite it appeareth but vaine, to bestowe any time priuately in learning of that thing, that euerie childe may and doth learne at al times and houres, when he doeth any thing himselfe alone, and much more when he talketh or reasoneth with other.

Master. Loe, this is the fashion & chance of al them that seeke to defend their blinde ignorance: that when they thinke they haue made strong reason for themselues, then haue they proued quite contrarie. For if numbring be so common (as you graunt it to be) that no man can doe any thing alone, and much lesse talke or bargaine with other, but

he shall still haue to doe with Number: this proueth not number to be contemptible and vile, but rather right excellent and of high reputation, sith it is the ground of all mens affaires, so that without it no tale can be tolde, no communication without it can be long continued, no bargaining without it can duely be ended, or no businesse that man hath, iustly completed. These commodities (if there were none other) are sufficient to approue the worthinesse of Number. But there are other innumerable far passing al these, which declare Number to excede all praise. Wherefore in all great workes are Clerkes so much desired: Wherefore are Auditors so richly fed? What causeth Geometrians so highly to be enhaunced? Why are Astronomers so greatly aduaunced? Because that by Number such things they finde, which else should farre excel mans minde.

Scholer. Verily Syr if it be so, that these men by numbring their cunning doe attaine, at whose great workes most men doe wonder, then I see well I was much deceiued, and numbring is a more cunning thing than I toke it to be.

Master. If Number were so vile a thing as you did esteeme it, then nede it not to be vsed so much in mens communication. Exclude Number, and answere to this question. Howe many yeres olde are you?

Scho

Scholer. Mum.

Master. Howe many dayes in a weeke? howe many weekes in a yere? What landes hath your father? howe many men doeth hee keepe? How long is it sith you came from him to me?

Scholer. Mum.

Master. So that if Number want, you answere all by Mummes: How many mile to London?

Scholer. A poake full of Plummes.

Master. Why, thus you may see, what rule number beareth, and that if number be lacking, it maketh men dumbe, so that to most questions, they must answere Mum.

Scholer. This is the cause sir, that I iudged it so vile, because it is so common in talking euerie while: For plentie is not deintie, as the common saying is.

Master. No, nor store is no sore: perceiue you this? The more common that a thing is, being needefully required, the better is the thing, and the more to be desired. But in numbring as some of it is light and plaine, so the most part is difficult, and not easie to attaine. The easier part serueth all men in common, and the other part requireth some learning. Wherefore as without numbring a man can doe almost nothing, so with the helpe of it, you may attaine to all things.

Scholer. Yea sir: why then it were best to

learne the Arte of Numbring first of all other learning, & then a man nede learne no more, if all other come with it.

Master. Nay not so: but if it be first learned, then shall a man be able (I meane) to learne, perceiue, and attaine to other sciences, which without it, he should neuer get.

Scholer. I perceiue by your former words that Astronomie & Geometrie depende much of the helpe of Numbring: but that other sciences, as Musicke, Phisicke, Lawe, Grammer, and such like, haue any helpe of Arithmetike, I perceiue not.

Master. I may perceiue your great Clerkliuesse by the ordering of your sciences: but I will let that passe nowe, because it toucheth not the matter, that I intend, and I wil shew you howe Arithmetike doth profit in al these, somewhat grossely, according to your small understanding, omitting other reasons more substantiall.

First (as you recken them) Musicke hath not onely great helpe of Arithmetike, but is made, and hath his perfectnesse of it: for all Musicke standeth by number and proportion.

And in Phisicke, beside the calculation of Criticall daies, with other things which I omit, howe can any man iudge the pulse rightly, that is ignorant of the proportion of Numbers?

And as for the Lawe, it is plaine, that the man

man that is ignorant of Arithmetike, is neither méet to be a Judge, neither an Aduocate, nor yet a Proctour. For howe can he wel vnderstand another mans cause appertaining to distributiõ of goods, or other debts, or of sums of money, if he be ignorant of Arithmetike? This oftentimes causeth right to be hindred, when the Judge either delighteth not to heare of a matter that he perceiueth not, or cannot iudge it for lack of vnderstanding: This commeth by the ignorance of Arithmetike.

Nowe as for Grammer, méethinketh you should not doubt in what it néedeth number, sith you haue learned that Nounes of al sorts, Pronounes, Verbes, and Participles, are distinct diuersly by numbers: besides the varietie of nounes of Number, and Aduerbs. And if you take away number from Grammar, then is all the quantitie of Syllables losse. And many other wayes doeth number helpe Grammar. Whereby were al kindes of méeters found and made? Was it not by Number?

But howe néedefull Arithmetike is to all parts of Philosophie, they may some sée, that doe reade either Aristotle, Plato, or any other Philosophers writing. For al their examples almost, and their probations, depende of Arithmetike. It is the saying of Aristotle, that he that is ignorant of Arithmetike is méete for no science. And Plato his master

wrote a like sentence ouer his Schoolehouse dore. Let none enter in hither (quoth he) that is ignorant of Geometrie. Seeing he woulde haue all his scholers expert in Geometrie, much rather he woulde the same in Arithmetike, without which Geometrie cannot stand.

And howe needefull Arithmetike is to Diuinitie, it appeareth, seeing so many Doctors gather so great mysteries out of number, and so much doe write of it. And if I should goe about to write all the commodities of Arithmetike in ciuil actes, as in gouernance of common weales in time of peace, and in due prouision and order of armies in time of warre: For numbring of the hoste, summing of their wages, prouisions of victuals, viewing of Artillerie, with other armour: Beside the cunningest point of all, for casting of ground, for encamping of men, with such other like. And howe many wayes also Arithmetike is conducible for all priuate weales, of Lordes and all possessioners, of merchants, and all other occuppyers, and generally, for all estates of men, besides Auditors, treasurers, receiuers, stewards, bailifes & such like, whose offices without Arithmetike is nothing. If I should (I say) particularly repeat al such cõmodities of this noble science of Arithmetike, it were ynough to make a verie great booke.

Sholer. No, no sir, you shal not nede: For I doubt not, but this that you haue said, were

ynough

inough to perswade any man to thinke this Art to be right excellent and good, and so necessarie for man, that (as I thinke nowe) so much as a man lacketh of it, so much he lacketh of his sense and wit.

Master. What, are you so farre chaunged since, by hearing these fewe commodities in generall? By likelihode you would be farre chaunged, if you knewe all the commodities particular.

Scholer. I beseech you sir, reserue those commodities that rest yet behinde vnto their place more conuenient. And if ye will be so good as to vtter at this time this excellēt treasure, so that I may be somewhat inriched thereby, and if euer I shall be able, I will requite your paine.

Master. I am verie glad of your request, and I will doe it spedily, sith that to learne it you be so readie.

Scholer. And I to your authoritie my wit doe subdue, whatsoeuer you say, I take it for true.

Master. That is too much, and more for no man to be beleeued in all things, without shewing of reason. Though I might of my scholer some credence require, yet except I shewe reason, I doe it not desire. But nowe sith you are so earnestly set this Arte to attaine, best it is to omit no time, least some other passion cole this great heate, and then

Perseuerance in studie.

you leaue off before you see the ende.

Scholer. Though many there be so inconstant of minde, that flitter and turne with euerie winde, which often begin, and neuer come to the ende, I am none of their sort, as I trust you partly knowe. For by my good will what I once begin, till I haue it fully ended, I would neuer blin.

Master. So haue I found you hitherto in deede, and I trust you will increase rather than goe backe. For better it were neuer to assay, than to shrinke and flie in the midle way. But I trust you wil not so doe, therefore tell me briefely. What call you the science that you desire so greatly?

Scholer. Why sir? you knowe.

Master. That maketh no matter. I would heare whether you knowe, and therefore I aske you. For great rebuke it were to haue studied a science, and yet cannot tell howe it is named.

Scholer. Some call it Arsmetrike, & some Augrime.

Master. And what doth those names betoken?

Scholer. That if it please you, of you would I learne.

Master. Both names are corruptly written, Arsmetrike for Arithmetike, as the Grecians call it, and Augrim for Algorisme, as the Arabians founde it, which both betoken

the

the science of Numbring. For Arithmos in Greeke, is called number: and of it commeth Arithmetike, the Art of Numbring. So that Arithmetike is a science or art teaching the maner and vse of Numbring. This art may be wrought diuerslye, with Penne or with Counters. But I will first shewe you the working with the Penne, and then the other in order.

Scholer. This I wil remember. But how many things are to be learned, to attaine this Art fully?

Master. There are reckoned commonly seuen parts or workes of it.

Numeration, Addition, Subtraction, Multiplication, Diuision, Progression, and Extraction of rootes: to these some men adde Duplation, Triplation, and Mediation. But as for these last three, they are contained vnder the other seuen. For Duplation and Triplation, are contained vnder Multiplication, as it shall appeare in their place. And Mediation is contained vnder Diuision, as I will declare in his place also.

Scholer. Yet then there remaine the first seuen kindes of Numbring.

Master. So there doeth: Howbeit, if I shall speake exactly of parts of numbring, I must make but fiue of them: For Progression is a compound operation of Addition, Multiplication and Diuision. And so is the Extraction

tion of rules. But it is no harme to name them as kindes seuerall, seeing they appeare to haue some seuerall working. For it forceth not so much to contende for the number of them, as for the due knowledge and practising of them.

Scholer. Then you will that I shal name them as seuen kindes distinct. But nowe I desire you to instruct me in the vse of each of them.

Master. So will I, but it must be done in order: for you may not learne the last as sone as the first, but you must learne them in that order, as I did rehearse them, if you will learne them spedily and well.

Scholer. Euen as you please. Then to begin, Numeration is the first in order: what shall I doe with it?

Master. First you must knowe what the thing is, and then after learne the vse of the same.

Numeration.

NUmeration is that Arithmeticall skill, whereby we may duely value, expresse and reade any number or summe propounded: or else in apt figures and places, set downe any number knowne or named.

Scho-

Scholer. Why: then me thinketh you put a difference betwéene the value and the Figures?

Master. Yea so doe I: For the valewe is one thing, and the figures are an other thing: and that commeth partly by the diuersitie of figures, but chiefly of the places wherin they be set.

Scholer. Then I must knowe here thrée thinges: the Value, the Figure, and the Place.

Master. Euen so: but yet adde Order to them as the fourth. And firste marke, that there are but ten figures, that are vsed in Arithmetike, and of those ten, one doth signifie nothing, which is made like an o, and is priuately called a Cypher, though all the other sometime be likewise named. The other nine are called signifying figures, and be thus figured. A cipher.

1 2 3 4 5 6 7 8 9

And this is their value.

i. ij. iij. iiij. v. vj. vij. viij. ix.

But here you must marke, that euerie Figure hath two values: One alwayes certain that it signifieth properly, which it hath of his forme: and the other vncertaine, which hée taketh of his place.

A place is called the seate or rowme that a figure standeth in. And loke howe many A place.

figures

gures are written in one summe, so many places hath that whole number. And the first place must be called that that is next to the right hand, & so reckoning by order towards the left hand, so that that place is last, that is next to the left hand. As for example: If there stode before you five men in a rowe, side by side, and you should tell them as they stand in order, beginning with the man that were next to your right hand: then he that were next him should be called the second, and so forth to the furthest from your right hande, which is the first and the last.

Scholer. Yes, I perceiue you well: so might I reckon letters or any other thing. As if I should write eight letters after this order, a,b,c,d,e,f,g,h, then must I say, h is the first, g the ij, f the iij, e the iiij, d the v, c the vj, b the vij, and a the viij.

Master. That is wel done. And after the same sort vse hereafter, that what I declare by one example, doe you expresse by another, and so I shall perceiue whether you vnderstand it or no. And so passe ouer nothing, till you perceiue it well, and be expert therein.

Scholer. Sir, I pray you howe many of these places be there in all?

Master. There is no certaine number of them, but they are sometimes more and sometimes fewer, according to the summe that is expressed. For so many as the figures are, so

many

many are the places: and the last place is so called, not because it is last of all other, but it is the last of that present summe, and it may be the midle place in an other summe.

Scholer. Me seemeth I perceiue this verie wel, as touching the order of reckoning of the places: But as for the number of them, you say there is no certaintie. Nowe there resteth to declare the value of the figures by the diuersitie of places, which you called the Value vncertaine. Value vncertaine.

Master. But first let me heare whether you knowe perfectly the certaine value.

Scholer. Yes sir, as you wrote them, so I marked them.

Master. Howe write you then fiue?

Scholer. By this figure 5.

Master. And howe fiue?

Scholer. Thus, 6.

Master. Write these three numbers each by it selfe as I speake them vij. iij. ij.

Scholer. 7.4.3.

Master. How write you these foure other, ij, j, ir, viij?

Scholer. Thus (I trowe,) 2,1,6,8.

Master. Nay, there you misse: Loke on mine example againe.

Scholer. Sir, truth it is, I was to blame, I toke 6 for 9, but I will be warer hereafter.

Master. Nowe then take heede, these certaine

taine values euerie figure representeth, when it is alone written without other figures ioyned to him. And also when it is in the first place, though many other doe followe: as for example: This figure 9 is ix. standing nowe alone.

Scholer. Howe? is he alone and standeth in the midle of so many letters?

Master. The letters are none of his fellowes. For if you were in Fraunce in the midle of a M. French men, if there were no English man with you, you woulde reckon your selfe to be alone.

Scholer. So it is. Then 9 without more figures of Arithmetike, betokeneth ix, whatsoeuer other letters be about it.

Master. Euen so, and so doeth it, if it bée in the first place ioyned with other, how many soeuer doe followe, as in this example, 3679. you sée 9 in the first place, and doth betoken nine, as if he were alone.

Scholer. I perceiue that. And doeth not 7 that standeth in the second place, betoken lxx: and 6 in the third place, betoken vj? And so 3 in the fourth place, betoken three?

Master. Their places be as you haue said, but their values are not so. For as in the first place, euerie figure betokeneth his owne value certaine onely, so in the second place euerie figure betokeneth his owne value certaine ten times: as in the example, 7. in the second

place

place is seuen times x, that is, lxx. And in the third place, euerie figure betokeneth his owne value a hundred times, so that 6 in that place betokeneth vj. C. And in the fourth place, euerie figure betokeneth his owne value a M. times, as in the foresaide number 3 in the fourth place, standeth for 3 M. And in the fifth place, euerie figure standeth for his owne value x. M. times. And in the vj. place a C M. times: and in the vij. place a M. M. times: and in the viij. place x. M. M. so that euerie place exceedeth the former x. times.

Scholer. As thus: if I make this number at all aduentures, 91359684, here are eight places. In the first place is 4, and betokeneth but foure: in the seconde place is 8, and betokeneth x. times 8, that is 80, In the third place is 6, and betokeneth sixe hundred: In the fourth place 9 is nine thousand. And 5 in the fifth place is x. M. times 5, that is, fiftie M. So 3 in the sixt place, is a C. M. times, that is CCC. M. Then 1 in the seuenth place, a M. M. And 9 in the eight place ten thousand thousand times 9, that is xc. M. M. But now I cannot easily nor quickely reade it in order.

Master. What shall you practise by this meanes. First put a pricke ouer the fourth figure, and so ouer the seuenth. And (if you haue so many) ouer the tenth, thirteenth, sixteenth, and so forth, still leauing two figures betweene each two prickes. And those two

rowes

rones betweene the pricks, are called Ternaries. Ternaries.

Then begin at the last pricke, and see how many figures are betweene him and the end, which cannot passe three: reckoning himselfe for one: then pronounce them as if they were written alone from the rest, and adde at the ende of their value so many times thousande as your number hath prickes.

After that come to the next three figures, and sound them as if they were apart from the rest, & adde to their value so many times thousandes, as there are prickes betweene them, and the first place of your whole number. And so doe by euerie other three figures following, if you haue moe. As in example, 91359684. this was your number.

Put a pricke ouer 9 in the fourth place, and ouer 1 in the seuenth place, and then no more, (for your places come not to ten) as thus: 91359684.

Now goe to the last pricke ouer 1, and take it and the figure 9 that followeth it, and value them alone.

Scholer. 91 that is xci.

Master. So is it: but then adde for the number of your prickes twice M.

Scholer. That is xci, thousand thousand.

Master. So is it. Then take the three other figures from one to the next pricke, and value them.

Scho-

Scholer. 359. that is CCC. lix.

Master, Now adde for the one pricke, that is betweene them and the first place, M.

Scholer. CCC. lix. thousande.

Master. Then come to the other three figures that remaine.

Scholer. 684, that is, vj. C. lxxiiij.

Master. Nowe haue you valued all. And at the ende of the last number you shall adde nothing, because there remaineth no pricke nor number after it: yet proue in an other number, as thus 2308640891053400.

Scholer. 2308640891053400. I haue pricked them as you taught me: but I am in doubt, whether I haue done wel or no, because of the Cyphers: for I remember, you tolde me that they doe signifie nothing, and therefore I doubt whether I shoulde recken them for a figure in setting of the prickes: and againe, I knowe not wherefore they serue.

Master. That wil I tel you nowe. Indeede they are of no value themselues, but they serue to make vp number of places, and so maketh the figure following them to be in a further place, and therefore to signifie the more value: as in this example 90, the Cypher is of no value, but yet he occupyeth the first place, and causeth 9 to be in the second place, & so to signifie ten times 9, that is, 90. so that two Ciphers thrusteth the figure following them, into the third place, & so forth.

Scholer. Then I perceiue in the example aboue, I haue pricked wel ynough: for though that Cipher that is pricked signifie nothing, yet must he haue the pricke, because he came in the xiy. place. Then wil I prowe to number that summe. First there is 230. M. M. M. M. and then followeth 864 M.M.M. And what shall I nowe doe? There is a Cipher in the third place, and no figure after him, but they that I haue reckoned.

Master. He did serue for them that you haue already reckoned, to make them in a place further than they should be if he were away: and therefore nowe you shall let him goe. And so doe alwaies when he occupieth that place next before any pricke, which is the last of that Ternarie, and a Cypher in the last place doth nothing.

Scholer. Then shal I say but 89.M.M.

Master. So, but goe forth.

Scholer. 105 thousand. Nowe are all my prickes spent, and yet remaine 340, so that I must value them CCC.xl. onely.

Master. Nowe can you recken after this sort: and remember, that euery such roume so parted, is called a Ternarie or Trinitie: for you haue numbered or valued the summe most truely, and by the aide of the prickes eache denomination is distinct most plainely.

Scholer. What call you Denominations?

ons?

Master. It is the last value or name added to any summe. As when I say: CCxxx. pounds: pounds is the Denomination. And likewise in saying: 25 men, men is the Denomination, and so of other. But in this place (that I spake of before) the last number of euery Ternarie, is the Denomination of it. As of the first Ternarie, the denomination is Unites, and of the seconde Ternarie, the denomination is thousands: and of the thirde Ternarie, thousand thousandes, or Millions: of the iiij. thousande thousande thousandes, or thousand Millions: and so forth.

Scholer. And what shall I call the value of the iij. figures that may bee pronounced before the Denominators? as in saying: 203000000, that is CCiij. millions. I perceiue by your words, that millions is the denominatiõ: but what shall I call the CCiij. ioyned before the millions? Numerator. Summe.

Master. That is called the Numerator or valuer, and the whole summe that resulteth of them both, is called the summe, value or number.

Scholer. Nowe is there any thing else to be learned in Numeration? or else haue I learned it fully?

Master. I might shew you here who were the first inuentors of this art, & the reason of

all these things that I haue taught you, but that I will reserue till ye haue learned ouer all the practise of this art, least I should trouble your wit, with ouer many things at the first.

Three kindes of number.

But yet this must you marke, that there are three kindes of Number: one called Digits, an other Articles, and the thirde mixt numbers.

Digit.

A Digit is any number vnder 10, as this:

1,2,3,4,5,6,7,8,9.

Article.

And 10 with all other that may be diuided into ten parts iust, and nothing remaine, are called articles: such are 10,20,30,40,50, &c. 100, 200, &c. 1000, &c.

Mixt or cõpound.

And that number is called mixt, that containeth articles, or at the least one article, and a digit: as 12, 16, 19, 21, 38, 107, 1005. and so forth. And for the more ease of vnderstanding and remembrance marke this: The digit number is neuer written with more than one figure, but the article and the mixt number are euer written with more than one figure. And thus they differ, that the article hath euermore this Cipher 0, in the first place: and the mixt number hath euer there some digit.

Scholer. By these last words, I perceiue it much better than I did before, and now (I thinke) I wil neuer misse to know those three asunder.

Ma-

Master. If you remember nowe all that I haue saide, you haue learned sufficiently this first kinde of Arithmetike, called Numeration. Howbeit, I wil exhort you nowe, to remember both this that I haue saide, and all that I shall say, and to exercise your selfe in the practise of it: For rules without practise, are but a light knowledge: and practise it is, that maketh men perfect and prompt in all things

And as you haue learned to gather and expresse the value of a summe propounded, and set downe before you: so must you practise to marke, note, or write downe, with apt figures, and in due places, any number, onely named or recited to you: or of your selfe imagined: as for a proofe. Howe note you, or write downe this summe, fiue thousand, two hundred, fiftie and seuen.

Scholer. This troubleth me nowe, whether I should begin at the first figure or at the last. For reason (me thinketh) should cause me to begin at the first: and yet if I write it as you speake it, I must begin at the last.

Master. When you know your places perfectly, you may begin where you list. But the more ease for your hande is to begin with the last, that is to say, as I did speake them. Yet for the more suretie, a while you may begin with the first, repeating my words backward thus: Seuen, fiftie, two hundred, fiue thou-

sand: or else sounding them all by their digit or valuer: as thus: seuen, fiue, two, fiue: for that way is easiest. But then must you loke well, whether there be any Cipher in your summe that he may be set in his place. As if your last valuer of your summe (as you speake it) be aboue 9, then is there a Cipher in the first place. And if it be a hundred or aboue, then is there two ciphers, one in the first place, and an other in the second, and so forth.

But because this thing is such that cannot be set forth without many wordes, I thinke best here nowe at the ende of Numeration to adde a table easie & ready for the first exercise of it.

Loe this is the Table.

The Table.

The right side or hande.

The names of the digits, values certaine, or valuers.

The Denominatours of the place or value vncertain.	Vnites.	Tennes.	Hundreds.	Thoulandes.	Tenthoulandes.	C. of thoulads.	Millions.	P. of millions.	C. of millions.	M. of millions.	P. M. of millios.
Nine.	9	9	9	9	9	9	9	9	9	9	9
Eyght.	8	8	8	8	8	8	8	8	8	8	8
Seuen.	7	7	7	7	7	7	7	7	7	7	7
Sixe.	6	6	6	6	6	6	6	6	6	6	6
Fiue.	5	5	5	5	5	5	5	5	5	5	5
Foure.	4	4	4	4	4	4	4	4	4	4	4
Three.	3	3	3	3	3	3	3	3	3	3	3
Two.	2	2	2	2	2	2	2	2	2	2	2
One.	1	1	1	1	1	1	1	1	1	1	1
Cipher.	0	0	0	0	0	0	0	0	0	0	0
The order of the places.	First.	Second.	Third.	Fourth.	Fift.	Sixt.	Seuenth.	Eighth.	Ninth.	Tenth.	Eleuenth.

The left side or hande.

This table (as you may see) hath eleuen places, and in each of them are set all the digits, whose certaine value is written in the right hand of the table, and the value vncertaine on the left hande. So that by this table you may learne both howe to expresse any number that you list, (if that it exceede not

eleuen places) that is to say, lxxxx. thousand millions, and so may you by the helpe of it, value all summes proposed vnder the sayde number.

For example: take the summe that I proposed before, which was fiue thousand, two hundred, fiftie and seuen. And if you will expresse it, take the first number (as I speake it) which is fiue M. whose valuer or certaine value is v, and his vncertaine value or denomination is M. First you shall seeke at the right hande of the valuer 5. Then seeke along vnder the title of Denomination toward the left hand, till you find thousands, and vnder it right at the foote of the table, is the number of the place, that is the fourth, wherin you must write your digit or valuer 5.

Afterward come to the second part of the number, two hundred, whose valuer is 2, and his denomination C. Seeke two at the right hand of the table, and goe along vnder the denomination toward the left hande, till you come vnder C: then looke to the table, and there you shal see the number of the place, that is to say, three, wherein you must set your digit 2.

Then doe so by your other two numbers that remaine, and you shall finde 5 in the second place for your fiftie, and 7 in the first place for your seuen. And thus may you doe with other numbers.

Scho-

Scholer. Master I thanke you heartily. I perceiue you seeke to instruct me most plainly and briefly, and not to hide your knowledge with subtile wordes as many doe. For this rule is so plaine, that I can desire it no playner. And though it seeme somewhat long, yet I perceiue it to be a sure way.

Master. So is it, and though it be long, yet it is neither to long, neither to plaine for young learners that lacke practise: for this table is in stead of a teacher, to them that lacke one. But nowe I truste I haue saide ynough of Numeration: which after you haue wel practised, then may you learne forth.

Scholer. Yet I pray you in one thing to tell me your iudgement. Why doe men reckon the order of the places backeward, from the right hand to the left?

Master. In that thing all men doe agree, that the Chaldeyes, which first inuented this arte, did set these figures as they set all their letters: for they write backewarde as you terme it, and so doe they reade. And that may appeare in all Hebrue, Chaldey, and Arabike bookes, for they be not onely written from the right hand to the left, and so must be read, but also the right ende of the booke is the beginning of it: whereas the Greekes, Latines, and all Nations of Europe, doe write and reade from the left hande towarde the right: And all their bookes begin at the

left

left side.

Scholer. That reason doth satisfie me.

Master. It neither satisfieth me, neither liketh me well, because I see that the Chaldeys and Hebrues doe not so vse their owne numbers, as at an other time I will declare. But this plaine reason may best satisfie you presently: That seeing in pronouncing of numbers we keepe the order of our owne reading, from the left hand to the right: And againe, we doe euer name the greater numbers before the smaller: it was reason, that the lesser places containing the lesser numbers, should be set on the right hand, and the greater places containing the greater numbers, to proceede toward the left hande.

Scholer. This reason is to me so plaine, that it seemeth nowe against reason to make a doubt of that order. So that now for Numeration I am satisfied: hoping that practise shall make me fully readie and expert in it. And in the meane season, I desire to learne the other kindes of Arithmetike.

Master That is wel said: but what should you next learne, can you tel?

Scholer. I remember you saide that Addition was next.

Master. Euen so, and what that is must you first knowe.

Addi-

Addition.

Addition is the gathering together and bringing of two numbers or more into one totall summe: as if I haue 160 bookes in the Latine tongue, and 136 in the Greeke tongue, and would knowe how many they be in all, I must write these two numbers one ouer an other, writing the greatest number highest, so that the first figure of the one, be vnder the first figure of the other. And the second vnder the second, and so forth in order.

When you haue so done, drawe vnder them a right line, then will they stand thus. Nowe begin at the first places, toward the right hand alwayes, and put together the two first figures of those two numbers, and loke what commeth of them, write vnder them, right vnder the lyne. As in saying, 6 and 0, is 6. Write 6 vnder 6: as thus.

160	
136	
296	

And then go to the second figures, and doe likewise: as in saying, 3 and 6 is 9: write 9 vnder 6 and 3, as here you see.

160	
136	
296	

And likewise doe you with the figures that be in the third place, saying: 1 and 1 be 2: write 2 vnder thē, and then will your whole summe ap-

160	
136	
296	

appeare thus.

So that nowe you see, that 160, and 136 doe make in all, 296.

Scholer. What? this is verie easie to doe, me thinketh I can doe it euen sith.

There came thorough Cheapeside two droues of cattell: in the first was 848 sheepe, and in the second was 186 other beasts.

Thole two summes I must write as you taught me, thus.

	848
	186

Then if I put the two first figures together, saying: 6 and 8 they make 14. That must I write vnder 6 and 8, thus.

	848
	186
	14

Master. Not so, and here are you twice deceiued. First, in going about to adde together two summes of sundrie things, which you ought not to doe, except you seeke only the number of them, and care not for the things. For the summe that should result of that addition, should be a summe neither of sheepe, nor of other beasts, but a confused summe of both. Howbeit sometimes ye shal haue sums of diuers denominations to be added, of which I will tell you anon: but first I will shewe you, where you were deceiued in an other point, & that was in writing 14. (which came of 6 and 8) vnder 6 and 8, which is impossible. For, howe can two figures of two places be written vnder one figure, and one place?

Scho-

Scholer. Truth it is: but yet I did so vnderstand you.

Master. I saide indéede, that you should write that vnder them, that did result of them both together: which saying is alwaies true, if that summe doe not excéede a digit. But if it be a mixt number, then must you write the digit of it vnder your figures, as I haue said before: but and if it be an article, then write o vnder them, and in both sorts you shal kéepe the article in your minde. And therfore when you haue added your second figures, which occupie the place of tennes, you shall put that 1 thereto, which you kept in your minde: for though it were ten indéede, yet in that place it is but as one, because, that euery 1 of that place, is ten, for it is the place of tens. And in like maner: if you haue in the second place so great a number, that it amounteth aboue 9, then write the digit, and reserue the article in your mind, euer adding it to the next place following: and so of all other places, how many soeuer you haue. And if you haue a mixt number, when you haue added your last figures, then write the digit vnder the last figures, and the article in the next place beyond them: so shall your number resulting of Addition, haue one place more than the numbers which you should adde together.

Scholer. Now doe I perceiue you, and the reason of this is, (as I vnderstande) because that

that no one place can containe aboue 9, which is the greatest figure that is, and then all tens or articles must be put to the next place following: for euerie place (as I may sée) exceedeth the other place next before him, by 10.

Now (if it please you) I wil returne to my example of Cattell. But I remember you saide, I might not adde summes of sundrie things together, & that might I sée by reason.

Master. Trueth it is, if you séeke the due summe of any thing, but if you onely séeke a bare summe, & haue no respect to the thing, then were it better to name the summe onely without any thing, as in saying 848, without naming shéepe, or any thing else. And likewise 186, naming nothing.

Nowe let me sée: howe can you adde those two summes?

Scholer. I must first set them so, that the two first figures stand one ouer an other, and the other each one ouer his fellow of the same place: then shall I drawe a line vnder them both. And so likewise of other figures, setting alwayes the greatest number highest, thus, as followeth.

Then must I adde 6 to 8, which maketh 14, ỹ is mixt number: therfore must I take the digit which is 4, & writing it vnder 6 & 8 kéeping ỹ article 1 in my minde thus.

Next that doe I come to the second

848
186

848
286

4

cond figures, adding them together, saying, 8 and 4, make 12, to the which I put the one reserued in my minde, and that maketh 13, of which number I write the digit 3 vnder 8, and 4, and keepe the article inmy minde thus.

Then come I to the thirde figures, saying: 1 and 8, make 9, and 1 in my minde maketh 10. Sir, shall I write the cipher vnder 1 and 8?

Master. Yea.

Scholer. Then of 10 I write the cipher vnder 1 and 8, and keepe the article in my minde.

Master. What needeth that, seeing there followeth no more figures?

Scholer. Sir, I had forgotten, but I wil remember better hereafter. Then seeing I am come to the last figures, I must write the cipher vnder them, & the article in a further place after the cipher, thus.

```
  848
  186
 1034
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Master. So nowe ye see, that of 848, and 186 added together, there amounteth 1034.

Scholer. Nowe I thinke I am perfect in Addition.

Master. That will I proue by this example.

There are two armies of souldiers: in the one are 106800, and in the other 9400. How many are there in both armies say you?

Scho-

Scholer. First I set thē one ouer an other, beginning with the first numbers at the right hand, thus.

106800
9400
116200

But the neather number will not match the ouer number.

Master. That forceth not.

Scholer. Then doe I adde 0 to 0, and there amounteth 0, that must I write vnder the first place, thus.

106800
9400
0

Master. Well saide.

Scholer. Then likewise in the seconde place I adde 0 to 0, and there ariseth 0, which I write vnder the second place, thus.

106800
9400
0200

Then I come to ỹ third place saying: 4 & 8 make 12, of which I write the digit 2, and keepe the article 1 in my mind, thus.

106800
9400
200

Then I adde 9 to 6, which maketh 15, to that I adde the article 1 that was in my minde, and it is 16. I write 6 vnder 6 & 9, and keepe one in my minde, thus.

106800
9400
6200

Master. Why doe you not write both figures, seeing you are come to the last couple of numbers.

Scholer. May reason sheweth me, that I must adde that article that is in my mind, vnto the next figure of the ouer summe, though there

there be no more in the neather summe.

Master. That is well considered: then doe so.

Scholer. Then say I, 0 in the ouer sum, and 1 in my minde, maketh 1, that write I vnder 0: Then followeth there yet one more in the ouer summe, which hath none to be added to it, for there is none in the neather summe, nor yet in my minde, therefore I thinke I must write that euen as it is.

Master. Yea.

Scholer. Then doth my whole summe appeare thus.

106800
9400
116200

Master. If you marke this, you haue learned perfectly the common addition of all summes which are of one denomination: so that ye obserue this also, that in Addition you must haue two numbers at the least, or else howe can you say that you doe adde? And euer let the greatest number be written highest: for that is the best way, though it be not necessarie.

And forget not this, that if you haue many numbers to adde together, you shall haue oftentimes an article of a greater value than 10, sometimes 20, sometimes 30, sometimes more, yea peraduenture 100. Therefore as you did with the article 10, so doe with them, reseruing them in your minde, and adding to the number next following, so many as their

valuer or value certaine is: that is to say, 2 for 20, 3 for 30, 5 for 50, 10 for 100, 12 for 120. and so forth of other like. So that if the Article be 100, then must you set downe the 0, and keepe 10 in minde to be caried to the next rowe of figures or place, if any such happē to come, which for your better vnderstanding take this example for al.

I would adde these fyue sums into one, which I set after this maner: then doe I begin and gather the summe of the first rowe of figures which commeth to 107, for I take 9 there v. times and that is 90, then 9 and 8 is 17, that is in all 107, of which summe I write the 7 vnder the first rowe of figures, and then for that 100 is v. tens I keep v. in minde: which ten I must adde vnto the next rowe of figures which are in the second place,

4889
4599
2299
3699
1399
4090
1099
3198
299
699
899
499
389
57

which second rowe of figures when they are added together with the v. that I had in my minde, maketh in all 125, of which summe, I write the digit 5 vnder the second rowe, and then for that 120 conteineth fyue tens. I keepe fyue in minde to be added to the third place or rowe of figures, which being added together, make in all 60, the cipher 0, I set downe vnder the rowe of figures in the third place.

And

And the figure 6 I kéepe in minde to be added to the rowe of figures in ỹ fourth place, which when they are added together, make 29: The figure or digit 9 I set downe vnder the fourth place, and because it is my last worke, I set downe the 2 also that I haue in minde to the 9 in the fifth place: so those summes doe make in all 29057.

*But for your more ease in worke when you haue an addition of so many summes to be added together, you were best part that summe into two or three parts, and worke them seuerall, and so put their additions together, and this were the best thing you could doe, when ouer many summes fall to be added.‡

4889
4599
2299
3699
2399
4090
1099
3198
299
699
899
499
389
29057

Scholer. This séemeth somewhat hard by the reason of so many numbers together. Howbeit I thinke if I doe often proue euen with this same example either by working of it alone, or else by parting it as you said euen nowe, that I shal be able to do so shortly with any other summe.

Master. So shall you. For it is often practise that maketh a man quicke and ripe in all things: But because as wel in great summes as in small there may chaunce to be some er-

rour, I will teach you howe you shall proue whether you haue done well or no.

Scholer. That were a great helpe and ease.

Master. Begin first with the highest number, and then to all the other orderly, and adde them together, not hauing regarde to their places, but as though they were all vnites: and still as your number increaseth aboue 9, cast away 9. Then goe forth, euer casting away 9, as often as it amounteth thereto: & so doe till you haue gone ouer all the numbers that you intended first to adde, and whatsoeuer remaineth after such addition and casting away of 9, write it in some voide place by the ende of a line for the better remembrance: and thus is the first part of your worke proued. Then secondly put together the figures that result of the Addition vnder the line, still casting away 9 also. And then that that remaineth, write at the other ende of that line: and if those two figures be like, then haue you well done, but if they be vnlike, then haue you misled. As for example in this present summe: The first figure of the ouer line is 9, let him go: then 8 and 8 is 16, take away 9, resteth 7, adde that 7 to 4 that followeth, & it maketh 11, from which if you take 9, there resteth 2. Then come to the next rowe, whose first and second numbers are 9, therefore ouerpasse them both, and take the 5 to the

the 2 which did remaine in the first row, that maketh 7, put thereto the 4 following, and that maketh 11, thence take 9, and there remaineth 2. Next to that goe to the thirde line, whose two first numbers you may let passe because they are nines, then take the two figures of 2, which with the other two that remained in the seconde rowe make 6. Then goe to the fourth rowe, whose two first numbers let goe, and take the 6 to the 6 that remained, and that maketh 12, take away 9, and there resteth 3, which with the 3 that is next, maketh 6. And so goe through al the other numbers, and you shall finde that there remaineth 5, after you haue cast away 9 as often as you can finde it: therefore write 5 at the ende of a line in a voyde place, thus 5———

Then gather all the figures of the totall summe which is vnder the lowest lyne, and cast away 9 as often as you can finde it, as thus: 7 and 5 make 12, take away 9, there resteth 3: to that if you adde the 2 that is last (for you may omit ỹ 9) then doth it make 5, which 5 you must write at the other ende of the line that you made in the voide place thus. 5———5

And then you see that those two figures be like, whereby you may knowe that you haue done wel, and so may you proue in any other.

Scholer. If it please you, I will proue in

another summe.

Master. With a good will.

Scholer. Then will I take one of your former examples, which was this.

First in the highest line, 8 and 6 make 14, then 9 taken away, there remaine 5, to which I adde the 1 that followeth, and that maketh 6. Then come I to the seconde line, where I finde first 4 which with 6 maketh 10, from that I take 9, and there resteth 1, the next figure is 9, and therefore I let him alone, so finde I one remaining, which I set at the end of a line thus. 1——

1 0 6 8 0 0
9 4 0 0
1 1 6 2 0 0
1 1 6 2 0 0

Then I come to the totall summe, and there I finde that all the figures put together make 10, from which I take 9, and there resteth 1 also, which I put at the other ende of the line thus. 1———1

And because they be like, I know that I haue well added.

Master. So you knowe nowe both how to adde two summes or more together, and also howe to proue whether you haue done wel or no; and nowe will I teach you howe to adde summes of diuers denominations together: which thing can neuer be but when the one denomination is such that it containeth the other certaine times. And yet you shal ad them to the other, not after this sorte as you did them that were of one denomination, but after

Addition of numbers of diuers denominations.

ter such a sort as I wil nowe shewe you, that is to say.

If you haue a summe of diuers denominations, then loke that ye set euerie denomination by himselfe, with some note or figure of his denomination, as they be wont to be written. Then write your other summes so vnder that first, that euerie one be set vnder the other of the same denominations, as for example: if your denominations be pounds, shillings, and pence, write poundes vnder pounds, shillings vnder shillings, and pence vnder pence, and not shillings vnder pence, nor pence vnder pounds.

Scholer. Nowe that you haue spoken it, me thinketh it needeth not to warne me of it, for it were against reason so to confound sums: but yet if you had not spoken of it, peraduenture I should haue bene deceiued in it.

Master. If you do say it is so plaine, I will speake no more of it, but with an example make the matter to appeare euidently.

First, one man oweth me 22 l, 6 s, 8 d. An other oweth me 45 l, 16 s, 6 d. And an other oweth me 4 l, 3 s, I would know what this is altogether. Therefore must I first set downe my greatest summe, and then the other, euerie one vnder his denomination agreeing to the greatest summe, as here you se with a line

l	s	d
22	6	8
45	16	6
4	3	

line vnder them.

Then must I begin at smallest numbers, (which must alwayes be set next the right hande) and adde them together, and if the summe of them will make 1, or 2, or 3 of the next denomination, then must I kepe it in my minde til I come to that place, and vnder that first place must I note the residue, if there remaine any of the same denomination, but if there remaine none, then nede I to write vnder it nothing. And this is all that you must marke in this Addition: for all other things are like to the other maner of Addition before mentioned. Therefore the chiefest point of this Addition is, to knowe the values of common coynes and rated summes. As how many shillings be in a pounde: how many pence in a shilling, of which & of other like things, I will instruct you hereafter in teaching of Reduction: But now I may not disturbe your wit from the thing that we are about.

Therefore let vs returne to that former example which I propoled of three debters, which summes when I had set orderly, they stode thus with a line vnder them.

	£	s	d
	22	6	8
	45	16	6
	4	3	

Then to adde them vnto one sum, I must begin at the right hande, where the smallest denomination is, and adde them together first, saying: 6 and 8 make 14. Nowe seing these

14

14 are pence, which containe one shilling and 2 pence. The 2 pence I set down vnder the line of pence. And the one shilling I kéepe in my minde to carie to the nert rowe being the place of shillings.

l	s	d
22	6	8
45	16	6
4	3	
		2

Then doe I adde the shillinges together, saying 1 in minde and 3 make 4, and 6 make 10, and 6 make 16, and 1 in the second place which standeth for 10, make 26. which is 1 pound, 6 shillings. The 6 shillings I set downe vnder the place of shillings, as appeareth in the example. And the one pound I kéepe to cary to the pounds.

l	s	d
22	6	8
45	16	6
4	3	
	6	2

Then come I to the pounds, adding them altogether, saying 1 that I kéepe, and 4 make 5, and 5 make 10, and 2 make 12. The figure or Digit 2 I set downe right vnder that place or rowe of poundes where I gather them. And the Article 1 I kéepe to carie to the nert place, saying 1 in minde, and 4 is 5, and 2 is 7, which 7 I set downe directly vnder that rowe also. And then appeareth my whole summe thus.

l	s	d
22	6	8
45	16	6
4	3	
72	6	2

And thus must you doe with any such like summes whatsoeuer, whether they be money, weight,

weight or measure, which if you practise your selfe well therein by setting downe of diuers summes, you shall be well acquainted with the feate of Addition.

But nowe can you tell howe to proue this Addition or such other like of diuers denominations, and to trie whether you haue well done or no?

Scholer. I would I could.

Master. That shal you do by this means.

You must make a crosse which shall haue so many lines as you haue sundrie denominations in your addition: As if you haue but two denominations, then you may make it thus, that the ouer part and the neather part may serue for one denomination. And if you haue three denominations, as poundes, shillinges and pence, then must you make three lines thus. The vpright line may serue for poundes, and the highest thwart line for shillings, and the lowest for pence: as for example by our summe which we last wrought,

£	s	d
22	6	8
45	16	6
4	3	0
72	6	2

For the proofe of the which, because it containeth three denominations, I must make a crosse of 3 lines, as in ỹ example before. Then I recken first at the right hand the pence: 6 & 8 make 14, from which I take 12 for the next denomination, ỹ is to say, a shilling, & there resteth 2, which I must write at one end of the nether thwart lyne.

After that I gather the summe of the shillings, 3, 16, 6, which maketh 25, to them I put 1 that I toke of the pence, and that maketh 26: from those I take 20, the quantitie of the next greater denomination, that is to say, a pound, and there resteth 6, which I write at the ende of the highest thwart line.

Thirdly, I adde together the pounds, 4, 5, and 2, which make 11, to them I adde the 1 that came of the shillings, and they make 12, from whence I cast 9, and there resteth 3. That three I ioyne to the 4 in the next place, and they make 7, then the 2 more make 9, which I cast away, & so is there left 0, which 0 I set at the vpper ende of the crosse also. (And so should there be set also any number that were left vnder 9.) And thus is the first part of my worke proued.

That done, I come to the totall summe vnder the line, and examine it, beginning at the pence, where I finde but 2, and cannot take 9 from him, therefore I set him at the other ende of the nether thwart line: Then I

come

come to the shillings, where I finde onely ſ. 6 which because it is lesse than 9, I set it at the other ende of the line of shillings, that is, the ouermost thwart line.

Last of all, of the 72 l, I take eight times 9, which is 72, and there remaineth 0, which I write vnder the vpright line, either else I may reckon them simply without any respect of their valuation or place: in saying 2 and 7 make 9, which I cast away, and so resteth 0:

Then I consider euerie number, comparing it to the number that is against it, and because I finde them to be euery one like his match, I knowe that I haue well done.

Scholer. This crosse I perceiue doth serue for those three denominations, poundes, shillinges, pence. But what if I had l, s, d, ob. and q.

Master. These lines as I haue saide, doe serue for three denominations, such as they be: as here they doe serue for pounds, shillings, and pence: but if ye haue no pounds in your sum, then may they serue for shillings, pence, and halfe penies: yea for d, ob, & q.: or in weight for C. q. and l, or in measures for Elles, quarters, and nayles, if you haue no greater denomination, so that you remember that the vpright line serueth for the greatest denomination, and the highest thwart line for the next, and the lowest for the least.

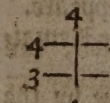
And so if you haue foure Denominations

you

you must make your crosse with so many lines. And if that your summe be of more denominations, make so many lines in your crosse. And thus will I make an ende of Addition, saving that here for the better vnderstanding of this Rule, I haue set you downe certaine examples both of money, weight, and measures, with their workes and proofes.

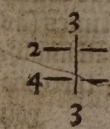
Examples of Addition.

l	s	d	l	s	d
23	10	4	130	17	10
45	6	8	28	6	8
37	2	9	13	13	4
15	13	6	120	0	0
121	13	3	292	17	10

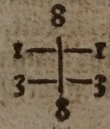


The proofes.

C	q	l	yards	q	nailes
24	1	3	17	3	3
12	2	2	35	2	1
7	3	4	26	1	3
13	0	13	54	2	0
57	2	22	134	1	3



The proofes.



Subtraction.

Scholer.

When haue I learned the two first kindes of Arithmetike: nowe as I remember, doeth followe Subtraction, whose name me thinketh doeth sound contrarie to Addition.

Master. So is it in deede: for as Addition encreaseth one grosse summe by bringing many into one, so contrary ways, Subtraction diminisheth a grosse summe by withdrawing of other from it, so that Subtraction or Rebating is nothing else, but an Art to withdrawe and abate one summe from an other, that the Remainer may appeare.

Scholer. What doe you call the Remainer?

Master. That you may perceiue by the name.

Scholer. So me thinketh: but yet it is good to aske ỹ truth of all such things, least in trusting to mine owne coniecture, I be deceiued.

Master. So is it the surest way. And as I see cause, I will still declare things vnto you so plainely, that you shall not neede to doubt. Howbeit, if I doe ouerpasse it sometimes (as the maner of men is to forget the small knowledge of them to whom they speake) then doe

you

you put me in remembrance your selfe, and that way is surest.

And as for this worde that you last asked me, take you this description: The Remainer is a summe left after due Subtraction made, which declareth the excesse or differēce of the other two numbers: as if I woulde abate or subtract 14 out of 18, there should remaine 4, which is called the Remainer, and is the difference betweene those two numbers 14 and 18. Remainer

Scholer. I perceiue then what Subtraction is: Nowe resteth to knowe the order to worke it.

Master. That shal you doe by this means. First you must consider, that if you should goe about to rebate, you must haue two sundrie summes proposed, the first which is your grosse summe or summe total: (and it must be set highest) and then the rebatement or sum to be withdrawen, which must be set vnder the first (whether it be in one parcel or in many) and that in such sort, that the first figures be one iust ouer an other, and so the seconde and third, and all other following, as you did in Addition: then shall you drawe vnder them a line, and so are your summes duely set to begin your working.

Then begin you at the right hande (as you did in Addition) and withdrawe the neather number out of the higher, and if there remaine

main any thing, write that right vnder them beneath the line: and if there remaine nothing (by reason that the two figures were equall) then write vnder them a cipher of nought. And so doe you with all the other figures, euermore abating the lower out of the higher, and write vnder them the Remainer still, till you come to the ende. And so will there appeare vnder the line what remaineth of your grosse summe, after you haue deducted the other summe from it, as in this example.

I receiued of your father 48 s, of which I haue laide out for you 36 s: nowe woulde I knowe what doeth remaine, and therfore I set my numbers thus in order: First I write the greatest summe, and vnder him the lesser, so that the figures at the right side be euen one vnder an other, and so the other, thus.

	s
	48
	36

Then doe I rebate 6 out of 8, and there resteth two, which I write vnder them right beneath the line, thus,

	48
	36
	2

Then I goe to the second figures, and doe rebate 3 out of 4, where there remaineth 1, which I write vnder them right, and then the whole summe and operation appeareth thus.

	48
	36
	12

Wherby it appeareth, that if I withdraw

36

36 out of 48, there remaineth 12.

Scholer. Nowe wil I proue in a greater summe: And I will subtract 2367924 out of 3468946. Those sums I set in order thus.

	3468946
	2367924

Then doe I begin at the right side, and deduct 4 out of 6, and there resteth 2, which I write vnder them. Then goe I to the second figures, and withdrawe 2 out of 4, and there remaine two, which I set vnder them also: then I take 9 out of 9, and there resteth 0, which I write vnder them: for you say, that if the figures be equall, so that nothing remaine, I must write this cipher 0 vnder them.

Master. It was well remembred, now goe forth.

Scholer. Then I come to the fourth place and drawe 7 out of 8, and there remaineth 1, which I write vnder them also. Then in the fifth place I take 6 from 6, and there resteth nought, for it I write vnder them a cipher, 0: Then in the 6 place 3 rebated from 4, there remaineth 1, which I write vnder them: and likewise in the vy. and last place, 2 taken from 3, there is left 1, which I write vnder them: so haue I done my whole working, and my summes appeare thus. Whereby

	3468946
	2367924
	1101022

I see, that if I rebate 2367924, out of 3468946, there remaineth 1101022.

Master. This is well done. And that you may be sure to perceiue fully the Art of Subtraction, let mée sée howe can you subtract 52984732 out of 8250003456.

Scholer. First I set downe the greatest summe, and after that I write vnder it the lesser number, beginning at the right side: and then my figures will stand thus.

8250003456
52984732

Then take I 2 from 6, and the rest is 4, which I write vnder them: then doe I withdrawe 3 from 5, and there remaine 2, which I write vnder them. Then take I 7 out of 4, but that I cannot, what shall I nowe doe?

Note. Master. Marke well what I shall tell you nowe, howe you shall doe in this case and in all other like. If any figure of the neather summe be greater than the figure of the summe that is ouer him, so that it cannot be taken out of the figure ouer him, then must you put 10 to the ouer figure, and then consider howe much it is, and out of that whole summe withdrawe the neather figure, and write the rest vnder them. Can you remember this?

Scholer. Yes, that I trust I shall. Nowe then in mine example where I shoulde haue taken 7 out of 4, and could not, I put 10 to

that

that 4, which maketh 14, from it I take away 7, and there resteth 7 also, which I write vnder them.

Master. So haue you done well, but nowe must you marke another thing also: that whensoeuer you doe so put 10 to any figure of the ouer number, you must adde one still to the figure or place that followeth next in the nether line, as in this example there followeth 4, to which you must put 1, and make him 5, and then go on as I haue taught you. Note.

Scholer. Then shall I say: 4 & 1 (which I must put to him for the 10 that I added to 4 before) make 5, which I should take out of 3, but that cannot be, therefore must I put to it also 10, and then it will be 13, from which I take 5, and there resteth 8 to be written vnder them: and because of that 10 added to the 3, I must adde 1 to 8 that followeth in the nether line, and that maketh 9, which I shold take out of 0, and cannot, therefore I put thereto 10, and that maketh 10: from 10 I take 9, and there remaineth 1, which I write vnder them.

Then doe I adde one likewise to the next figure beneath, which is 9, and that maketh 10, that 10 should I take out of the figure aboue, but I cannot, for it is 0, therefore I put 10 to it, and so take I 10 out of 10, and there resteth 0 to be written vnder them.

Then come I to the next figure which is 2, and to him do I adde 1, which maketh 3, that 3 I can not take out of nought, therefore of that nought I make 10, and thence doe I take 3, so remaineth there 7 to be written vnder them. Likewise doe I put 1 to 5 that followeth, and then is it 6, that would I take out of 5, and cannot, therefore I adde 10 to that 5, and make it 15, from which I rebate 6, and there remaineth 9, which I write vnder them. Nowe haue I spent all the nether figures, and what shall I doe more?

Master. You should haue added 1 to the neext figure following (if there had bene any) because you added 10 to the last figure before of the ouer line: but seeing there is no figure following, you must adde that 1 to the place following, and then deduct that 1 from the number aboue.

Scholer. Then shall I say because I borowed 10 to the ouer 5, I must put 1 in the next place beneath, that is vnder 2: then must I subtract that 1 from 2, and there resteth 1, to be written vnder that, in the ninth place. Nowe I haue no more to subtract, for there is not any figure remaining beneath, neyther yet any vnite to be added, because I borowed not 10 to the figure last before, and yet is there 8 remaining in the ouer line, which (I thinke by reason) shoulde be set at the ende of the figures in the lowest rowe

which

which is vnder the line, for because there was nothing taken from it.

Master. That is well considered, and reason teacheth so in déede.

Scholer. But sir, I beseech you, shall I alwaies when any number so remaineth alone (as this 8 did) write him vnder the line straight against his owne place?

Master. Yea, what else? Whether they be one or many: and this well remembred, you haue sufficiently learned Subtraction. Howbeit, because of certaine thinges that might deceiue you, if you did not take good héede to your working, I will propose to you another example of many numbers to be subtracted, as thus.

I receiued of a friende of mine to kéepe 2869 crowns, of which at one time I deliuered him again 500, at an other time 368, and at another time 440, and an other time 80, and an other time 64: now would I know how many doth rest behinde. Therfore first I set downe my grosse summe, and a line vnder it: and vnderneath it I set all the parcels, thus: and vnder them a double line.

Then first I begin at the first place, and gather togither the summe

2869	*Crowns receiued.*
500	*Deliuered.*
368	
440	
80	
64	

of all those lines (saue the ouermost) in their first figures, and so I doe with all the figures of the second place, and so forth, as I did in Addition, saue that I leaue out the highest
Note. rowe of numbers (as the line warneth me) and that summe so gathered betwéen the double line, is the summe deliuered in all, which summe I doe afterwards subtract out of the highest rowe of numbers, and the remainer doe I set vnder the nethermost line: as for example.

I set the summes as before: then doe I gather the first figures of all the parcels deliuered together, where I finde but 4 and 8, that make 12 (for thée Cyphers increaseth no summe in Addition, as you learned before) of the 12, therefore doe I write the digit 2 betwéen the double line, and kéepe the article 1 in my minde, till I come to the second place, where I finde 6,8,4,6, that make 24, to them I put the article in my minde, and it is 25, of which I write 5 vnder the seconde place, and kéepe the digit 2 in my minde for the thirde place, where I finde 4, 3, 5, that make 12, to the which I adde the 2 in my minde, and it maketh 14, thereof I write the 4 vnder the third

2869	*Receiued.*
500	*Deliuered.*
368	
440	
80	
64	
1452	*Deliuered in all.*
1417	*Rest behinde.*

third place: and because there remaineth no more figures to be added, I write the digit 1 in the fourth place, as you see in the example, and so it appeareth I haue deliuered in all 1452 crownes.

Then come I to the subtracting of this summe betweene the lines, for by Addition it is equall to the fiue parcels ouer it. Therefore I proceed to subtract it from the ouermost summe, saying: 2 from 9, remaine 7, to bee written vnder them beneath the lowest lyne. Then in the second place I take 5 from 6, and there resteth 1, to be written vnder them. Then in the third place, 4 from 8, resteth 4. Last of all in the fourth place, 1 from 2, remaineth 1. And thus I see that after those 5 summes are subtracted from 2869, the Remainer is 1417.

Master. This is a verie sure & easie way for a learner: therefore I counsell you to practise it well.

Scholer. This I perceiue: but is there no shorter way and more spedier?

Master. Yes, when you are a while exercised in it: for you may as fast as you can gather the numbers together, withdrawe them out of the highest summe. But if in quantitie those numbers added together, excede the highest summe or vpper number (then shal you as before hath bin taught you) imagine to borowe 10, 20, or 30, or moe as nede shall require,

An abridgement of the former maner of Subtraction.

quire, and put them to the vpper number to helpe to further the abatement, reseruing or restoring the Articles that you borowed to the next place againe, and so still goe forward till you haue ended your worke, as for example: In the last summe proposed, I gather first in the first place 4 and 8, that maketh 12: which 12 I should deduct or take out of 9 in the vpper number aboue the line. But I cannot, therefore I adde to 9 an Article of 10, and that maketh the vpper number 19: from whence I take 12, then there resteth 7. Then for the Article 10, I adde 1 to the next place of money deliuered: saying, 1 that I bring, and 6 make 7, and 8 make 15, and 4 make 19, and 6 make 25, which 25 I should take out of 6 in the vpper number, but I cannot, Therefore I adde 2 tennes or 20 vnto 6 in the vpper number, and that maketh 26, then 25 out of 26 resteth 1, then the 2 tens which I borowed, or haue in minde, I adde to the nert rowe, or summe deliuered, saying: 2 that I bring, and 4 make 6, and 3 make 9, and 5 make 14, then 14 out of 8 I cannot take, but 14 out of 18, resteth 4. Nowe because there are no more places to be added, the 1 that I borowed or haue in my minde, I rebate from 2 in the vpper line, and there remaineth 1 which I set downe in the remainer-line, and so my summe appeareth as before to be 1252 crowns. Lo thus haue you now a shorter way.

Scho-

Scholer. I like both waies well, and I perceiue both well, yet as in the one the worke king seemeth somewhat long, so in the other it leaneth verie much (me seemeth) to remembrance, and therefore may cause errour quickely, except a man haue a quicke and an exercised remembrance. But yet for the sharping of my wit, by your patience, if you will giue me leaue, I will trie what I can doe in such a like summe, to worke it the shortest way: whereupon I woulde subtract out of 40308964, these three parcels

	40308964
20003428	20003428
10002342	10002342
10101461	10101461

Therefore I set them first in order due: & then I gather the parcels of the first place, which are 8, 2, 1, that is 11, which I should take or deduct out of 4, which is ouer him, but I can not, therefore I adde an article or one ten vnto 4, which maketh 14, then 11 out of 14, there resteth 3, to be written vnder the first place betweene the two lines.

Then come I to the second place, saying: 1 that I borowed or haue in minde, and 6 make 7, and 4 make 11, and 2 make 13, which I cannot take from 6, therefore I adde 10 to 6, which maketh 16, then 13 from 16, resteth 3, which I write vnder the second place betweene the

the two lines.

Then come I to the third place, saying, one that I borowed or haue in minde, and 4 make 5, and 3 is 8, and 4 make 12, which I take out of 9 that is ouer them, but I cannot, therefore I adde 10 to 9, which make 19, then 12 out of 19, resteth 7.

Then come I to the fourth place, saying, one in minde and 1 is 2, and 2 make 4, and 3 make 7, which because it is a digit, I take from 8, and there remaineth 1.

After that I come to the fifth place where are onely three ciphers, which make nothing, then shold I take that, that is to say nothing, from the figure ouer them, which is also a cipher, therefore I must say thus: if I take nought from nought, there remaineth nought: so must I write a cipher vnder them. Then in the first place I finde but 1, which I take out of 3 ouer him, and the Remainer is 2, that must be written betweene the two lines in the first place. So I goe to the seuenth, where I finde onely ciphers, and in the grosse summe ouer them a cipher also, therfore must I write their remainer (which is nothing) with a cipher also. Then in the eight and last place, I gather 1, 1, 2, that make 4, which if I take out of that 4 that is ouer them, there will nothing remaine. And that must be noted with a cipher betweene the two lines, as I haue often saide, and so haue I ended my worke, and

the

the figures stand thus.

But syr, I remember you taught me that ciphers should not come in the last place, for because they serue onely to increase the value of other figures which follow thē, and serue not those figures that goe before them: and now in my example I haue set two ciphers in the two last places.

	40308964
	20003428
	10002342
	10101461
	00201733

Master. I commend you for your remembrance. And trueth it is, you should not haue set them here, but onely because that I would make you plainly to perceiue the Art of Subtraction. Therefore seeing that you doe nowe perceiue it, whensoeuer you shold write down a cipher, loke whether any other figures be yet behinde. And if not, then let goe the cipher also, for it needeth not to write him in any latter places, where noother figure doeth folow, except it be (as I did now suffer you) to teach the vse of Subtraction the playner.

Therefore your figures must stād thus when the worke is ended.

40308964	*Charge.*
20003428	*Discharge.*
10002342	
10101461	
201733	

Scholer. Syr I doe thinke with that ỹ you taught me before, and by these two summes that you taught

taught me last also, that nowe I could subtract any summe.

Master. So may you if you haue marked what I haue taught you. But because this thing (as all other) must be learned surely by often practise, I will propound here two examples to you, wherein if you often exercise your selfe, you shalbe ripe and perfect to subtract any other summe lightly, for in them is contained all the obseruances of whole number. And because you shall perceiue somewhat both how to doe it, and also whether it be wel done when you haue proued to doe it, therefore haue I written vnder them both the Remainers.

30606 *Lent.*		30606 *Debt.*	
10354	*Repaide.*	10354	*Paide.*
10249		10249	
163		163	
20766 *Paide in all.*		9840 *Rest.*	
9840 *Rest to pay.*			

Scholer. Sir, I thanke you. But I thinke I might the better doe it, if you did shewe me the working of it.

Master. Yea, but you must proue your selfe to doe some things without my aide, or else you shall not be able to doe any more than you were taught: And that were rather to learne

learne by rote (as they call it) than by reason. And againe there is nothing in this example or any other of whole number, but I haue taught you the rules of them alreadie, sauing I haue not yet shewed you how you shal proue this worke.

Scholer. Sir I pray you then shewe it me also.

Master. For the performance whereof if you marke well what I saide in Addition, you may easily perceiue what is to be done for the proofe of Subtraction, what is onely performed by the aide of Addition, thus. Drawe vnder the lowest number (which is your Remainer) a line, & then adde the sum paide in al (which before was made by the particulars) and the remainer together. And if these twain being added together in one summe, do make the contents of the vpper number aboue the line, then is the Subtraction well wrought, or else not.

For the triall whereof I will propone one of our examples which are done, which was of 2869 crownes receiued to keepe of a frind.

Crownes	2869	*Receiued.*
Of which at sundrie times I deliuered these particular sums	500	
	368	
	440	
	80	
	64	

The proofe of Subtraction.

which

which particulars in one Addition make 1452 *Deliuerd in al.*

So resteth to my frind 1417 *Rest to pay.*

2869 *Proofe.*

Nowe put or adde this remainer, and the summe deliuered together, saying: 7 and 2 make 9, and 1 and 5 make 6, and 4 and 4 is 8, and 1 and 1 is 2, which in one summe amounteth to 2869, and is equall with the summe 2869 aboue the line, whereby I knowe the subtraction is well wrought, and this is the order and proofe hereof: and of all such like workes of subtraction. ‡

Scholer. Then I trust by practise to attaine the vse of it. And this is al that I shal learne of Subtraction.

Master. Yea, sauing that (as you haue seene in Addition) there are numbers of diuers denominations, in which the working is not much vnlike, yet without some instructions be giuen of it, it might seeme to a learner more difficult, than in deede it is. Therefore I wil briefly shewe you the vse of it onely, by one example or two.

A certaine man owed to me 14 ℔, 12 ʃ, 8 ɗ of which he paide me at one time 4 ℔, 6 ʃ, 8 ɗ, at an other time 3 ℔, and at an other 2 ℔, 3 ʃ, 4 ɗ, and last of all, 6 ʃ, 8 ɗ.

Now would I knowe what remaineth vnpaide yet, therefore I set my summes thus, e-

uery

uerie one in their due place: As pounds vnder pounds, shillings vnder shillinges, pence vnder pence.

li	s	d
14	12	8
4	6	8
3	0	0
2	3	4
	6	8

Scholer. Sir, I pray you why doe you write 2 li, for the common speach vseth rather to say 40 s.

Master. We must here vse the denomination that is greatest in any summe, so that wée may not write according as we vse to speake, saying: 16 d, 18 d, or likewayes, 7 grotes, 8 grotes, 24 s, 40 s, 48 s, and such other, but we must write euery denomination that is in any summe by it selfe, namely shillings and pounds. So must we write for these last summes nowe named, 1 s, 4 d: 1 s, 6 d: 2 s, 4 d: 2 s, 8 d: 1 li, 4 s: 2 li, 8 s: and so forth of other like.

Note how the pen differeth from common order of counters.

Scholer. So that we may not write in Arithmetike pence, when the summe amounteth to shillings, nor shillings when the summe maketh pounds. Nowe if it please you, end your example.

Master. When my summes are so set as I shewed, then according to the rules of Addition I gather all the particular summes which he paide me into one totall summe: directly to be set vnder them betwéene the two lines, not medling with the 14 li, 12 s, 8 d, as the lyne warneth me, therefore must I first begin with the

the smallest denomination, saying, 8, 4, 8, is 20 d, which maketh one shilling and 8 pence: the 8 d I set down vnder the place of pence, and the one shilling I keepe in minde to carie to the next denomination of shillings. Then come I to the shillings, and say, one that I bring or haue in minde, and 6 is 7, and 3 is 10, and 6 make 16, which because it conteineth not one pound, I set directly vnder the place of shillings. Then come I to the pounds whose parcels are 2, 3, 4, that is in all 9, that 9 doe I set downe directly vnder the pounds: And so the totall or whole Addition of all the particulars paide, amounteth to 9 l, 16 s, 8 d.

	l	s	d	
	14	12	8	
	4	6	8	
	3			
	2	3	4	
		6	8	
	9	16	8	
	4	16	0	*Rest.*

Nowe for the worke of Subtraction I must rebate that totall summe of Addition out of the higher number, that is to say, from the 14 l, 12 s, 8 d.

Therefore to performe the worke, I say 8 d out of 8 d, remaineth or resteth nothing, therefore in the place of rest or remaine, right vnder that denomination I set downe a 0. Then comming to the shillings, where I finde 16, which should be taken out of 12, but I cannot, therefore I imagine to borowe one of the next de-

denomination, that is of the 9 ℔, and put that one pound so borowed vnto 12 s̄, that maketh 32 s̄.

Nowe 16 s̄ out of 32 s̄ resteth 16 s̄, which 16 s̄ I set downe directly vnder the place of rest.

Lastly comming to the pounds, saying, one pound in minde that I borowed, and 9 make 10, then 10 out of 14, there resteth 4.

So doth my whole rest or remaine appeare to be 4 ℔, 16 s̄, 0 d̄

This I account the easiest way for a yong beginner to practise, though it be something long.

Scholer. Is there then any shorter way for this worke also?

Master. Yes, as in this last example I wil also shew you, for you may adde together the particular summes as they are set in order: beginning with the pence, saying, 8, 4, 8,

	℔	s̄	d̄
	14	12	8
make 20 d̄, which 20 d̄ you	4	6	8
should take out of the 8 d̄ a-	3	0	0
boue the line, but you cannot:	2	3	4
therefore shall you borow one	0	6	8
of the next denomination,	4	16	0

ỹ is to say, one of the shillings, and put it to the 8 d̄ that maketh 20 d̄, nowe 20 d̄ out of 20 d̄, rest 0, which cipher I set downe directly vnder them.

Then one shilling that I borowed or had

in minde, and 6 make 7, and 3 make 10, and 6 make 16, then 16 out of 12 I cannot take, therefore of the next denomination I doe borowe one pound, and put it to 12 s, which maketh 32 s, then 16 s out of 32 s, resteth 16 s.

Lastly I come to the pounds, saying: one pound in minde or that I borowed, and 2 make 3, and 3 is 6, and 4 is 10, then 10 out of 14, there resteth 4.

So doeth my remaine or rest appeare as before to be 4 l, 16 s, 0 d.

Scholer. This doe I perceiue very well, and if there be none other thing to be learned in Subtraction, then may I come to Multiplication, for that you reckened to be the next order.

Master. We haue done in déde with the Art of Subtraction as touching the working. But yet before we goe to Multiplication, I will instruct you howe to examine your worke whether it be well done or not. For the performance whereof, if you marke what I said right nowe in the last maner of proufe, you may eastlie perceiue what is to be done for this proufe, which is onely made (as before was taught you) by the aide of Addition, thus.

The proof of Subtraction.

Drawe vnder the lowest number (which is your Remainer) a line, and then adde the summe paide in all, and the Remainer together. And if these twaine added together in one summe make the contents of the vpper num-

number aboue the line, which in this example is 14 ℔, 12 s̄, 8 d̄. Then is the Subtraction well wrought, or else not.

As for example, in our first summe which stode thus.

℔	s̄	d̄	
14	12	8	
4	6	8	
3	0	6	
2	3	4	
	6	8	
9	16	8	*Paide in all.*
4	16	0	*Rest.*
14	12	8	

Where in the title of pence I finde 8 and 0: the 8 I set downe directly vnder him in ỹ place of pence. Then in the place of shillinges I finde 16 and 16, which make 32 shillings, wherein is conteyned one pound and 12 s̄, the 12 s̄ I set down directly vnder them in their due place of shillings, and one pound I keepe.

Then comming to the pounds, saying: one that I keepe, and 4 is 5, and 9 is 14, which 14 in due order I set downe directly vnder them, as this figure sheweth. Which is also 14 ℔, 12 s̄, 8 s̄, agreeing with the vpper number aboue. So I finde the worke is good, and the Subtraction well wrought.

Now for the proofe of the latter Subtraction which we reckoned for the shorter worke. Drawe vnder the remainer also a line, then adde that remainer and al the other numbers that ye did subtract before together, & write that that amounteth vnder the lowest line.

And if the summe that commeth thereof be equall to the highest number aboue, then is the Subtraction well wrought, or else not.

As for example also in the last summes which stood thus.

	£	s	d
	14	12	8

First in the title of pence I adde 8, 4, 8, that maketh 20 d, which containeth one shilling and 8 pence.

	4	6	8
	3	0	0
	2	3	4
	0	6	8
	4	16	0

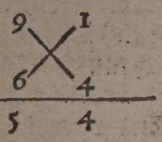
The 8, I set down vnder the lowest line in the rowe or title of pence, and the one shilling I keepe to carie to the next denomination or place of shillings.

Then returning to the shillings, saying: one in minde, or that I keepe, and 16 make 17, and 6 make 23, and 3 make 26, and 6 make 32 shillings, which amounteth to one pound, 12 s, the 12 s I set downe vnder the title of shillings, and 1 pound I keepe or haue in minde to carie to the next denomination or place of pounds. Then come I to the pounds, saying, one that I bring, and 4 make 5, and 2 make 7, and 3 is 10, and 4 make 14, then doe I write 14 vnder the pounds, and so haue I ended the Addition, and I see that the lowest line is like vnto the vppermost line in number, wherefore I knowe that I haue wel done.

	14	12	8

And thus nowe haue I taught you the art of

ber the digits thus. And so appeareth the multiplication of 6 times 9, to be 54. Thus I see the feate of this maner of multiplication of digits.



Master. Nowe might you goe straight to the multiplication of greater numbers, saue that both for your ease and suretie in working, I will drawe you here a Table, whereby shall appeare the multiplication of all digits, and this is it that followeth.

1	1	2	3	4	5	6	7	8	9
	2	4	6	8	10	12	14	16	18
		3	9	12	15	18	21	24	27
			4	16	20	24	28	32	36
				5	25	30	35	40	45
					6	36	42	48	54
						7	49	56	63
							8	64	72
								9	81

In which Table when you would knowe the product in any multiplication of Digits, seeke your first or last digit in the greater figures, and from it goe right forth toward the right hand, till you come vnder the number of your second digit, which is in the highest row: and then the number that is in the meeting of the rowes of litle squares (which

come directly from both your propounded digits) is the multiplication that amounteth of them. As if I would knowe by this Table the multiplication of 7 times 9, seeke first 7 in the greater figures, and then go right forth toward the right hand, till you come vnder 9 of the highest rowe, in which place where you so come vnder the other digit (as here for example you come vnder 9) is alwaies contained the ofcome or product, which you seeke: and that place we terme to be in the common angle, in respect of the two numbers so taken on ỹ outsides, as here in that common angle, where the rowes of litle squares) directly proceeding frō 7 ⁊ 9) do meet, you haue 63, which 63 is the sum of the multiplication of 9 by 7.

Scholer. This is verie good and ready. And so may I finde the multiplicatiō of any digits. But now how shal I doe in greater summes?

Master. When you would multiplie any summe by an other, you shall marke that it is the meetest order to set the greatest number highest, which is the place of the number that must be multiplied: and likewise the lesser number vnder it, for that is the place of the Multiplier, or Multiplicatour, that is to say, the number by which multiplication is made: and is in English alwaies put before this word, Times: in such speaking when I say, 20 times 70. And the number that followeth this word times, is that which must be multiplied.

of Subtraction, and the meanes to proue whether it be well wrought or not.

Many other workes and proofes might be shewed of Subtraction, but one of these whether of them you please, are most aptest & best allowed of any other worke or proofe, whether it be of l,s,d, or any other grosse summe whatsoeuer.

Scholer. Sir, I thanke you most heartily, for nowe I vnderstand well both the feate of Subtraction and his proofe.

Master. Therefore here will I make an end of Subtraction, and will instruct you in Multiplication.

Multiplication.

Multiplication is such an operation, that two summes produceth the third: which thirde sum so many times shall containe the first, as there are vnites in the second. And it serueth in the stead of many Additions. As for example. When I would knowe how many are 30 times 48: if I should adde 48 thirtie times, it would be a long worke. Therefore was this worke of Multiplication deuised, which shall doe that at once, that Addition

dition should doe at many times.

Scholer. I perceiue the commoditie of it partly, but I shall not see the full profite of it, till I knowe the whole vse of it. Therefore Sir I beseech you, teach me the working of it.

Multiplication of Digits.

Master. So I iudge it best, but because that great summes can not be multiplied, but by the multiplication of digits, therefore I thinke it best to shewe you first the way of multiplying them: As when I say, 8 times 8, or 8 times 9, &c. And as for the small digits vnder 5, it were but follie to teach any Rule, seeing they are so easie, that euerie childe can doe it. But for the multiplication of the greater digits, thus shall you doe.

First set your digits one ouer the other right, then from the vppermost downeward, and from the neathermost vpwarde, drawe straight lines, so that they make a crosse, commonly called Saint Andrewes crosse, as you see here. Then loke howe many each of them lacketh of 10, and write that against each of them at the end of the lines,

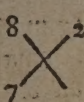
The difference.

and that is called the differences, as if I would knowe howe many are 7 times 8, I must write those digits thus.

Digit difference.

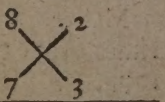
8 ×
7

Then doe I loke howe much 8 doth differ from 10, and I finde it to be 2, that 2



doe I write at the right hand of 8, at the ende of the line, thus.

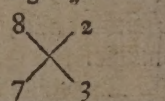
After that, I take the difference of 7 likewise from 10, that is 3, and I write that at the right side of 7, as you see in this example.



Then doe I drawe a lyne vnder them, as in Addition, thus.

Last of all I multiplie the two differences, saying: 2 times 3 make 6, that must I euer set vnder the differences, beneath the line: then must I take one of the differēces (which I will, for all is like) from the other digit (not from his owne) as the lines of the crosse warne me, and that that is left, must I write vnder the digits. As in this example. If I take 2 from 7, or 3 from 8, there remaineth 5: that 5 must I write vnder the digits: & then there appeareth the multiplication of 7 times 8, to be 56. And so likewise of any other digits, if they be aboue 5, for if they be vnder 5, then will their differēces be greater than themselues, so that they cannot be taken out of them. And againe, such litle summes euerie childe can multiplie, as to say: 2 times 3, or 4 times 5,

Digit difference.

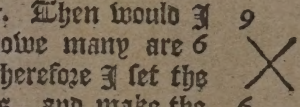


and such like.

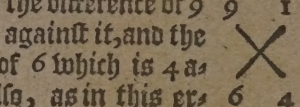
Scholer. Truth it is. And seeing me seemeth that I understand the multiplying of the greater digits, I will prowe by an example howe I can doe it. I would knowe howe many are 9 times 6.

Master. It is all one in value to say 9 times 6, or 6 times 9: but yet the order is best to put the lesse summe first, saying: 6 times 9, and so of all other summes.

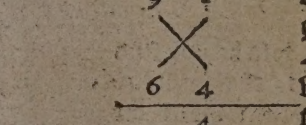
Scholer. Then would I knowe, howe many are 6 times 9: therefore I set the digits thus, and make the crosse thus.



Then doe I set their differences at the right side: the difference of 9 which is 1 against it, and the difference of 6 which is 4 against it also, as in this example.



And vnder them I drawe a line. Then doe I multiplie the digits together, saying: one time 4 maketh 4, that 4 doe I write vnder the differences thus,



Then take I one of the differences from the other digit, as one from 6, or else 4 from 9, and eache waies there resteth 5, which I doe write vn-

der

tiplied.

Therefore when I would multiplie one number by another, I must write the greatest highest, and the lesser vnder it, as in Addition. And vnder them must I drawe a line. As for example: If I would multiplie 264 by 29, I must set them thus.

264	
29	

* Of which numbers thus set downe to be multiplied, may be formed a question as thus: There are 29 men, and each man hath 264 lambes. The question is howe many lambes they haue in all. ‡

To the performance whereof I must multiplie euerie figure of the higher rowe by euerie figure of the nether rowe: and that that amounteth, I must set vnder the line, as thus. First I do multiplie 4 by 9, saying: 9 times (or 4 times 9, which is all one) and that maketh 36, as the Table before of digits doth declare, of that 36 I must write the 6 that is the digit, vnder the 9, & the Article 3 I kéepe in minde to carie to the next place.

264	
29	
6	

Then come I to the second figure of the higher rowe, which is 6, and say: 9 times 6 make 54, & with the 3 in my minde make 57, the 7 I set downe vnder the 2, and the 5 I kéepe in minde.

264	
29	
76	

After that I come to the next figure which is 2, and doe multiplie it by 9, and that maketh 18, and with 5 that I haue in minde maketh 23, wherefore because it is the last worke of that multiplier, I set it downe in order as you see.

264
29
2376

And so haue I ended the first figure of the multiplier. Wherefore I giue it nowe a fine dash with my pen.

Then begin I with the next figure, and multiplie it into all the higher figures, as thus.

264
29
2376
8

First 2 times 4 make 8, that 8 doe I write vnder the second place: for euermore the digit or first figure of Multiplication that amounteth of the first figure of the higher number, must be set vnder the multiplier of it, and the other in their order, toward the left hand.

Scholer. I vnderstand you thus: that the digit of the summe amounting of the multiplication of the first figure of the higher rowe, by the first figure of the lower rowe or multiplier, must be set vnder the first place: and that that amounteth of the same first figure by the second multiplier, must be set vnder the second place, and so of the other, if there be more multipliers.

Master. So meane I indeede: and if there amount but a digit, then must it be set vnder

the

the multiplier.

And nowe to goe forth: I multiplie by the same 2, the second figure of the higher rowe, which is 6, saying: 2 times 6, make 12: whereof I write the digit 2 vnder the third place, and the article 1 I kéepe in minde.

Then doe I multiplie the last figure of the higher summe by that same 2, saying: 2 times 2 is 4, & with the one that I haue in mind maketh 5, which 5 I write vnder the fourth place. And so haue I ended the whole multiplication: wherefore I also giue the 2 a dash with my pen, thus: and so I doe euer as soone as I haue dispatched any Digit by which I multiplie: And the summes stande thus.

Then must I drawe a line vnder all those sums that amount of the multiplication, and must adde all them into one summe, as in this Example you may sée.

264	
2ϑ	
2376	
28	
264	
2ϑ	
2376	
528	
264	
2ϑ	
2376	
528	
7656	

Where in the first place I finde but 6, and therefore write I it vnder the line. Then in the second place 8 and 7 make 15, whereof I write 5, and kéepe one in my minde, and so forth, as you learned in Addition. And so appeareth the whole summe to be 7656, which amoun-

amounteth of the multiplication of 264 by 29, and is the iuſt number of Lambes the 29 men had.

Scholer. If there be no more to be obſerued in it, then can I doe it, I ſuppoſe, as by this example I ſhall proue.

* There is a peece of ground which conteineth 1365 yards in length: And 236 yards in breadth, I would knowe how many yards ſquare there is in al this peece of ground, which numbers I ſet downe with the greater aboue, & the leſſer vnder, as you ſee. ‡

Then doe I multiplie 5 by 6, ſaying: 6 times 5 make 30: of which I write the cipher in the firſt place, and the article 3 I keepe in minde to carie to the nexte place.

```
1365
 236
   0
```

Then doe I by the ſame 6 multiplie the ſecond figure of the higher ſumme, which is 6, ſaying: 6 times 6 make 36, and 3 in my mind make 39, of which I write the 9 vnder the ſecond place, and the Article 3 I keepe in minde.

```
1365
 236
  90
```

Then do I multiply the third figure which is 3, by the ſame 6, and that maketh 18: and with 3 in my mind make 21. Then 1 I ſet downe, & 2 I keepe in minde.

```
1365
 236
 190
```

Then

Then come I to the last figure of the higher summe, and multiplie it by 6, saying: 6 times 1 make 6, and 2 in my minde make 8, that 8 doe I write vnder the fourth place. And so haue I ended the first multiplier, and dash him slightly with my pen.

Then begin I with the second multiplier, and say first 3 times 5, that maketh 15, of which I set the 5 vnder the second place, because ỳ the multiplier is there, & the article 1 I kẽpe in minde.

Then come I to the second figure, that is 6, and multiplie it by 3, which maketh 18, & with one in minde maketh 19, the 9 I set downe vnder the third place, and 1 I kẽpe in minde.

Then come I to the third figure, which is 3, and multiplie it by 3, saying: 3 times 3 make 9, and with one in my minde make 10, the cipher I set vnder the fourth place, and the Article 1 I kẽpe in minde.

And then comming to the last figure 1, I multiplie it by 3, and it maketh 3, and with the one in mind it maketh 4: which 4 I set in the fift place, and then haue I

1365
236
8190

1365
236
8190
5

1365
236
8190
95

1365
236
8190
095

1365
236
8190
4095

ended

ended two of the multipliers, and the summes stand as you may see in the latter ende of the page going before. And then I giue 3 his dash.

Then come I to the thirde multiplier, and multiplie it into euerie figure of the higher sum, and first I say: 2 times 5 make 10, of which I set the cipher vnder the multiplyer in the thirde place, and the Article 1 I kéepe in minde.

1365
236
8190
4095
0

And so multiplying the second figure 6, by that same 2, there amounteth 12, and with 1 in my minde make 13, whereof I write the digit 3 vnder the fourth place, & the article 1 I kéepe in minde.

Then do I multiplie the said 2 by the third figure of the higher summe, which is 3, and that maketh 6, and with the one in minde make 7, which 7 I set downe vnder the fifth place, as appeareth by the example.

1365
236
8190
30

1365
236
8190
4095
730

1365
236
8190
4095
2730

Then come I to the last place, and multiplie that 1 by 2, and there amounteth 2, which I set in the first place, and then doeth the summe stand thus.

And so haue I ended the whole

whole multiplication.

But now (as you taught me) to knowe what this whole sum is, I must adde all those parcels togither, and then vnder the line will appeare as you may see, the grosse or totall summe, that is, 322140. Wherby I know there is so many yards square in that peece of ground.‡

	1365
	236
	8190
	4095
	2730
	322140

Master. This is well done, and like a workeman.

Scholer. Then me thinketh I could call it well done, when I knowe whether I had well done or no.

Master. It is to be proued by 9, as Addition was, but the surest proofe is by Diuision, and therefore I wil reserue that proofe by Diuision till you haue learned the Art of Diuision. And anon I will shewe you howe it is commonly proued.

*But first for your further instruction in this exercise of Multiplication, I will with one example more trie your cunning, and so make an ende. And the question is this. I would knowe howe many daies it is since the Natiuitie of our Lorde and Sauiour Iesus Christ vnto this present yere 1586. Which to performe, you must multiplie by the daies in one whole yere, which are 365.

Scholer. Now for that you haue giuen me

so much light into the question, you shall see I will handsomely finish the work, for according to your former instructions I set them down with a line under them, thus.

1586
365

Then say I 5 times 5 make 30, the 0 I set down vnder the first place, & the article 3 keepe in minde to be added to the next multiplication, then 5 times 8 make 40, and 3 in minde make 43, the 3 I set downe in the second place and 4 I keepe.

1586
365
7930

Then say I 5 times 5 is 25, and 4 in minde make 29, the digit 9 I set downe in the third place, and 2 I keepe in minde. Then comming to the last figure, I say: once 5 is 5, or 5 times 1 is 5, and 2 in minde make 7, that doe I write vnder the fourth place, and thus haue I ended my first multiplier, and therfore I giue it a dash with my pen.

Then come I to the second multiplier, which is 6, and doe multiplie it into the vpper number, saying: 6 times 6 make 36, the 6 I set downe in the second place vnder the multiplier, and the article 3 I keepe in minde. Then say I 6 times 8 make 48, and 3 in minde make 51, the digit 1 I set downe in the third place, and 5 I keepe in minde. Then say I 6 times 5 make

1586
365
7930
9516

make 30, and 5 in minde make 35, the digit 5 I set downe in the fourth place, and 3 I kéepe, then say I 6 times 1 is 6, or once 6 is 6, and 3 in minde is 9, which I set down next to 5, so haue I ended two multipliers. Wherefore I dash the 6 with my pen.

Then begin I to multiplie the third multiplier into the ouer number, saying: 3 times 6 make 18, the digit 8 set downe in the third place vnder the multiplier, and the article 1 I kéepe. Then 3 times 8 make 24, and 1 in minde make 25, the 5 I set downe in order next, and 2 I kéepe: then say I 3 times 5 is 15, and 2 is 17, the 7 I set downe, and 1 I kéepe. Lastly I say, once 3 is 3, and 1 I kéepe, make 4, which I set orderly downe next, and so haue I ended the multiplication, and my figures stande thus.

```
1586
 365
7930
9516
4758
```

Master. I commend you for your diligence, the worke is very perfectly done, which parcels if you nowe adde together into one sum, it will be 578890, which is the grosse or totall summe of that Multiplication, and declareth the number of daies since our Lorde and Sauiour his Incarnation, vnto the ende of 1586 yeres, besides 396 daies and 12 houres, for leape yeres.

Scholer. This is maruelous me thinketh, that such great matters may so easily be atchieued

chieued by this Art, which heretofore I euer thought had beene vnpossible, as infinite sorts of people are of that minde.

Master. Truth it is that knowledge hath no greater enemie than ignorance, for this is one of the least of ten thousand things that may be done by this Art, as hereafter you shall be able to iustifie.

Scholer. This maner of Multiplication I perceiue, if there be no more in it.

Master. Yes there are other formes and helpes for ease and shorter labour of the work of Multiplication, but I will remit them till you haue a litle tasted Diuision, where also the like helpe into Diuision may be vsed, and so therefore vnder one example for both, will I shewe you both ease in Multiplication and also in Diuision.

But sith the other formes and workings doe nothing differ from these workes in effect, but onely in setting of the numbers, I will ouerpasse them till a more meeter place and time. And nowe will I instruct you in Diuision, so that you thinke your selfe sufficiently to perceiue what I haue taught you.

Scholer. Yes Sir I thanke you, but I doe not perceiue howe to examine my worke, to trie whether I haue well done or no, therefore as you promised me while ere, I pray you first shewe me howe I shall proue it.

Master. That is commonly vsed by the proofe

proofe of 9, as you learned before in Addition, saving that it differeth from that forme in diuers respects: As for example.

First you must make a crosse after this maner.

Then must you examine your sum that should be multiplied, and loke what remaineth after casting away of 9, that set you at the one side of the crosse: then examine the multiplier, and whatsoeuer remaineth in it, after casting away 9 so often as you can, write that at the other side of the crosse: then must you multiplie those two numbers together, and loke what amounteth thereof, if it be vnder 9, write it at the higher part of the crosse: but if it be aboue 9, then take thence 9 as often as you can, and write the rest at the head of the crosse. As first for example we wil proue the example you put forth of the peece of ground that contained 1365 yards in length and 236 yards in breadth.

Therefore first I cast away all the nines from the summe to be multiplied, saying: 5 and 6 make 11, cast away 9, rest 2, then 3 and 2 make 5, and 1 is 6, that 6 I write at the one side of a crosse thus.

Then doe I examine the multiplier which is 236, wherein when the 9 is cast out, there remaineth 2, that 2 therefore I set

at the other side of the crosse.

Then doe I multiplie 6 by 2, and it maketh 12, from which 12 I withdrawe 9, then resteth 3, that 3 doe I set at the head of the crosse. Then doe I examine the grosse summe amounting of the Multiplication, which is 322140 where I finde 9 once, and 3 remaining, that 3 I set at the foote of the crosse, and then I see it to agree with the other 3 at the top of the crosse, and so know I that I haue done well: for if they two did differ, the were my worke vaine, and the multiplication false.

This is the common proofe, but the most certaine proofe is by Diuision, of which I will anon instruct you.

Scholer. Sir, what is the chiefe vse of Multiplication?

Master. The vse of it is greater than you can yet vnderstand: howbeit, these plaine commodities it hath, that if you would resolue any great and whole value into many small and lesse portions: as if you would change pounds into shillings, pence or any other greater or smaller parcels, by Multiplication, ye shall doe it spedily and easily. Also if you should nede to adde one summe to it selfe, or to any other oftentimes, you shall doe it by Multiplication much more spedily, readily, easily and surely, than by often and sundrie

Ad-

Additions. Take you these commodities grossely shewed for an aunswere at this time, and hereafter I will more abundantly make you to perceiue the vse of it.

Diuision.

Scholer.

WEll Sir, then in Diuision I pray you to instruct me. But me thinketh by the name of it, that it should be all one with Multiplication: for I call that Diuision, when any thing is parted into diuerse and many parts.

Master. You take it as it is taken commonly: howbeit, if you marke well, you shall perceiue that it is quite contrarie to Multiplication, and doth not part one thing or fewe things into many, but contrarie waies it bringeth many parcels into fewe, but yet so, that these fewe taken together, are equall in value to the other many; for by Diuision pence are turned into shillings, and shillings into pounds: as for example of 120 shillings, it maketh 6 pounds, so are 120 turned into 6, which is a smaller number: but then if you consider the denominators, you shall see that

they are such, that one of the latter is equall to 20 of the first, and so in value the summes are one, though in number they doe farre differ, and the latter summe is the lesser, and so is it alwaies in Diuision: howbeit, yet in the working, the summe is parted by another, and thereof doth it take the name.

Scholer. I thinke I shall better vnderstand the reason of the name, when I knowe the vse of the worke, therefore now would I gladly learne that.

Diuision what it is.

Master. Diuision is a distributing of a greater summe by the vnites of a lesser. Or Diuision is an Arithmeticall producing of a third number, in respect of two propounded numbers: which third number shall so often containe an vnite, as the greater of the two propounded numbers doth containe the lesser. So that, euen as Multiplication did seeme to serue in stead of many Additions, so Diuision may seeme to be in place of many Subtractions: Because that third number briefly expresseth, howe many times the lesser of your two propounded numbers may be subtracted from the greater: as in practise will more plainely appeare. Therfore (as you may perceiue) vnto Diuision are required three numbers: the first, which should be diuided, and that must (generally) be the greater: and the second, by which the other must be diuided, and that is (generally) the lesser, and is called the

the Diuisor. And the third which answereth to the question, How many times: and therefore is called the Quotient.

The first must be first written, and the second so set vnder it, that the last figure of the lower number be right vnder the last of the higher, contrariewaies to the worke of the other kindes of Arithmetike: for in them the two first figures were set euer mete one vnder the other, but in Diuision the last figures must be set mete, except it chance so, that the last figure of the Diuisor be greater than the last of the higher number, for then you shall set the last of the Diuisor, vnder the last (saue one) of the higher number, as for example. A generall rule for placing y figures. An exception.

If you should diuide 365 (which are the summe of the daies of a yere) by 28, which are the daies of a common moneth, then shoulde you set them thus,

365	
28	

But if you would diuide those 365 daies, by 52, which is the number of wekes in one yeare, then should you set them thus.

365	
52	

Likewise if I would diuide the same 365 by 4, which is the summe of the quarters of a yere, then must I set them thus.

365	
4	

Scholer. Syr, this doe I vnderstand, but howe nowe should I doe to diuide the one by the other?

Ma-

☞ Maſter. You muſt begin with the laſt figure next the left hande, and ſée howe many times the laſt figure of the Diuiſor may be taken out of the laſt figure of the ouer number, and that ſhal you note within a croked line toward your right hand. As for example.

I would diuide 365 by 28, then ſet I thoſe two ſummes thus.

365 (
28

And I loke how many times I may finde 2 (which is the laſt figure of the Diuiſor) in 3, (which is the laſt of the number to be diuided) and conſidering that I can take 2 out of 3 but once, I make a croked line at the right hand of the numbers, and within it I ſet 1, and that is called the Quotient number, as I tolde you. Then becauſe that when 2 is taken out of 3, there remaineth 1, I muſt write that 1 ouer 3, and deface or cancell the 3 and the 2, then will the figures ſtand thus.

Quotient numbers.

1
365 (1
28

Then muſt I goe to the next figure of the Diuiſor, and take it likewaies ſo many times out of the figures that be ouer it, and loke what doth remaine, that I muſt write ouer them, and cancel them, as in this example.

Therefore now doe I take once 8 out of 16, and there remaineth 8, which I muſt ſet ouer the 6, and cancel or croſſe out the 16, and the 8 of the Diuiſor: And then will the figures

gures stand thus.

And so haue haue I once wrought. 28 / 365 (1

Scholer. So I perceiue that you take the nether figure not onely out of the other that is right ouer him, but out of that with ỹ other also that remaineth before, and are written toward the left hand.

Master. So must you doe: for you must so take the diuisor out of the ouer number, that there remain not ouer it so great a summe as it selfe is, for then were your worke in vaine.

But yet againe here must you marke, that when you seeke howe many times the last figure of the diuisor may be found in the number ouer him, that you loke also whether you may as often finde all the figures following in those that are aboue them, (considering all the remainders if there be any) if not, take your Quotient lesse by one, and then proue againe, and so still, till you finde a méete Quotient: And by that méete Quotient must you alwayes multiplie your diuisor, and the product set vnder your diuisor, so that his first figure stand vnder the first figure of your diuisor, and the second vnder the second, and so forth: and then subtract that product from the number to be diuided, that standeth directly ouer it, as you haue seene me doe. Note.

When you haue thus wrought once, then must

must you begin againe, and write your diuisour a newe, nearer toward the right hande by one place, as in this example, you shall set 2 vnder 8, and 8 vnder 5, thus.

```
x8
365 (1
x88
 2
```

Then (as before) seeke howe many times you may take your diuisor out of the number ouer him nowe.

Scholer. That may I doe here 4 times.

Master. Truth it is that you may finde 2 foure times in 8: but then marke whether you can finde the figure following so many times in the other that is ouer him. Can you finde 8 foure times in 5?

Scholer. No, neither yet once.

Master. Therefore take 2 out of 8, once lesse.

Scholer. That is 3 times.

Marke howe to consider this kinde of remainer.

Master. Well, then 3 times 2 make 6: if I take 6 out of 8, there remaineth 2: which 2 with the 5 following, make 25, in which summe I finde 8 iij times also, and therefore I take 3 as a true quotient, and write it within the crooked line of the Quotient, before the one thus.

```
x8
365 (13
x88
 2
```

Then say I: 3 times 2 make 6, then 6 out of 8 resteth 2, therefore I cancel the 2 and the 8, and write ouer it the 2 that doeth remaine, thus.

```
x8
365 (13
x88
 2
```

Then

Then doe I take 8 as many times out of 2, saying: 3 times 8 make 24, and if I take 24 out of 25, there remaineth 1: so then I cancell 25 and 8, and ouer the 5 I set 1, thus.

Or you mought (after you found 3 to be a fit Quotient) straight way haue multiplied the whole diuisor 28, by that 3, at once: which giueth 84, which being set vnder 28, and duely subtracted from 85, of the number diuided, giueth 1, the remainer of the whole diuision: as before you had. Worke which way you list: here you see also the forme.

2	
281	
365	(13
288	
2	
81	
365	(13
288	
24	
8	

And nowe haue I done with diuiding, for I can finde my diuisor 28 no more in the ouer summe.

Scholer. No except you would part the 1 that remaineth, into 28 parts.

Master. That is well saide, and so must we doe in such cases, when there remaineth any thing: but I will let that passe now, and will make you perfect in diuision of whole numbers, and will hereafter teach you particularly of broken Numbers, called Fractions.

Nowe if you doe perceiue the order of Diuision, then do you diuide this sum 136280 by 452.

Scholer. First I set downe the number that

that should be diuided, then doe I set the diuisour vnder it, so that the last figure of it bee right vnder the last figure of the ouer number. Then will it bee thus.

136280	
452	

Master. Can you take the last of your diuisour (which is 4) out of 1, which is the last of the ouer number?

Scholer. I had forgotten, because the last of the diuisor can not be taken out of the last of the ouer number, in so much as it is the greater, therfore must I set the diuisor one place more forward, toward the right hand thus.

136280	
452	

And then must I loke howe often I may finde the last figure of the diuisour (that is 4) in 13, which thing I may doe 3 times, therefore doe I say: 3 times 4 is 12, which I take out of 13, and there remaineth 1. Then doe I make at the right hande of my summes a croked line, and write before it my quotient 3: and I cancel 13 and 4, and ouer the 3 I set the 1 that remaineth, and then the figures stand thus.

1	
136280	(3
452	

Then doe I multiplie the same Quotient into euerie figure of the diuisor, & withdrawe the summe that amounteth out of the numbers

bers ouer them, as first I say: 3 times 5 make 15, which I take from 16, and there resteth 1, I cancell therefore 16 and 5, and write ouer the 6 that 1 that remaineth, thus.

```
      11
1362 80 (3
  452
```

Then doe I say likewaies, 3 times 2 makes 6, which I take out of 12, and there resteth 6, therfore I cancel the 12 and the 2, and ouer the 2 I write the 6 that remaineth, thus.

```
      116
1362 80 (3
  452
```

Then shoulde I set forward the diuisor, into the next place toward the right hand, thus.

```
      116
1362 80 (3
  4522
    45
```

Master. But you may see, that ouer the 4 is no figure, therefore must I set the diuisor yet forewarder by an other place.

And marke, whensoeuer it chaunceth so, that you should set forward the diuisor, & that it cannot stande there, because there is no number ouer the last place, or if there be any, it is lesser than the last figure of the Diuisor, then must you remoue the diuisor yet once againe: and because that his first place of remouing serued not to subtract him so much as once, therefore you shall write in the quotient a cypher 0. And if you should by chance néede to doe so oftentimes, for euerie time write a ci-

cipher in the quotient. The reason of this, will I shewe you hereafter.

Scholer. Then must I set my summes thus.

And because I remoued the diuisor, so that I ouerskipped one place, I must write a Cipher in the quotient: & then must I seek a new quotient, as in this example I must say, Howe many times 4 is there in 6 (and sith it can be but once, therefore doe I write 1 in the quotient: and then say I: 1 time 4, taken out of 6, remaineth 2, I cancell the 6 and the 4, and write 2 ouer them, thus

Then say I againe, once 5 out of 28, remaineth 23, I let the 2 stand as it did, and ouer that 8 I set 3, cancelling the 8 and the 5 vnder it, thus.

```
xx6
xz6z80
45z52 (0
  4

  2
xx6
xz6z80 (301
45z52
  4

  2
xx6
xz6z80 (301
45z52
  4

  2
xx63
xz6z80 (301
45z52
  4
```

Master. You might as well haue saide, once 5 out of 8, and so remaineth 3, but now goe forth.

Scho-

Scholer. Then once 2 out of 0, can not be, what shall I nowe doe?

Master. Borowe of the next number that is behinde, (for there is 230) and doe as you learned in Subtraction in a like case.

Scholer. Then must I borowe 1 of the 3 comming behinde next, and make that 0 to be 10: and then take I 2 out of 10, and there resteth 8. And because I borowed one of the 3, I must cancel the 3, and write 2 ouer it: thē doth the figure stand thus.

```
        22
  136280 (301
  452
  ----
   4
```

Master. Nowe haue you done, and yet remaineth 228, & your quotient sheweth you, that if you diuide 136280 by 452, you shall finde your diuisor in your greater number 301, that is CCC times and once, and 228 remaining.

And in the other example, where I diuided 365 by 28, the quotient was 13, and 1 remained: whereby I knowe that in a yeare (which containeth 365 daies) there are 13 monthes reckoning 28 daies (or 4 wekes) iust to a moneth, and 1 day more.

Scholer. Why then doe we call a yeare but 12 moneths.

Master. Of that at a more conuenient time will I fully instruct you: but nowe it is not conuenient to entangle your minde with o-

ther things, than doe directly pertaine to your matter. Therefore if you remember what you haue heard, you haue learned a short maner of diuision, which I would haue you often to practise, so that you may be perfect in it, and hereafter I will shewe you certain other proper points touching it.

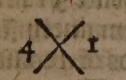
Scholer. Then I pray you, yet tel me, how I shall examine and trie my worke, whether I haue done well or no, that though no man be by me to tell me, yet I may perceiue it my selfe.

Master. Some men (yea and commonly) doe trie it by the rule of 9, as in all the other kindes, saue that their order is. First they cast away 9, as often as they can, out of the diuisor, and that remaineth, they set at one side of a crosse: As in our first example, the diuisor was 28, from which you may take 9 three times and 1 remaineth, which they set by a crosse, thus.

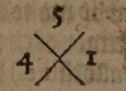
Then doe they likewise examine the quotient (which in our example is 13) and from thence they cast away 9 as often as they can, and the remainer they set at the other side of the crosse, and then multiplie they together those two remainers: and to it that amounteth, they adde the remainer of the Diuision, if there were any, from that whole sum they withdrawe 9 as often as they can,

and

and the reſt they ſet at the head of the croſſe: as in our example the quotient is 13, from which take 9, and there remaineth onely 4, and therfore muſt you ſet 4 at the other ſide of the croſſe, thus.



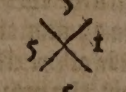
Then multiplie 4 by 1, and it yéldeth but 4, thereto adde the remainer of the diuiſion (which was 1) and it will be 5, which ſumme doth not amount to 9, & therefore muſt be ſet wholly at the head of the croſſe, as you ſée here.



And this number on the head of the croſſe, is the firſt proofe, to which if you finde another like in the number that was diuided, then haue you done well.

Therefore nowe ſhall you likewiſe examine the whole ſumme that was diuided, and take away 9 as often as you can, and that that remaineth, ſet at the foote of the croſſe: and if it be equall to that in the head of the croſſe, then haue you well done, elſe not.

As in our example the whole ſum was 365, which maketh 14, from that take 9, and there reſteth 5, which ſet at the foote of the croſſe, thus.



And you ſhall ſée that they agrée: therefore haue you wel done.

Scholer. Nowe will I likewiſe examine

our second example, where the diuisor was 452, which maketh 11: from thence I take 9, and the 2 that remaineth I set at the right side of a crosse, thus.

Then examine I the quotient, which was 301, where I finde but only 4, that doe I set at the other side of the crosse thus.

Then doe I multiplie 4 by 2, and it maketh 8, to that doe I adde the remainer of the diuision (which was 228, and maketh 12) & they two make 20, wherein I finde twise 9, and 2 remaining, that 2 must I set at the head of the crosse, thus.

Then doe I examine the whole number to be diuided, which was 136280, where I finde twise 9, and 2 remaining, which I set at the foote of the crosse, thus.

And because that it doeth agrée with the figure at the head of the crosse, I knowe that the diuision was well wrought.

Master. This is the common proofe, howbeit, the more certaine working is by the contrary kinde, as to proue Diuision by Multiplication, thus.

Againe of the proofe of diuision

Multiplie the quotient by the diuisor, and if

if the summe that amounteth be equal to the summe that should be diuided, then haue you wel diuided, else not.

Howbeit, this must you marke, that if there remained any thing after the Diuision, that must you adde to the sum that amounteth of the Multiplication: as in our first example the quotient was 13, and the Diuisor was 28: Nowe multiplie the one by the other, and the summe will be 364: to that if you adde the one that remained after the Diuision, then will it be 365, which was the summe that should be diuided, and therefore I knowe that I haue well done.

Scholer. Nowe will I proue the same in the second example, whose diuisor was 452, and the quotient 301: these doe I multiplie together, and there amounteth 136052 to which if I adde the 228 that remained, then will it be 136280, which was the whole summe to be diuided, and therefore I perceiue that I haue well done.

Master. This is the surest way, to examine Diuision by Multiplication: and contrarie waies the surest proofe of Multiplication, is by Diuision.

And therefore according to my promise, nowe will I shew how you may proue Multiplication by Diuision. A proofe of multiplication by diuision

When you haue ended multiplication, and would knowe whether you haue well done

or not, set the grosse summe that amounteth of the multiplication ouermost, and diuide it by the multiplier: and if the quotient be the same number that should be multiplied, then haue you well wrought, else not: as in that example where we multiplied 264 by 29, the grosse summe was 7656.

Nowe if you will know whether that multiplication be true, you shal diuide that 7656 by the multiplier 29, and you shall perceiue that the quotient will be 264, and that is a token that you haue well wrought.

Scholer. By your patience I will proue that: and first I set downe the grosse summe and the multiplier, not after the rule of multiplication, but after the rule of diuision, for nowe that number is become the diuisor, that was before the multiplier, I shall set them therefore thus.

7656	
29	

Then shall I seeke howe many times 2 in 7, that may be 3 times, and 1 remaineth: but then may not 9 be found so often in 16, therefore must I take a lesser quotient, that is to say, 2: then say I twise 2 maketh 4, which I take out of 7, and there remaineth 3, then doe I cancell 7 and 2, and ouer 7 I write 3, and in the quotient I set 2, so the figures stand thus.

3	
7656 (2	
29	

Then say I forth, 2 times

9 make

9 make 18, which I abate out of 36, & there resteth 18, then cancell I 3, and ouer him set 1, and likewise I cancell 6 and 9, and ouer them I set 8, so that thus stand the figures.

1
3 8
7656 (2
2 9

Then doe I set forward the diuisor by one place, and seeke a newe quotient, that is to say, howe many times 2 are in 18, which I finde to be 9 times, but then can I not finde 9 so many times in 5, therefore I take a lesser quotient, as to say, 8: but yet is that to great: for if I take 8 times 2 out of 18, there remaineth but 2, and I can not finde 8 times 9 in 25, therefore yet I take a lesser quotient, that is 7, which is also to great, for if I take 7 times 2 out of 18, there resteth 4, but now I cannot take 7 times 9 out of 45, therefore yet I seeke a lesser quotient, as to say, 6: then say I, 6 times 2 make 12, that I take out of 18, and there remaineth 6, so I cancell the 18 and the 2, and write 6 ouer 8, thus.

1 6
3 8
7656 (26
2 9 9
2

Then say I forth: 6 times 9 maketh 54, that take I out of 65, and there remaineth 11, and the figures stand thus.

1
16
381
36556 (26
299
2

Then must I set forth the diuisor againe, and seeke a newe quotient, which will be 4: for though

though I may finde 2 in 11 fiue times, and 1 remaine, yet I cannot finde 9 so often in 16, therfore I set the figures thus.

And the 4 in the quotient I multiplie into the figures of the diuisor, saying: 4 times 2 maketh 8, which I take out of 11: and there resteth 3, therefore I cancel the 11 and the 2, and set 3 ouer the first place of 11, thus.

And then doe I say forth, 4 times 9 maketh 36, which I take from 36, and there remaineth nothing, so that the quotient of this Diuision, where 7656 is diuided by 29, is 264, which doeth declare that if 264 be multiplied by 29, the summe will be 7656. And thus I perceiue nowe howe both Multiplication is proued by Diuision, & Diuision also by Multiplication.

Master. Nowe haue I ended the fiue common kindes of Arithmetike: for as touching Mediation, Duplation, Triplation, and such other, they are no seuerall kindes of Arithmetike, but are contained vnder the other: for Mediation is contained vnder Diuision, and is nothing else but diuiding by 2: & so are Duplation & Triplation contained vnder Multiplication: for Duplation is nothing else but multiplying by 2, & Triplation is multiplying by

1 16 381 7656 (264 2999 22 1 163 381 7656 (264 2999 22

by 3, of which I will onely propose an example, for the rules you haue heard alreadie.

If you would mediate or diuide into 2, this sum, 4 5 3 1 0 1 0, you shal set 2 for the diuisor, & worke as you learned before, as thus.

An example of Mediatiõ.

```
4 5 3 1 0 1 0 ( 2
2
```

Then I finde 2 in 4 two times, therefore my quotient must be 2: so I cancell 4 and 2, and remoue the diuisor forward, thus: as the worke requireth, and as before in diuision hath beene declared.

```
4 5 3 1 0 1 0 (2
  2 2
```

Which mediation or diuision by 2, beeing finished, you shall haue for your Quotient 2 2 6 5 5 0 5, which is ỹ halfe of 4 5 3 1 0 1 0, as you may trie by Duplation: for double that Quotient, or multiplie it by 2, and the same number will amount.

Duplatiõ.

I will no longer tarie about these, seeing they are but members of ỹ other kindes. But here now according to my promise I wil teach you certaine easie formes both of Multiplication and of Diuision, & first of Multiplication.

If you would therefore multiplie any sum by 10, you shall neede to doe no more but adde a cipher before his first place, as for example: 36 multiplied by 10, make 360.

Easie formes of Multiplication. ✝

Likewise if you would multiplie any sum by 100, put two ciphers at his beginning.

So if you wold multiplie any sum by a thousand, adde three ciphers to the beginning of it.

Scho-

Scholer. This doe I well perceiue, and also the reason of it.

Master. I will omit all reasons til our next meeting, when I shall tel you the reason of al other parts of Arithmetike also: and as to our matter now, loke (as I haue tolde you) that you both remember it, and also often practise it.

And nowe haue you learned howe to multiplie easily by 10, 100, 1000: and of like maner may you do with any other of that sort.

But nowe if you will multiplie by 20, 30, 40, and so forth: or by 200, 300, & such like, where there is one cipher in the first place, or many orderly in the first places, you shal take away those ciphers, and multiplie the summe onely by the other figure or figures, (if they be many) and then at the beginning of the sum that amounteth, you shal set so many ciphers as you toke away.

Example of 2873, which I would multiplie by 300. First I omit the 2 ciphers from the multiplier, and I multiplie the summe by onely 3 that is left, & it amounteth to 8619: before which I put the two ciphers that I before omitted or toke away, and then is it 861900. And that is the sum that amounteth, when 2873 is multiplied by 300.

Scholer. And if there were two or more figures beside the ciphers, I must onely take away the ciphers, and multiplie by the other figures

figures, as I learned before: as if I woulde multiplie 93648 by 25000, I should take away the three ciphers, & multiplie the same by 25, and then at the beginning of that totall summe, should I adde the three ciphers againe.

Master. Euen so: but and if it chance the number that should be multiplied, or both the summes as well the number that shoulde be multiplied as the multiplier, to haue ciphers in their first places, euermore omit the ciphers, and worke by the rest. But remember to restore as many ciphers to the amounting summe, as you bated before, as in this example: 30200 shall be multiplied by 206: I shall onely take away the two ciphers from the greater number, and then multiplie 302 by 206, and afterward adde the two ciphers againe. But if I would multiplie the same 30200 by 2060, I shall not onely take away the two ciphers from the number that should be multiplied, but also I may take away the one cipher from the multiplier, and then must I adde 3 Ciphers to the sum that amounteth: but take heed that you take away no cipher, that commeth after any signifying figure, as in this last example, you may not take away that in the fourth place of the higher number, neither that in the third place of the multiplier: howbeit, yet this you may doe: If one cipher or more come in the middest

of

of your summes, you may multiplie by the other figures, and ouerskippe them, but so, that you giue euerie figure his due place, as thus: I will multiplie 3026 by 2004, therefore I set them thus.

3026
2004
24
1208

And thus doe I multiplie them: First 4 times 6 make 24: I set the 4 vnder the first place, and kéepe the 2 still in my mind: Then say I againe: 4 times 2 maketh 8, and the 2 that is in my minde maketh 10, I set downe the cipher 0, and kéepe the article 1 in my minde. Then 3 times 4 is 12, and the 1 in my minde maketh 13, I set downe the figure 3, and kéeping the 1 still in my minde hauing no more places of the vpper number to multiplie it withall: I put it downe next 2 in the fift place.

But nowe when I come to the next place being a cipher 0: I let it goe, because it multiplieth nothing: & likewise the second cipher.

But then when I doe come to the 2, and multiplie it into the 6 of the ouer number, you must take héede (according as I taught you in Multiplication) that the first number amounting of the multiplication, be set right vnder the multiplier, and the other orderly toward ỹ left hand, according as you may sée in this example.

3026
2004
12104
6052
6064104

which being finished with the Addition thereof gathered togither, will stand as this example sheweth.

Which is indéede wrought so much soner and shorter by ouerskipping of the two ciphers: which otherwise if the same example were wrought at length, it would haue had two worckings more as by the same example here also set downe doth appeare.

	3026
	2004
	12104
	0000
	0000
6052	
6064104	

Scholer. Sir, I thanke you: for I sée great ease in this way of Multiplication, and if you can shewe me such like in Diuision, you shall greatly further me.

Master. Yes, I will teach you some easie waies in Diuision also, and first this: If you would diuide any summe by 10, you shal onely with your pen make a square line, betwéen the first figure of your summe and the second, and then haue you done: for the whole number that followeth the line, standeth for the quotient and the figure that is before the line, is the remainer: as for example, 3648 diuided by 10, wil stand thus.

364	8

Where 364 is the quotient, and betokeneth that so many times are 10 in 3648: and the 8 after the line, is the remainer, which cannot be diuided into 10, but by breaking it into

into fractions, wherwith I wil not medle yet.

And so likewise if you would diuide any summe by 100 with your pen, you shall cut away the two first figures, and if ye would diuide by 1000, you must cut away the 3 first figures, and so of any other diuisor, whose last figure is 1, and the other be ciphers, loke how many ciphers the diuisor hath, and so many figures at the beginning shall you cut away with the squire line, and they stand alwaies for the remainer because they are lesse than the diuisor, and cannot be diuided by it, and the other figures that be behind the line, stand for the Quotient.

But nowe if your diuisor haue any other figure in his last place than 1, and in all his other places haue ciphers, loke howe many ciphers there be, cut away so many of the first figures of the number that should be diuided, and diuide the rest that followeth the line, by that figure that is in the last place, as if it were the whole diuisor.

Example of 6 4 2 8 4, which I would diuide by 300, here must I cut away the two first figures, (for so many ciphers my diuisor hath) and must diuide the rest by 3 which is the figure in the last place of the diuisor. First therefore I part away the two first figures, and the summe standeth thus.

6 4 2 | 8 4

Then doe I diuide 6 4 2 by 3, and the quotient

tient is 214, for in 6 I finde twice 3, and in 4 once, and 1 remaining, which 1 with the 2 next before, doth make 12: wherein I finde 3 foure times: and this is a readie way to turne shillings into pounds: for sith one pound doth contain 20 shillings, I must diuide the whole number of shillings by 20 therefore easilie to doe it, I see that my diuisor hath one cipher, and therefore I cut away one figure from the beginning of the whole summe of shillings, and then I doe mediate or diuide by 2 the other figures or summe that followeth.

Scholer. I will put an example.

If you would diuide 64287 shillings by 20, that is to say: If I would turne so many shillings into pounds, I must cut away the first figure, that is 7, and diuide the rest, that is 6428 by 2, so shall the quotient be 3214, whereby I knowe that 64287 shillings make 3214 pounds, and 7 shillings remaining.

Master. Nowe proue by Multiplication whether you haue well done or no.

Scoler. The quotient is 3214, which I doe multiplie by the diuisor 2, and it doeth amount to 6428.

Master. Hereby may you perceiue not onely that you haue well done, but also howe by Diuision you may turne shillings easilie into pounds: And contrarie waies, by Multiplication you may turne pounds into shillings.

But

But here shall you see amongst diuers men, diuers formes of such diuision, but if you mark what I haue tolde you, you shal perceiue easilie all their waies: for some men doe not cut away so many of the first figures of the sum that they would diuide, as there are ciphers in the first places of their diuisor, but they set all their ciphers orderly vnder the first places of the number that they would diuide, and then with the other figure (or figures if they be many) they diuide the rest of their summe.

An other maner of the abridgement.

Example. If they would diuide 725931, by 3400, they set their summes thus.

725931	
34 00	

And then doe they diuide orderly till they come to the ciphers: for there they stay and ende their worke, as in this example: They seeke howe often 3 may be found in 7, which is 2 times, and one remaining, therefore they set 2 in the quotient, and cancel 3 and 7, and ouer 7 they set the 1 that remained, thus.

1	
725931 (2	
34 00	

Then do they goe forth saying: two times 4 maketh 8, which they take out of 12, and there remaineth 4, thus.

x4	
725931 (2	
34 00	

Then renewe they the diuisor forward, and seeke howe often 3 may be found in 4, which

is

is but once, and 1 remaineth, then set they 1 in the quotient, and cancell 3 and 4, & ouer them they set that 1, thus.

Then take they once 4 out of 15, and there resteth 11: Or else more easilie: Take once 4 out of 5, and there resteth 1, so they cancel the 4 and 5, and set 1 ouer them, thus.

```
   1
  x4
725931 (21
344 00
  3
```

```
   1
  x41
725931 (21
344 00
  3
```

Then set they forth the diuisor againe, and seeke howe many times 3 are in 11, which they finde 3 times, and 2 remaining: so they set 3 in the quotient, and cancel 11 & 3, and ouer them setteth 2, thus.

Then doe they multiplie 4 by 3, which maketh 12, that withdraw they out of 29, and there resteth 17, of which the 7 must be set ouer the 9, and the 1 ouer the 2, thus.

```
   x2
  x41
725931 (213
344400
  33
```

```
   1
   x2
  x417
725931 (213
344400
  33
```

And now are the two ciphers next ensuing, so that the diuisor can no more be set forward, and therefore is the diuision ended, and the

remainer is 1731.

Nowe the quotient, which is 213, doth declare, that if you diuide 725931, by 3400, you shall finde it therein 213 times, and there remaineth 1731, so shall you finde it, if you woulde as I taught you, by cutting away the two first figures, because of the two ciphers.

Note. But this must you marke (as you may perceiue by this last example) that if there be left any other remainer in the summe that was behinde the squire line, that the remainer must be set to the latter ende of the first remainer, which was cut away with ỹ squire line: as if you would diuide 725931 by 3400, after the forme that I taught you, then would your summes appeare thus.

1	
22	
442	7
7233	31 (213
3444	
33	

So that 17, which remaineth after the line, must be set to the 31 (that was cut away with the line) in higher places, as you see here: where that 17 with the 31, doe make 1731.

¶ And here will I make an end of Diuision, saying that I doe request you to exercise your selfe well herein by many summes, till you haue attained some expertnesse therein. For the reasons and conclusions thereof are so many and so aunsweable for all sorts of men whatsoeuer, that if I should speake of the infinite

finite vses therof, I should rather lacke woods than matter, and therefore recommending it to your iudgement hereafter vpon your further trauell into the Art, I will here end this treatise, representing vnto you one example or simple question of Diuision and Multiplication in stead of many, which is this.

There are 4 brasse peeces: The first of them at a shot spendeth 9 pound of poulder, the second spendeth 5 pound, the third 4 pound, and the fourth 2 pound. They are all appointed against the batterie of a holde, and there is allowed by the maister gunner 700 pounde of poulder to be spent by these 4 peeces in this assault. The question is two folde: The first howe many shot each peece shall iustly make about with this 700 pound of poulder. And lastly how many pound of poulder ought iustly to be allowed to each peece for his true proportion.

Scholer. Why sir you make me smile, to beare me in hande, that these two demaunds may be simplie resolued by Multiplication and Diuision.

Maister. Truely that they may, and that may you by and by worke your selfe with a litle labour: First adde together their quantities of poulder, that is 9 pound, 5 pound, 4 pound, and 2 pound, all which make 20: Diuide the 700 pound of poulder by that 20, and your quotient giueth 35, as here appeareth,

a rule. brasse peeces

which sheweth for most certainty that they shall make iust 35 shottes about.

1
700
200 (35
2

Scholer. Syr all this haue I done, and I see it is so, but whether it be true or not, I cannot tell.

Master. To trie the trueth of the same, multiplie the first peece, that spendes 9 pound by 35, and you shall see his allowance, which is 315 pound of poulder. Multiplie also the second peece that spends 5 pound by 35, and you shall finde 175 pound his allowance: then 4 by 35, and you shall finde 140 pound his allowance. Lastly multiplie 2 by 35, and you shall finde 70 pound his allowance. Al which 4 particular summes you shall adde together by Addition, as here appeareth, and it maketh iust 700 pound, and so is the question truely absolued.

315
175
140
70
700

Scholer. Truely Syr these excellent conclusions do wonderfully more and more make me in loue with the Art.

Master. It is an Art that the further you trauell, the more you thirst to goe on forward. Such a fountaine that the more you draw, the more it springs. And to speake absolutely in one word (excepting the studie of diuinitie, which is the saluation of our soules) there is no

no studie in the world comparable to this for delight in wonderfull and godly exercise: For the skill hereof is well knowen, immediately to haue flowed from the wisedome of God into the hart of man, whom he hath created the chiefe image and instrument of his praise and glorie.

Scholer. The desire of knowledge doeth greatly encourage me to be studious herein: and therefore I pray you cease not to instruct me further into the vse hereof. ‡

Master. With a good will. And now therfore for the further vse of these two latter, that is Multiplication and Diuision, I wil briefely shew you the feate of Reduction.

Reduction.

REduction is, by which all summes of grosse denomination may be turned into summes of more subtile denomination. And contrarie waies, all summes of subtile denomination, may be brought to summes of grosser denomination.

Scholer. What call you grosse denomination, and subtile denomination? Grosse denomination.

Master. That I cal a grosse denomination

which doth containe vnder it many other subtiller or smaller: As a pound in respect to shillings, is a grosse denomination: for it is greater than shillings, and containeth many of them. And shillings in comparison to pounds, are a subtile denomination, for because they are lesser than pounds, and many of them are contained in one of the other: as so, likewise of other things, whatsoeuer thing is compared to other, if it be a greater and containeth many of them, it is a grosser denomination: but if it be lesser, so that many of them are in the other, them are they called subtile denominations: whereby you may perceiue, that one denomination may be called a grosse denomination, and also a subtile (that is to say, a great and a smal) in diuers comparisons. For shillings compared to pounds are a subtile or small denomination: but compared to pence, they are a grosse or great denomination.

Subtile denominations.

Scholer. Now I vnderstand the name, I pray you teach me the vse.

Maister. The vse is easily learned, if you remember what you haue learned before. For if you will reduce any summe of a grosse denomination, into a summe of a smaller or subtiller denomination, you must consider howe many of that subtiller denomination do make one of the grosser denomination, and by that number or numerator doe you multiplie the other sum: as if you would reduce 20 pounds into

To reduce grosse denominations to subtile.

into shillings, you must consider that in a pound are included 20 shillings, therfore multiplie the one 20 by the other 20, and there will amount 400, whereby you may knowe, that in 20 pound are contained 400 shillings. Likewise if you would reduce 30 shillings into pence, considering that in 1 shilling are 12 pence, you must multiplie 30 by 12, and it will be 360: whereby you finde, that in 30 shillings, are contained 360 pence. And thus may you reduce any grosse denomination into a more subtiller, by multiplication, if you knowe howe many of the lesser doe make the greater: of which thing I will anon giue you a briefe table for the most accustomed kindes of monie, weights, measures and time, and such like, whereby you may knowe how often each subtile denomination is contained in the Grosser, when you shall nede it for the foresaide kinde of Reduction. And also the same shall serue you, if you would reduce any sum of a subtiler denomination, into a summe of a grosser denomination: For in such Reduction you must consider (as in the other forme) how many of the smaller do make the greater, and by that number must you diuide the other sum, and the quotient will declare howe many of the greater denomination, are comprehended in that summe, as for example: If you would knowe howe many shillings are contained in 3240 pence, consider that 12 pence do make

To reduce subtile denominations to grosse.

1 shilling: you must diuide that 3240 by 12, and your quotient will be 270, whereby you knowe that so many shillings are in 3240 pence. But and you would knowe further, how many pounds are in those 270 shillings, seeing that euerie pound containeth 20 shillings, diuide that 270 by 20, and it will be 13, and 10 remaining, whereby you may knowe that in 3240 pence, or 270 shillings, are 13 pounds and 10 shillings. For euermore the remainer must be named by the name or denomination of the sum that was diuided, which in this place were shillings. And thus may you doe with any other kindes of denominations.

* Wherefore to the intent you may haue a certaine light or knowledge in most common coines, weights and measures (which is the most chiefe and principallest thing in traffike to be knowen) I haue in each Reductions as they come in order, set downe certaine instructions incident thereunto. And first I haue hereunto added this Table, wherein is comprehended not onely our currant and common coines, but also of the most part of the vsuall coines of Christendome, with their iust waights and value currant in this Realme of England, entending at the latter end of my Addition to this booke, to write of the ordinary mony vsed in diuers places, and their common values currant for traffike, with the maner of their exchanges from place to place, &c.

A Table of the names and valuation of the moſt vſuall Gold Coines throughout Chriſtendome, with their ſeuerall weight of Pence and Graines: and what they are worth of currant money Engliſh.

The names and titles of the Golde.	The weight. Pence.	grains	The value. Shil.	Pence.
Royall	4	23	15	0
Halfe Royall	2	11	7	6
Olde Noble	4	9	13	4
Halfe old Noble	2	4	6	8
Angel	3	8	10	0
Halfe Angel	1	16	5	0
Salute	2	5	6	4
2 parts of Salute	1	11	4	2
George Noble	3	0	9	0
Half George Nob.	1	12	4	6
Firſt crowne K. H.	2	9	6	4
Baſe crowne K.H.	2	0	5	0
Great Soueraigne	10	0	30	0
Souera. K. H. beſt	3	14	10	8
Soueraigne K.H.	4	0	10	0
Edward Souer.	3	14	10	0
Elizab. Soueraine	3	14	10	0

Eli-

The names and titles of the golde.	The weight. Peny.	Grains.	The value. Shil.	Pence.
Elizab. crowne	1	19	5	0
Vnicorne of Scot.	2	10	6	0
Scottish crowne	2	5	6	0
French Noble	4	16	13	4
All the sortes of French crownes	2	5	6	0
Old French crown	2	5	6	0
Flaunders Rider.	2	6	6	6
Gelders Rider.	2	2	3	5
Philips Royall	3	10	10	0
Philips crowne	2	5	5	0
Collyn Gilden	2	2	4	8
New Andr. Gyld.	2	3	5	0
Flaunders Noble	4	10	12	0
Halfe Flaun. noble	2	6	5	0
Flem. Angel best.	3	6	9	0
Fland. royal or Key.	3	10	10	0
Carolus Gilden.	1	21	3	6
Flanders royal.	2	6	5	0
Saxon Gilden.	2	2	4	8
Flanders crowne.	2	5	6	0
Phillips Gilden.	2	3	4	2
halfe Phil. Gilden.	1	1	2	1
Golden Lion.	2	16	7	8
3 parts of gol. Ly.	0	21	2	5

The names and titles of the Golde.	The weight. Pence.	grains	The value. Shil.	Pence.
2.3 parts of gol. Li.	1	19	4	11
Dauids Gylden	2	2	4	0
Horne Gylden	1	12	4	11
Old Andre, Gyld	2	3	4	10
Crusado long cros	2	6	6	0
Crusado short cros	2	6	6	2
Myl rayes	4	20	13	4
Half Mill rayes	2	10	6	8
Portigue. 1. ounce	2	16	68	0
Golden Castilio	2	23	8	10
Ducket of Aragon	2	6	6	6
Hungarie Ducket	2	7	6	4
Double Pistolat	4	8	11	8
Single Pistolat	2	4	5	10
Ducket of Floren.	2	5	6	4
Double Ducket	4	11	13	0
Single Ducket	2	6	6	6
Dou. duc. of Rome	4	13	12	8

Of Siluer coynes currant in this Realme.

The Edwarde Crowne of 5 s.
The Edwarde halfe Crowne of 2 s. 6 d.
The Edward Shilling, halfe Shilling, and the Thrée-pence.
Phillip and Maries Shilling, & halfe Shilling.
The Marie Groat, and Marie Two-pence.
Quéene Elizabeths shilling, 6 d, 4 d, 3 d, 2 d, 1 d, 3 ob, 3 q, and ob.

Note. It is to be vnderstoode (gentle Reader) that wheras the waight is called by the name of a penny, it is not meant a penny of siluer mony, but a penny of Goldsmithes waight, which is 24 Barly cornes drie. And 20. of these pence make an Ounce: and 12 of these ounces make a Pound Troy: So that if a man haue not the waight wherwith to wey any péece that may come to his hand, he may do it with the Barly graines or cornes being drie, and taken out of the middle of the eare. ‡

Here would I nowe expresse the values of sundrie other coines of diuerse Countries, but for thrée causes I now refraine. The first and chiefest is, because they are not currant by the Statutes of this Realme. An other cause is, by reason they are so vncertaine, that they be neuer long at one rate. And againe they are so different in so many places, that it were mat-

mattter ynough for a great booke, to speake sufficiently of them all. Howbeit, yet because you shall not be altogither ignorant of them, I will shewe you the values of some that are most in vse, and first of Fraunce.

The most common money are Deniers, Soulx and Franks. 12. Deniers make 1 s̄, 20 Soulx make one Frank, so that as you see, these three kinds are like in the rate to pence, shillings, and pounds with vs, but that this is the difference, that their Denier is but the ninth part of our penie, and so their Soulx (commonly called Sowses) goe 9 to our shilling, and 9 of their Frankes to an English pound of money: So that 3 of their Frankes make a Noble. And by those 3 may you practise howe to reduce French money into English money, according as I haue set forth here following, French coines.

2160 deniers make 240 d̄, or 20 s̄.
3240 deniers make 360 d̄, or 30 s̄.
8352 deniers make 929 d̄, or 2 l̄, 17 s̄, 4 d̄
2160 soulx make 240 shillings. And so of other in like rate. As for the rest of their coines I omit them till hereafter that you haue some vnderstanding in broken numbers.

But nowe as for the coynes of Flaunders they be so changeable, that you must knowe them from time to time, else you cannot reduce them into our money certainely. But yet that you may haue an example of their money Flanders coines.

money to exercise you withall, you shal take those that be most common: as Stiuers both single and double, Groates Flemish, Carolus, and Gyldens. A Flemish Groate is a litle aboue three Farthings English. A single Stiuer is 1 d, ob. q, halfe farthing. The double stiuer is 3 d, farthing. The double stiuer Carolus is 4 d, ob. halfe farthing. Then there is also the Carolus Gylden which is worth 20 stiuers. And the Flemish Noble is worth 3 Carolus Gyldens, and 12 Styuers.

So that if you would conuert Flemish money, or any other kinde of money whatsoeuer it be, iustly into sterling, you must reduce it first into the smallest part of English money that is in that coine, as for example. If I would reduce 368 double stiuers into English money, considering that a double stiuer containeth 3 d farthing, you shall first loke howe many farthings be in the double stiuer, & you shall finde them 13, therefore multiplie the summe of the stiuers by 13, & then haue you their value in farthinges, which is 4784. Notwithstanding if you diuide that by 4, then there will appeare the number of pence: but better it were to diuide it by 48 (for so many farthings are in 1 shilling) and then will the quotient declare the summe of the shillings.

Likewise, if you would reduce any summe of single stiuers into English money, you must multiplie the summe first by 13, & then haue you

you reduced them into a certain summe, that is to wit, halfe farthings: which summe if you diuide by 8, then will amount the summe of pence: or if you diuide it by 96, the summe of shillings will appeare.

But marke this in all Diuision, when ye doe reduce to bring one denomination into an other, if there be any remainer after the diuision, that must be named by the denomination of the grosse summe that was diuided: as for example. I would bring 254 farthings into pence, therefore I doe diuide that 254 by 4, for so many farthings make 1 penie, and the quotient is 63, which is the summe of the pence, and then remaineth yet 2, which are farthings stil, as one may proue by diuiding. And this must be marked in all Diuision, namely when it is done for Reduction. Note well.

* Touching Danske mony they haue their Soulr, whereof 20 is a Liuer: which is 2 s̄ sterling. They haue also their Grash, whereof 30 make a Gilderne, which is 4 s̄ sterling. They haue also Dollors, and their common or olde Dollor is 35 Grash. Newe Dollors they haue, which be diuerse, some valued at 24 Grash, some at 26, and some at 30. And thus much I thought good to adde to the Author touching Danske money.

Concerning Spanish money, whereof the most commonly are Cornados, Marueides, Ryals and Duckets: 6 Cornados make a

Mar-

Marueide, 4 Marueides maketh 1 Ryal, and 11 Ryals maketh one Ducket, so the Ducket containeth 374 Marueides, which is about 5 s̄, 10 d̄ sterling. Therfore if you wold conuert 124 l̄ 5 s̄ sterling into Duckets, consider that pence is the least value or denomination named in this question, therefore reducing 124 l̄ 5 s̄ into pence, and it maketh 29820 pence: which if you diuide by the pence that a Ducket is worth, which is 70, you shall haue for your quotient 426 Duckets your desire. ‡

Weights. Thus much haue I saide of monie, nowe will I shewe you in like sort the distinction of weightes, after the statutes of Englande, where the least portion of weight is commonly a Graine, meaning a Graine of Corne or wheate, drie and gathered out of the midle of the eare. Of these graines in time passed, 32 wayed iust 1 penie of Troy, and then was but 20 pence in an Dunce. But nowe are there 46 pence in an Dunce, so that there are not fully 14 graines in one penie. But nowe of Dunces after Troy rate (which is the standard of England) 12 doe make 1 pound.

A graine.

A penie of Troy.

An ounce.

Haberdepoise weights. But cōmonly there is vsed another weight called Haberdepoise, in which 16 Dunces make a pound. Therefore when you would reduce Dunces into pounds, you must consider whether your weights be Troy weight or Haberdepoise: and if it be Troy weight you must

must diuide your Ounces by 12 to bring them to pounds, but if it be Haberdepoise, you must diuide them by 16. Nowe againe, there be greater weights which are called an hundred, halfe a hundred, and a quarterne, and also a halfe quarterne, &c. A hundred weight.

Scholer. Why: so there may be reckoned 20 pound, 40 pound, 200 pound and such innumerable.

Master. Al these are numbers of weight, but they haue not common weights made to their rate, as the other haue. And againe, these that I did name are not iust in number as they seeme by their name, for an hundred is not iust 100, but is 112 pound. And so the halfe hundred is 56: the quarter 28, and the halfe quarter 14. And these be the common weights vsed in most things that are solde by weight.

Howbeit there are in some thinges other names, as in wool, 28 pound is not called a quarterne but a Todde: and 14 pound is not named halfe a quarterne but a Stone, and the 7 pound halfe a stone. Other names because they differ in many places, and agree in fewe, I let them passe. Wooll weights. Todde. Stone.

But a Sacke of Wool by the statutes, is limited to be 26 stone. Sacke.

* Nowe in cheese, though it be solde by the hundreth, and by the stone in some places, yet the verie weights of it are Cloues Cheese weights.

and Weyes. So that a Cloue containeth 8 pound: and a Wey 32 Cloues, which is 256 pound, that is 12 score and 16 pound: And so much weyeth the Weigh of Suffolke cheese: And the like weight is or should be the barrel of Suffolke butter. ‡

The Wey of Essex Cheese containeth 16 score, and 16 pound: And so much is also the barrell of Essex Butter.

Measures for liquor.

Now of weights are made other measures, both for graine and liquor. For a pounde in waight maketh a Pint in measure, so that 8 pound (or 8 pintes) doe make a gallon: halfe a Gallon is named a Pottle: ‡ halfe a Pottle is called a Quart, which containeth two pintes. Nowe aboue a Gallon the next measure is a Firken: then a Tertian, a Kilderkin or halfe Barrell, and a Barrell. And by those measures are sold commonly Ale, Beere, Wine, and Oyle, Butter, and Sope: Salmon, Herrings, and Eeles.

A pinte.
Gallon.
Pottle.
Quart.

Firkin.
Tertian.
Kilderkin.
Barrell.

Ale measures.

But as these be vnlike things, so the measure of their vessels doe differ: for the measures of Ale are as followeth:

Of Ale, { the Firken / ỹ Kilderkin / the Barrell } conteineth { 8 / 16 / 32 } gallons.

Of Beere { the Firken / ỹ Kilderkin / the Barrell } conteineth { 9 / 18 / 36 } gallõs

Sope

Sope measures, both Firken, Kilderkin, and Barrel, should be equall to Ale measures. Howeuer the statutes doe limite the weight of euerie of those three vessels being emptie. Sope measures.

A Barrell	to weigh emptie	26	pounds.
Halfe Barrel		13	
A Firken		6½	

Herrings also be solde by the same measures that Ale and Sope be solde by. Herring.

Herrings also are solde by the tale, 120, to the hundred, ten thousand to the last.

Salmon and Eeles haue a greater measure. Salmon & Eeles.

Salmon & Eeles	the butte	holdeth	84 Gallons.
	ỳ barrell		42
	halfe bar.		21
	the firken		10½

Howbeit, some statutes did limit Eele vessells equall with Herring vessels.

Now as for wine vessels they are seldome smaller than hogs heads, which are of 63 gallons: euerie Hogges head is two Barels: yet there are many other wine vessels, but of them all, see this table, and marke the measures one to another. Wine measures.

Of wine and oile.	the Rondlet	holdeth	18½	gallons.
	the Barrell		31½	
	½ Hogs head		63	
	the Tertian		84	
	the Pipe		126	
	the Tonne		252	

Tertians. But you shall marke that there be other kindes of Tertians: for there be Tertians (that is to say) Thirdles of Pipes, of Hogges heads, and of Barrels, as wel of other things as of wine.

Butte. Also of Malueseys, and Secke, &c. the halfe Tonne is not called a Pipe, but rather a But.

And thus much haue I thought méete to tell you at this time.

Scholer. And is that alwaies true?

Master. I haue told you howe it should be, but howe it is I may not say: howe they doe differ daily from their iust measure, that Gagiers can tell you better than I. But I wil let this passe nowe, and speake briefely of the other measures.

Drie measures. And as of weights there did spring the liquide measures, (whereof I spake last) so of the same springeth drie measures: as Pecks, Bushels, Quarters, and such like, whereby are measured corne and like graines: also salt, lime, coales, and other like. And this is the order and quantitie of them.

A pecke. A Pecke is the measure of two Gallons.

A

A bushell containeth foure peckes. A Bushell.
A quarter holdeth eight bushels. Quarter.
A wey containeth fire quarters. Wey.
These are the common names and measures, but in diuers places there be diuers sorts.

The bushell in many places is 2 bushels: but then is the bushell there called a strike. Strikes.
And in some places halfe a quarter is called a Coxnoke. But these diuersities are too many to tell you briefely them all. And againe, sith they are against the lawe and statutes, I count them vnmete to be vsed.

But nowe remaineth yet an other kinde of measures, whereby men meate length and breadth, and thickenesse, and those are an inch, a foote, and such other: whose names and quantities this table sheweth. Measure to meate length, breadth, & thickenes.

3 graines of barly in length make an inch. An inch.
12 Inches make a foote. Foote.
3 foote make a yarde. Yarde.
3 foote and 9 inches make an Elle. Elle.
5 yards and a halfe, make a perch. Perch.
1 perche in breadth and 40 in length, doe make a rodde of lande, which some call a rode, some a yarde land, and some a Farthendele.
2 farthendels make halfe an acre of groũd.
4 Farthendels make an Acre. Acre.
* More 40 rods in length do make a Furlong: 8 furlongs make a mile, which conteineth 320 perches.

So that an English mile grounded vpon the statute, is in length 1760 yards, 5280 fote, and 63360 inches.

Somewhat greater than the Italian mile of 1000 paces, and 5 fote to a pace. ‡

Here might I tell you many thinges else touching measure, and also howe to reduce strange measures to our measures, but because it can not be well done without the knowledge of Fractions, which as yet you haue not learned, I will let them passe til an other time that I haue taught you the knowledge of broken numbers.

The parts of time. A day. An houre. Weeke. Moneth. Yeare.

Scholer. But yet sir of the parts of time I pray you tell me somewhat.

Master. You knowe that a naturall daie hath 24 houres, and euerie houre hath 60 minutes. It nedeth not to tell you, that 7 daies make a weeke, and 4 weekes make a comunon moneth, and 13 moneths make a yeare, lacking 1 day, and certaine houres, and minutes: But of that I shal instruct you hereafter.

Here wil I make an end of Reduction for this time, which though it be counted no kinde seuerall of Arithmetike, yet you see it is no lesse needefull to be knowen, or easier to be done, than of any of the other.

Scholer. Mary sir, it semeth vnto me much harder than any other sort, for it requireth the knowledge of so many thinges: but

but nowe sir when you see time, I am readie to learne forth: for as much of Reduction as you haue taught me, I remember, but and if I doe at any time forget, I shal haue recourse to the tables which you haue set forth for me.

Master. So doe you, for it will not be remembred without exercise. But in as much as you vnderstād so much as we haue intreated of, I wil now instruct you in Progression.

Progression.

ALthough vntill this day the most part of writers haue defined Progression as a compendious kinde of Addition, yet truely it is not so: for Progression (as the verie nature of the worde doth informe any man) is a going forward and proceeding in numbers, and that regularly and orderly, whose place is aptly chosen to be verie neare, or rather next after the exposition of the foure principall parts of Arithmetike, for in it after a most easie maner, are all the foure former parts exercised and practised: and not only Addition, as customeably is done. Which custome hath beene the cause, why it hath so specially beene named a kinde of Addition, and defined to be a quicke

and briefe Addition of diuers summes proceeding by some certaine and reasonable order.

You shall also vnderstand, that there are infinite kindes of Progressions, but for you (as yet) two are sufficient to be exercised in: of which the one I call Arithmeticall, and the other Geometricall.

Arithmeticall progression.

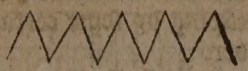
Arithmeticall Progression is a rehearsing or placing downe of many numbers, number after number, in such sort that betweene euerie two next numbers rehearsed or placed downe, the difference, diuersitie, or excesse, be equall and alike.

Scholer. Syr, I thanke you for that you haue both opened vnto me what Progression is, truely, and also why it is here placed. But I pray you with an example make plain your definition.

Master. Examples cannot want, seeing al reasonable creatures naturallie vse the order of one kinde of Arithmeticall Progression (which therefore is also named Naturall) whensoeuer they distinctly doe count or number any multitude by one, saying: 1. 2. 3. 4. 5. 6. whereby the proceeding from number to number, and euerie one surmounting and exceeding his fellowe next before by a like quantitie (which here is 1) declareth the same to be Arithmeticall Progression. And for the more plainnesse, I set it down in this maner.

The

The common excesse. 1 1 1 1 1



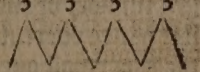
The Progression. 1 2 3 4 5 6

Scholer. This is most euident. And I thinke that I am able to tell you nowe of any Progression Arithmeticall propounded, what is that common excesse or difference whereby it procedeth if this order be kept in it.

Master. What say you of 3. 6. 9. 12. 15?

Scoler. They excede ech other by 3. And that may I set downe in such euident order, as you did your example of Natural Progression, in this wise,

The common excesse. 3 3 3 3



The Progression. 3 6 9 12 15

Master. And do you not also now perceiue, that the whole Table of Multiplication may be made by the order of Progression Arithmeticall? either if you will begin at the first number of any of them on the left hande, and so procede right ouerthwart: or at any of the first numbers of the vpper rowe, and goe directly downeward.

Scholer. I pray you let me consider the thing a litle, and I will answere you.

By

1	2	3	4	5	6	7	8	9	10
2	4	6	8	10	12	14	16	18	20
3	6	9	12	15	18	21	24	27	30
4	8	12	16	20	24	28	32	36	40
5	10	15	20	25	30	35	40	45	50
6	12	18	24	30	36	42	48	54	60
7	14	21	28	35	42	49	56	63	70
8	16	24	32	40	48	56	64	72	80
9	18	27	36	45	54	63	72	81	90
10	20	30	40	50	60	70	80	90	100

By this triall I perceiue it now verie wel: for the commun excesse or difference betweene any two next, is continually as much as the first number of euerie rowe, either from the left hande ouerthwart taken, or from any of the vppermost ouerthwart rowes downward.

Master. Now then if of any such Progression you would spedilie knowe the total sum, much quicklier than by common rules of Addition: first tell howe many numbers there are (which numbers here we call places or parcels) and if they be odde, write their sum downe by it selfe, as in this example, 2, 4, 6, 8, 10, 12, 14, where the numbers are 7, as you may see, therefore set downe 7 in a place alone: then adde togither the first number and the last, as in this example: adde 2 to 14, and that maketh 16, take halfe of it, and mul-

To knowe the totall summe of an Arithmeticall progression.

multiplie by the 7, which you noted for the number of the same places, and the sum that amounteth, is the summe of all those figures added togither, as in this example: 8 multiplied by 7 make 56: and that is the summe of all the figures.

Scholer. That will I worke by an other example. I would knowe how much this sum is, 5, 8, 11, 14, 17, 20, 23, 26, 29. I tel the places and they are 9, that I note. Then I put the first number 5, and the last 29, together, and they make 34. I take the halfe of it, that is, 17, and multiplie by 9, and it maketh 153. That you say is the summe of all the numbers.

Master. So shall you finde it if you trie it.

Scholer. Howe shall I trie it?

Master. By your common Addition: for if you adde all the parcels togither, you shall see the same summe amount, if you did worke well. And that maner of Addition trieth all kindes of summing any Progression.

Scholer. Then can I summe a Progression, if the numbers of the parcels be odde. But what if they be euen: as in this example, 1, 2, 3, 4, 5, 6, 7, 8?

Master. When the number of the parcels is euen, then note that also as you did before, and likewise adde the first summe to the last, and by the halfe of the number of the places doe you multiplie it: as in your example, the par-

parcels are 8, that I note: then adding the first summe to the last, there amounteth 9, that doe I multiplie by the halfe of parcels, that is by 4, and it maketh 36, which is the summe of the 8 parcels.

☞ But if you wil take one rule for these both, doe thus. Multiplie the halfe of the one by the other whole, and the summe will amount all one. For sometime it chanceth that the number of the parcels be odde, so that their halfe cannot be taken: and sometime it chanceth the Addition of the first number and the last, to bring forth an odde number, so that the halfe of it cannot be taken: but they will neuer be both odde.

A general rule.

Scholer. Then I perceiue this, if there be no more longing to it.

Master. This is inough for Progression Arithmeticall: howbeit there is one other maner of Progression called Geometricall, when the numbers increase by a like proportion, that is, if the second number containeth the first, 2, 3, or 4 times, and so forth: then the thirde containeth the second so many times also: and so the fourth the third, and the fifth the fourth, wherefore I set these three examples.

Progression Geometricall.

3,	6,	12,	24,	48,
1,	3,	9,	27,	81,
2,	10,	50,	250,	

Here in the first example you see, that euerie number containeth the other (that goeth

eth next before him) 2 times: and in the second example 3 times, in the third example 5 times. Nowe if you will knowe how to finde easily the summe of any such numbers, doe thus: Consider by what number they be multiplied, whether by 2, 3, 4, 5, or any other, and by the same number doe you multiplie the last summe in the Progression.

Scholer. I pray you worke it by this example, 2, 8, 32, 128, 512, 2048, which I haue framed by proceeding from 2, and continually multiplying by 4.

Master. Then must I multiplie the last summe (which is 2048) by 4 also, and it wil be 8192. Nowe must I bate from this sum the first number of the progression, which here is 2, then resteth 8190, which summe I must diuide by 1 lesse than was the number that I multiplied by. Seeing then I multiplied by 4, I must diuide by 3, so diuiding 8190 by 3, the quotient will be 2730, which is the summe of all the Progression. And nowe to proue whether you can doe the same, I giue you these numbers to adde by this rule, 3, 15, 75, 375, 1875, 9375, 46875.

Scholer. I cannot well tel by what number this Progression doth increase.

Master. In any such doubt, doe thus: Diuide the second number by the first, and the quotient will shewe you the number that engen-

gendreth the Progression.

Scholer. Then is that number in this example 5, for so many times is 3 in 15.

Master. So is it. Nowe worke as I taught you.

Scholer. The last number is 46875, which I multiplie by 5, & it yeldeth 234375, from which I abate the first number of the Progression, that is 3, and there resteth 234372, which I diuide by 4, for that is one lesse than 5, and the quotient is 58593, which is the whole summe of the progression.

Master. If you remember well this, you haue learned the Art of Progression both Arithmeticall, and also Geometricall, which you may prowe either by Subtracting of each number alone from the summe, and so will there nothing remaine: or else by adding together of all the parcels, for so will the same summe amount.

* And nowe for the vse and better vnderstanding of both these rules, I will propone vnto you certaine pleasaunt and necessarie questions Arithmeticall and Geometricall. And to the performaunce of their workings, such necessarie rules and documents, as are requisite for the vnderstanding of them, or any such like.

A certaine mercer solde 20 yards of veluet to be paide in 12 weekes, by Arithmeticall proportion; that is to wit, to receiue the first

weeke

wæke 6 shillings, the second wæke 12 shillings, the third wæke 18 shillings. And so forth encreasing the number of wækes by 6 shillings, till the twelfth and last wæke were expired. The question is howe many pounds he had for his 20 yards of veluet.

To the performance of this question and such other the like, I set forth the 12 payments in such sort, as for example here appeareth in the margent.

Then touching the adding together of these summes without the aide of Addition, according to the rules I taught you in Progression Arithmeticall, I note the number of the places, which are 12, then adding the last number of the Progression which is 72, and the first number together make 78, and multiplying 78 by halfe the number of the places, which is 6, amounteth to 468 shillings, and in pounds maketh 23 ℔, 8 ſ. And so much hath the mercer for his 20 yards of veluet, which is nie about 23 ſ, 5 d, a yard.

6	6
12	12
18	18
24	24
30	30
36	36
42	42
48	48
54	54
60	60
66	66
72	72

Scholer. I understand this worke verie well: But is there any proofe for the iustifieng hereof as you haue of other workes.

Master. The work of it selfe (being so perfectly wrought) that in your proceeding and going forward from number to number, each number exceeding his fellowe by an equall or like quantitie is all that is demaunded for iustifieng of the same; yet notwithstanding because

cause your request is reasonable, I will propone an example for the prowfe hereof.

A certaine man is bounde to pay for 20 yards of veluet, the summe of 23 ℔ 8 s̄. And it is to be paid wéekly in 12 wéekes or termes by Arithmeticall Progression. The question is therefore to knowe with what number the same progression is to be begun and continued in such equall proportion Arithmeticall, that in 12 wéekes the same may iustly be accomplished.

The profe of the last question.

For the solution whereof, and of all such other like, reduce 23 ℔, 8 s̄, all into shillings, which maketh 468 shillings.

Then adde 1 vnto 12 the number of the termes, it maketh 13, which 13 you shall multiplie by halfe the number of the termes which is 6, it maketh 78: then diuide 468 by 78, and you shall find 6 in the quotient which is the true number that shall begin and continue the saide progression. That is to say, the first wéeke, 6 s̄, the second 12 s̄, the third wéek 6 s̄ more, which is 18 s̄, and so euery wéeke as they rise, 6 s̄ more than the wéeke before, as is manifest in the question aforesaide.

A general rule.

A Farme is to be solde to be paide by the wékes in a yere, the first wéeke to pay 4 s̄, the second wéeke 8 s̄, the third wéeke 12 s̄. And so forth encreasing each number by 4 till the number of 52, which are the number of wékes in a yere be expired. The question is,

what

what the price of the farme commeth to.

Scholer. I doubt not but by that you haue already taught me, to ende this question verie well, wherefore I set the Progression with his excesse 52 times.

Master. May stay a while. And here for your further ease, (to abridge you of great labour that appeareth to fall out in this question, and so may doe in many other the like, if a question were proponed of 100 or 200 places or moe, and that this question nor any other the like can be ended, vnlesse you know absolutely what the last number of the Progression at the 52 place is or ought to be) I will giue you a generall rule howe to knowe the last number of any Progression Arithmeticall, aswell as if you had ordinarily proceeded by continuall Addition, till you had come to the last worke, which is this.

Multiplie the excesse by a number lesse by one than the number of the places, and thereto put the first number of the Progression, and you shall haue your desire. A generall rule.

Scholer. This rule is well worth the noting: for if I vnderstand you aright, I consider that my excesse is 4, which I multiplie by 51, which is one lesse than the number of the places, and it maketh 204: whereunto I adde the first number of the Progression which is 4, and then it is 208: which you say is or should be the last number of the Progression.

Master. This is a most approued truely if there were neuer so many places.

Scholer. This rule is so easie, that I were much to blame, if I doe not remember it. For by the benefite hereof I haue such an ease and light into this excellent art, that my first entrance doeth seeme to passe a great many mens further studie and longer continuance.

Master. Many moe considerations could I propound you in these Arithmetical progressions: but these are sufficient for a task, to giue you occasion to thinke that rules of knowledge and artes are infinite capable of enlargement.

Scholer. Happie were I, if I did but wel vnderstand that which is alreadie inuented and written. But these things in my simple fantasie offer themselues to be greatly beneficiall into the aide of Progression. Therefore nowe I will goe forward with your question.

Nowe considering that the 52 ℔ last place is 208, I adde thereunto the first number of the progression which is 4, it maketh 212, which I multiplie by halfe the number of the places, which is 26, ℔ it amounteth to 5512 shillings. And so much is the totall summe or Addition of this progression: which maketh 275 ℔, 12 s, as appeareth here by my tables.

Master. I like well your labour and commend you for your diligence. I will here propone one example more, and therewithall for this

this time will end progression Arithmeticall.

A certaine man bought 20 Elles of Holland to be paide in 17 weekes or termes by progression Arithmeticall. And the first weeke to pay 1 s, 8 d, the second weeke, 3 s, 4 d, the third weeke 5 s, the fourth weeke 6 s, 8 d, and so forth each weeke succeeding 20 pence more than the weeke before. The question is what the summe of his 20 elles commeth to.

Scholer. Because here is mention made both of shillings and pence, I feare there is some harder matter contained herein, than in the other before, therfore I pray you worke it your selfe, and I will diligently marke your labour.

Master. There is no more to be done in this, than in the other before, but because your request is so reasonable, be attentiue vnto me.

First by our generall rule I seeke to finde out the last number of the 17 place, what this progression ought to be. Therefore here in my tables multiplying the excesse 20 by 16, which is one lesse than the number of the tens termes or places, and it commeth to be 320, and thereunto adding the first number of the progression which is 20 pence, all is 340 d or 28 s, 4 d: for so much ought the last number of the paiments to be.

Then finallie to knowe what the whole 17 places amount vnto, I adde the first number

of the progression and the last together, both which make 360: Now because 17 is an odde number, whose halfe cannot be taken, I take the halfe of 360 which is 180: and multiplying 180 by 17, it commeth to 3060 pence, which maketh as you see by Diuision 27 l, 15 s. And so much is the buyer to pay for his 20 elles of Holland, which 3060 pence if you diuide by 20, the number of elles that was bought, you shall finde 12 s 9 d, and so much paide he for an elle one with another.

The Proofe.

A certaine man doth owe 27 l, 15 s, to be paide in 17 weekes or termes by Arithmeticall progression. The question is to knowe with what number he shall begin and continue the progression in such equall proportion, as the same may be truely paide and satisfied in 17 weekes.

The answere.

First I reduce 27 l, 15 s, all into pence: which as you see here in my Tables make 3060 pence, that I let stand by a while.

Then I adde 1 to 17 the number of the places or termes, which maketh 18, which I should multiplie by halfe the number of the weekes or termes, which is 8½, which 8½ multiplied

tiplied by 18, cannot well be done vnlesse you were acquainted with Fractions or broken numbers, therfore you shall let that passe, and multiplie 17 by the halfe of 18 : which is 9, (for that is all one with the multiplication of 8½: and the multiplication of 9 into 17 maketh as you see 153, with which number you shall diuide the 3060 pence before saide: and the quotient bringeth forth 20 pence, which is the first number or payment to begin the progression withall: and so each weeke succeeding to rise 20d more than the weeke before, and thereby in 17 weekes shall the 27l, 15s, be paide : as before was sufficiently declared. Thus much for progression Arithmeticall.

Scholer. Certainely sir I knowe not howe to render you condigne thankes for these benefites shewed me, which me thinketh are so easie, delightfull and pleasaunt, that I count my selfe happie to be in your companie.

Master. I am glad you delight so well herein, which is an arte of wonderfull dexteritie to al sortes of men of what degree or profession soeuer they be. And nowe will I propone a question or two of progression Geometricall.

A mercer hath 12 yards of satten which he valeweth at 16 shillings the yard, and selleth the same 12 yards to another man to be paide as followeth. That is to wit for the first yard to haue one shilling, for the second yard 2 s:

for the third yard 4 s, for the fourth yard 8 s, and so forth doubling each number following till the twelfth and last yard. The question is who hath made the better bargaine of the buyer or seller.

First you may set downe 12 the number of the yards, as you see here in this example. And against each number the number of shillings due to be paide, as the order of Duplation or the multiplication by 2 teacheth.

	s
1	1
2	2
3	4
4	8
5	16
6	32
7	64
8	128
9	256
10	512
11	1024
12	2048

Then resorting to the adding vp or summing of this progressiō, where I consider that the increase of this sum proceded by the multiplication of 2. And therefore after I haue drawen a line vnder the 12 worke, and multiplie the last summe by 2 also, it yeldeth 4096: from whence I abate the first number of the progression which is one, and then resteth 4095: which I should diuide by 1 lesse than I did multiplie by, but seeing it is one, I made not to diuide it: for 1 (as I haue saide before) doth neither multiplie nor diuide, therefore I take that summe 4095 for the whole summe of the shillings, which by Reduction

amoun-

amounteth to 204 l. 15 s. and so much hath the mercer for his 12 yards of satten: which is 17 l. 1 s. 3 d a yard. But I thinke you will buy none so deere.

Scholer. No sir by the grace of God this yeare. ‡

Master. Then what say you to this question. If I sold vnto you a horse hauing 4 shoes, and in euerie shoe 6 nailes, with this condition, that you shal pay for the first naile 1 ob: for the second naile 2 ob: for the third naile 4 ob: and so forth doubling vntill the ende of all the nailes. Now I aske you how much would the price of the horse come vnto.

Scholer. First to knowe the number of the nailes, I must multiplie 6 by 4, and it maketh 24. Then will I do thus: I will write the number of the nailes euerie one in order from 1 to 24, and against each number of the nailes the summe of halfe pence duely, as the order of Duplation or Multiplication by 2 teacheth, and as in this next figure following appeareth.

Then doe I resort to the rule of summing vp this progression, where I consider that the increase of this summe proceedeth by the multiplication of two, as the last example did. And therefore multiplying the last summe by 2 also, and it yeeldeth 16777216, from which 1 I abate the first number which is 1, & then resteth 16777215, which I should

1	1	diuide by 1 lesse than I
2	2	did multiplie: but seeing
4	3	that it is 1, I nede not
8	4	to diuide it, for 1 (as you
16	5	haue before saide) doeth
32	6	neither multiplie nor di-
64	7	uide, therefore I take
128	8	that number 16777215
256	9	for the whole summe of
512	10	the pence, which by Re-
1024	11	duction I finde to be
2048	12	699050s, and 7d ob:
4096	13	that is 34952l, 10s,
8192	14	7d, ob.
16384	15	Master. That is wel
32768	16	done, but I thinke you
65536	17	will buie no horse of the
131072	18	price.
262144	19	Scholer. No sir if I
524288	20	be wise.
1048576	21	Master. Well, then
2097152	22	answere me to this que-
4194304	23	stion.
8388608	24	A Lord deliuered to a

Bricklayer a certaine number of loades of bricke, whereof he willed him to make 12 walles, of such sort, that the first wall should receiue 2 thirdeles of the whole number: and the second 2 thirdeles of that that was left. And so euerie other 2 thirdeles of that that remained: and so did the bricke-

brickelayer: and when the 12 walles were made, there remaineth 1 loade of bricke.

Now I aske you, howe many load went to euerie wall, and howe many loade was in the whole?

Scholer. Why sir, it is vnpossible for me to tell.

Master. Nay, it is verie easie if you marke it well. Marke well that I saide, that euerie wall should receiue 2 thirdeles of the summe that was left. Nowe take away 2 thirdeles from any summe, and you must néeds graunt that that which remaineth is 1 thirdele of the summe last before: example of 9, from which if you take 2 thirdeles, there will remaine 3, which is one thirdele of 9. Likewise from 3 bate 2 thirdeles: and there will remaine 1.

Scholer. This is true, and nowe I perceiue, that the least wall had but two loade of bricke.

Master. And by the same reason may you knowe howe many loade euerie wall had, according as this figure following doth shewe, & likewaies what the whole summe of brickes was: for if you make 12 summes, multiplying by 3, still from the last remainer, as you may see here on the left side of the table, there will appeare all the remainers of the whole wall: and if you multiplie the last of those 12 summes by 2 also, then will that be the summe of the loades which were deliuered to the bricke-

brickelayer.

Againe, if you doe double euerie remainer, as you may see at the right side of this table, those numbers will shewe the summe of loades that went to each wall: whereby you may perceiue, that each wall was 3 times so great as the nert lesser.

Scholer. Loe, nowe it appeareth easie ynough. Nowe surely I see that Arithmetike is a right excellent arte.

Master. You will say so when you knowe more of the vse of it: For this is nothing in comparison to other pointes that may be wrought by it.

Scholer. Then I beseech you sir, cease not to instruct me further in this wonderful cunning.

The remainer after euerie wall.				Loades due to each wall.
1	1	2	2	
3	1	1	6	
9	1	0	18	
27	9		54	
81	8		162	
243	7		486	
729	6		1458	
2187	5		4374	
6561	4		13122	
19683	3		39366	
59049	2		118098	
177147	1		354294	
Summe of the	531441			Loads deliuered.

The

The Golden Rule.

Master.

By order of the Science (as men haue taught it) there should followe next the extraction of rootes of number, which because it is somwhat hard for you yet, I will let it passe for a while, and will teach you the feate of the rule of Proportions, which for his excellencie is called the Golden rule. Whose vse is, by three numbers knowen, to finde out an other vnknowen, which you desire to knowe, as thus. If you pay for your borde for three moneths 16 s, how much shall you pay for 8 moneths.

To knowe this and all such like questions, you shall consider which two of your 3 numbers be of one denomination, & set those two the one ouer the other, so that the vndermost be it that the question is asked of: as in my question 3 and 8 be both of one denomination, for they both be moneths, and because 8 is the number that the question is asked of, I set them one ouer the other, and 8 vndermost, thus, with such a crooked draught of lines. Then doe I set the

3
8

other

other number which is 16, a- gainst 3 at the right side of the line, thus.

And nowe to know my question, this must I doe: I must multiplie the lowermost on the left side, by that on the right side, and the summe that amounteth, I must diuide by the highest, on the left side. Or in plainer wordes thus: I shall multiplie the number of which the question is asked (which is called the third number) by the number of an other denomination, (which is called the second) and the summe that amounteth must I diuide by the summe of like denomination, which is called the first. Then for the knowledge of this question, I multiplie 8 into 16, and there amounteth 128, which I diuide by 3, and it yeeldeth 42 shillings, and 2 s remaineth, which I turne into pence, and they be 24 d, of which the third part is 8 d, so the thirde part of 128 s, is 42 s, 8 d: which summe I write at the right hande of the figure against 8, thus.

The third number. The second number. The first number.

Hereby I knowe, that if three monethes bording doe come to 16 s, that 8 monethes bording will come to 42 s, 8 d: and likewise of any other like question.

But here must you marke, that the first number and the third be of one denomination, and also the second and the fourth, the which

which you seeke: or else be of such denominations, that you in working may bring them into one. As if a man should aske me this question.

* Twelue weekes iourneying cost me 14 French Crownes at 6 s the peece, howe many pounds is that in one yere. Here you see no two numbers of one denomination, but yet in working, you may turne them into like denominations, as thus: turne the one yere into 52 weekes, and the fourth summe will be French Crownes, by the order of the working: Then to knowe this question, multiplie the third summe 52 by the second 14, and the summe will be 728: that diuide by your first number 12: and the quotient will be 60 Crownes: and 8 crownes remaining: which if you turne into shillings they will be 48 shillings, which if you doe diuide by your first number 12 the quotient will be 4: which signifieth 4 s, put these 60 French Crownes (which make 18 pounds) with the 4 shillings: for the summe that aunswereth to the question: and it is the iust expences of a yeare: And the summe will be thus.

12 — 14
52 — 60 — 4 s.

And take this euermore for a generall rule touching this whole Arte, that the doubtfull or vnknowen number that you would be resolued of, shall alwaies be set in the thirde place. Note also the first number & the third, must

muſt euer be of one nature and denomination, or elſe muſt in worʒking be brought to like denomination, and then of neceſſitie muſt the other number be in the ſecond place. ‡

Remember alſo, that the place of the firſt number is the higheſt on the left ſide: and the place of the ſecond right againſt it on the right ſide: the place of the third number is vnder the firſt, as by thoſe examples you haue ſeene.

Scholer. This I truſt I can doe.

Maſter. But and if the queſtion be aſked thus: In 8 weekes I ſpend 40 s̄, howe long will 105 ſhillings ſerue me? Here you ſee that 8 weekes anſweres himſelfe, and ſaieth 40 ſhillings. But howe long time 105 ſhillings will ſerue, you knowe not. Therefore you ſhall ſet 105 in the third place, according as I tolde you euen nowe. And the firſt place muſt alwaies be of the ſame nature or Denomination, that the third is of, which here is 40. Then muſt 8 needes be that other. Nowe multiplie 105 by 8, and it wil be 840, which if you diuide by 40, it will yelde 21, which is the fourth number, and ſheweth howe many weekes 105 ſhillings will ſerue, if you ſpende 40 s̄ in eight weekes.

The figure of this queſtion is this: as if you ſhould ſay: If 40 s̄ ſerue for 8 weekes, 105

Shillings.	*Weekes.*
40	8
105	21

ſerue

serue for 21 weekes.

Other diuersities there be of working by this rule, but I had rather that you woulde learne this one well, than at the beginning to trouble your minde with many formes of working, sith this way can doe as much as all the other, and hereafter you shall learne the other more conueniently.

* And for your further aid and instruction to make you better acquainted with this Golden Rule, I haue here proponed 6 questions and their answeres, which I thinke most conuenient and meete to preferre the desirous to perfect vnderstanding. The first foure are al braunches of one question sprung out of the best tree, (for a young learner to taste of) that groweth in this Ground of Artes, for that no maner of question in the Rule of 3 whatsoeuer it be, can be proponed, but it must be comprehended vnder the reason or stile of one of these foure.

The questions be these.

If 15 elles of Cloth cost 7 l, 10 s: what comes 27 elles to at that price: Aunswere. 13 l, 10 s.

If 27 elles cost 13 l, 10 s: what are 15 elles worth. Answere. 7 l, 10 s.

If 27 elles cost 13 l, 10 s: how many elles shall I haue for 7 l, 10 s. Answere. 15 elles.

If

If I sel 15 els for 7 ℔, 10 s: how many els are to be deliuered for 13 ℔, 10 s. Ans. 27 elles.

If 8 pound of any thing cost 16 s, 6 d: what money is to be receiued for 49 pounde: Answere. 5 ℔, 1 s, 0 d, ¾.

If 4 ℔ of any thing cost 17 d: what money will 8765 pounde of that commoditie cost. Answere. 155 ℔, 4 s, 3 d, q.

Of al which questions I omit the work of purpose, y you shold whet your wit therby at conuenient leisure, to climbe each braunch, and gather the fruite of them, and doe minde nowe, before we make an ende of this rule, to giue you some instruction of the Backer rule of 3, whose order is quite contrarie to this that you haue learned. ‡

For in this rule hitherto euermore loke howe much the third number is greater than the first, so much the fourth number is greater than y second. And contrarie waies: loke how much the first sum is greater than the third, (if it doe chaunce so) so much is the seconde summe greater than the fourth. But in this rule, there is a contrarie order, as this: That the greater the third summe is aboue the first, the lesser the fourth summe is beneath the second: and this rule therefore you may call the Backer rule, as in example.

The backer Rule.

Question of buying cloth.

If I haue bought 30 yards of cloth, of 2 yards breadth, and would buie canuas of 3 yardes broade to line it withall, howe many

yards

yards should I neede?

Scholer. Why, there is none so broade.

Master. I doe not care for that, I doe put this example onely for your easie vnderstanding: For if I should put the example in other measures, it would be harder to vnderstand. But nowe to the matter: If you would knowe this question, set your numbers as you did before: but you shall multiplie nowe the first number by the second, and that ariseth thereof, you shall diuide by the third: which thing if you doe here, I meane if you multiplie 30 by 2, it will be 60: which sum if you diuide by 3, there will appeare 20: whereby I knowe, that if 30 yards of cloath of two yardes broade,

	Breadth.	*Length.*
should be lined with canuas of three yards broad,	2	30
20 yards of canuas wold suffice, as this figure sheweth.	3	20

And nowe because ye found fault at my example, howe say you, perceiue you this?

Scholer. Yes sir, I suppose.

Master. Then answere me to this question: how many elles of canuas of elle breadth, will serue to line 20 yards of Say, of three quarters broade.

Scholer. In good faith sir, I cannot tell, for I knowe not howe to bring the summes to like denominations.

Master. Then will I tell you: sith there is mention here of quarters, and againe euerie one of the measures both elles and yards may be parted into quarters, doe you part them so both in the breadth and length, and then put forth the question by quarters.

Scholer. Then I shall say thus. How many quarters of canuas of fiue quarters broad, will line 80 quarters of 3 quarters broade.

Master. Now answere to the question.

Scholer. First I wil set them downe in their forme thus, for 5 is ioyned with the question, and is therfore the third number: then is 3 the number of the same denomination, I meane because they be both referred to breadth. Nowe I multiplie 80 by 3, and it is 240, which I diuide by 5, and it yeeldeth 48. Then say I, that 48 quarters of 5 quarters broade, will suffice to line 80 quarters of 3 quarters broade.

Breadth.	Length.
3	80
5	

Master. Turne the quarters againe into elles and yards.

Scholer. Then I say, that 9 elles & three quarters of a yard of elle broad will serue to line 20 yardes of three quarters broade, as this figure sheweth.

Breadth.	Length.
3	80
5	48

* Master. Nowe what say you to this que-

question: I lent my friend 400 l for 7 moneths, howe much money ought he to lende me againe for 12 moneths, to recompence my courtesie shewed him. Can you aunswere to this?

Scholer. Yes Sir I suppose, for I will set downe my Numbers thus: where I multiplie 7 into 400, and it maketh 2800, which I diuide by 12, and it yeldeth 233 l, and there is 4 l remaining of my Diuision, what shall I doe therewith.

Moneths.	Pounds.
7	400
12	

Master. Turne that same 4 l into s, and then diuide it by 12 as you did before.

Scholer. Well sir it shall be done, so haue I 6 s for my quotient, and yet remaineth 8 s vpon my diuision.

Master. You must also reduce that 8 s into pence, which maketh 96, and diuide that also by your first diuisor.

Scholer. So haue I done, and I finde 8 pence for my quotient, and nothing is left.

Master. This must you alwaies do, when any thing remaineth vpon your diuision: whether it be money, weight, measure, or any kinde of thing whatsoeuer. This rule is so profitable for all estates of men, that for this rule onely (if there were no more but it) all men were bound highly to esteeme Arithmetike.

By this rule may a Captaine in warres worke many things, as Master Digges in his Stratiaticos doth declare: Onely nowe in this my simple Addition, for a taste and incouragement, I will inlarge the Author with a question or two more, wishing you, and euerie my Countriemen or Gentlemen whatsoeuer, that by nature be any thing giuen to Militarie affaires, to be familiar and acquainted with this excellent Art, the which he shall finde not onely at the Sea, but also in the Campe and Fielde seruices, abounda[n]tly to aide him, either in fortification, paying of Souldiers wages, charges of ordinance, poulder, shot, munition, and instruments whatsoeuer, as for example.

Question of prouision touching an armie.

If it should chaunce a Captaine which hath 40000 Souldiers to be inclosed with his enemie, that he could haue no fresh purueyance of victuals, and that the victuals which he hath, would serue that armie but onely 3 moneths, howe many men should be dismisse, to make the victuall to suffise the residue 8 moneths?

Schol. As you taught me, I set the numbers, thus, saying: If 3 moneths suffise 40000, to howe many will 8 moneths suffice?

Moneths.	Men.
3	40000
8	

To knowe this, I multiplie the first number

ber 3 into the second 40000, and it yeldeth 120000, which summe I diuide by 8, and there will be in the quotient 15000, which if I doe subtract from 40000, the remainer wil declare that he must dismisse 25000 as this figure sheweth.

Moneths.	*Men.*
3	40000
8	15000

* Master. Nowe aunswere me to this question: If 136 Masons in a moneth be able to builde a fort, to preserue the souldiers from the enemie, and such expedition requireth that I would haue the same finished in eight daies, howe many workemen say you is there to be appointed?

Schol. As you taught me I set the numbers thus, saying: If 28 days require 136 Masons, what number of men by the like proportion will 8 daies require.

28	136
8	

To know this I multiplie the first number 28 into 236: and it yeldeth me 3808: which I diuide by 8. And my quotient is 476, which is the iust number of Masons that shall supplie this worke. And nowe me thinke these questions are verie easie.

Master. Truely if you take delectation herein, you shall finde this Art not onely easie, but wonderfull pleasaunt and profitable. Nowe therefore one question more I will pro-

pose, and so leaue of this rule in whole numbers, vntill we come to the vse of it in broken numbers, for had you the vnderstanding of broken numbers perfectly not onely in this rule, but in all other, the question that in sight or apparance seemeth to be 100 times more harder to absolue, may thereby be wrought as sone, or soner than this.

Scholer. Your words doth greatly encourage me to be studious to attaine whole numbers, but might I once attaine to be a practitioner in broken numbers, I should thinke my selfe happie.

Master. What say you then to this question: If 48 Joyners in 2 daies make 200 light horse mens staues (estéeming they worke but 12 houres a day) and such néede requireth that 384 Joyners are set to the finishing of those 200 staues, in what time say you, will they make them vp.

Scholer. I sée héere that I must turne my 2 daies into houres. And so doing I set my numbers thus.

48 — 24
384 —

Saying, if 48 men are 24 houres, 384 men will make an ende quickely. For it is grounded vpon an olde prouerbe, many hands make quicke spéede.

I multiplie 48 into 24, and it amounteth to 1152, which I diuide by 384: and my quotient

quotient is 3 houres which is my desire.

I take this for a note worthie the marking, either in the rule of three, forward, or backeward, when the two numbers are multiplied together, the product is of the same nature and denomination that the second number is of. ‡ Note.

Master. Well, sith you perceiue now the vse of this rule, I will shewe other which ensue of the same, and first the double rule, which is so called, because there is in it double working, by which thing onely it differeth from this. The double rule.

Scholer. Then by an example I shall vnderstand it well ynough.

Master. So shall you, and let this be the example: If the cariage of 1 ℄ weight (that is to wit 112 pounde) 30 miles, doe cost 12 d, howe much will the cariage of 5 ℄ weight cost, being caried 100 miles? Of cariage.

Scholer. I pray you shew me the working of it.

Master. You must make 2 workings of it: the first thus. If 1 ℄ weight cost 12 d, howe much will 5 ℄ weight cost? Set your figure thus.

℄ weight.	*Pence.*
1	12
5	

And multiplie 5 by 12, and thereof amounteth 60, which if you diuide by 1, the quotient will be still 60, that is the price of 5 ℄

weight for 30 miles.

Then begin the second worke, saying: If 30 miles cost 60 d, how much will 100 miles cost? Set your figure thus.

Miles. Pence.
30 — 60
100 —

Then multiplie a 100 by 60, whereof amounteth 6000, which being diuided by 30, will yeeld 200 d. Then you may say that so many pence shall cost the cariage of 500 pound weight 100 miles, after the rate of 12 pence for the 100, caried 30 myles.

Scholer. Nowe I perceiue it also.

* Master. These and such other like questions of the double rule of 3, are to be aunswered much sooner, at one onely working by the rule of 3 composed of 5 numbers, which anon I will shewe you, and then when you haue the vse thereof, you may vse which way you thinke good.

Scholer. Sir I thanke you much for your courtesie, and I long now till this rule be ended, that I may see howe I shall behaue my selfe with that newe rule of 5 numbers, for that I haue euer since you taught me hitherto, in the Golden Rule both forwarde and backeward wrought but with three numbers onely. ‡

Question of sowing.

Master. But yet a while we will goe on forward with this double rule of 3: therefore aunswere me to this question: 30 bushels of wheate

wheate sowed, yelded in one yere 360: how many will 80 bushels yelde in 7 yeare? I meane sowing euery yere of those 7, still 80 bushels.

Scholer. First I say, that if 30 bushels will yeld 360 in 1 yere, then 80 bushels will yeld 960 in 1 yeare. Then for the second worke I say: If one yere yeld 960, then 7 yere will yeld 6720: as these two figures doe shewe.

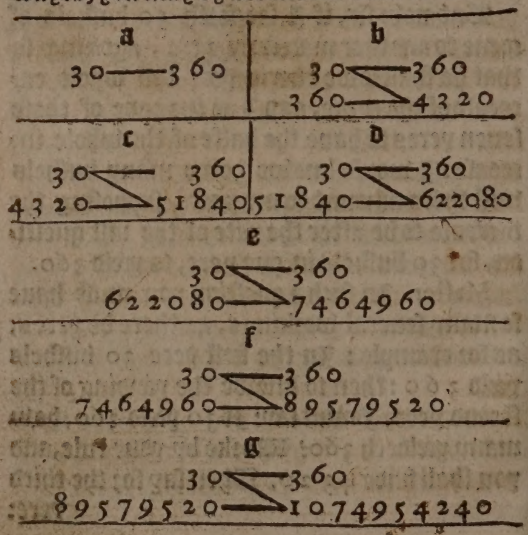
Seede.	Encrease.	Yeres.	Encrease.
30	360	1	960
80	960	7	6720

Question of corne.

But now sir, if I set forth 30 bushels of corne to another man for 7 yere, agreeing so that he shall sowe euerie yere the whole encrease of the corne, and I at the ende of those seuen yeres to haue the halfe of the whole increase: I would knowe howe many bushels will there amount to my part, supposing the increase to be after the rate of the last question, for 30 bushels in one yere, to yeld 360.

Master. In such a question you must haue so many seueral workings, as there be yeres, as for example: In the first yere 30 bushels yeld 360: then to knowe the yelding of the second yere, I must say: If 30 yeld 360, how many yeldeth 360? Worke by your rule, and you shall finde 4320. Then say for the third yere:

yere: If 30 yéld 360, how many wil 4320 yélde? you shall haue 51840, and so euery yere multiplying the whole increase by 360, and diuiding it by 30, the increase of the next yere wil amount, as these 7 figures following doe orderly declare: where I haue set 7 letters for the 7 yeres, of which the first is set without art, because that is the encrease which you doe presuppose: and the last number of each other doth shewe the increase of the yere that it standeth for, which the letters doe declare, so that the increase of the seuen yere, is 1074954240 bushels: how many quarters that is, and also howe many wayes, you may by Reduction some finde.



Nowe with one question more I wil proue you. If 6 Mowers doe mowe 45 acres in 5 daies, howe many mowers will mowe 300 acres in 6 daies?

Scholer. If 45 acres do require 6 mowers, then 300 acres requireth 40. Nowe againe: if 5 daies require 40 mowers, then 6 daies needeth but 33 mowers.

Master. Why doe you not make mention of the 2 that remaineth in the last Diuision? for the last part of the question is wrought by the Backer rule, where the first number 5, is multiplied into the second that is 40, whereof amounteth 200, which if you diuide by the third number 6, the quotient will be 33, as you saide, but then will there remaine 2, which cannot well be diuided into 6 partes: howbeit, you may vnderstand by the firt part of 2, the third part of one mans worke, which you must put to the 33, or else you must say, that 33 workemen will ende all the 300 acres in 6 daies, saue two mens worke for one day, or 2 daies worke for one man. But such broken numbers called Fractions, you shall hereafter more better perceiue, when I shall wholly instruct you of them.

Master. Yet one question more of fielde matters I will propone, and so I will make an end of this double Rule of 3.

Scholer. With all my heart sir I thanke you, and I will dispatch it as sone as I can, be-

because I would faine see the order of the next rule of 5 numbers.

Master. If a Captaine ouer a band of men, did set 300 Pioners a worke, which in 8 houres did cast a trench of 200 Rods: I demaund howe many Labourers will be able with a like trench in three houres to entrench a campe of 3500 Rods.

Scholer. I thinke I am now in the Backhouse ditch, for I knowe not well which way to goe about it: And besides that, truely I thinke I shall neuer come to preferrement that way my growth is so small.

Master. You knowe not howe God may raise you hereafter by knowledge and seruice into the fauour of your Prince, for the auaile of your Countrie.

Example for Nauigation: Sir Frauncis Drake, a man greatly honoured for his knowledge, is not the tallest man, and yet hath made as great aduenture for the honour of his Prince and Countrie, as euer English man did.

Scholer. Sir, I thanke you for your good encouragement. My minde, though I be litle, is as desirous of knowledge, as any other: I haue ponered now a litle of it, and thus I set forth the worke.

Rod.	Men.
200	300
Rod.	
3400	

Say-

Saying, if 200 rod require 300 men, what shall 3400 rod require? I multiplie 3400 by 300: and it yeldeth 1020000: which I diuide by 200, and my quotient is 5100 men.

Then must I say for my second worke, if in 8 houres 5100 men be able to discharge it, howe many shall performe the same in 3 houres? Nowe if I would worke by the Golden Rule of proportion forward, I should find a lesse number of men, because 3 houres is lesse than 8 houres: but because reason teacheth me that the lesser the time is, wherein the trench must be made, the more labourers I ought to haue, thereupon I vse nowe the Backer Rule, as in example. And I haue in my quotient 5000. So many Pioners must I haue, to entrench the Campe in three houres.

Master. You haue aunswered the question verie artificially: And truely I commend you for your diligence and apt vnderstanding: and nowe according to my promise, I will (in whole numbers) giue you a litle taste of the Rule of 3 compounded of 5 numbers.

The

The Rule of 3 compound of 5 Numbers.

THis Rule of 3 compoſed, is diſtinct for moſt needfull queſtions into two ſeueral parts or worckings: And there belongeth vnto it alwaies 5 numbers, whereof in this rule being the firſt part, the ſecond number and the fift are alwaies of one nature and like denomination, which rule is to be wrought thus: you muſt multiplie the firſt number by the ſeconde, and that ſhall be your diuiſor: Then againe, multiplie the other three numbers, the one by the other, & their product ſhal be your diuidend.

And nowe according to my promiſe, we will firſt worke the queſtion of weight and cariage which I deliuered you in the double rule of 3: to be abſolued by this rule, which was this.

If the cariage of 1 C weight 30 miles coſt 12 d, what will the cariage of 5 C weight ſtand me in being caried 100 miles?

C. weight.	*Miles.*	*Pence.*	*C. weight.*	*Miles.*
1	30	12	5	100

Nowe marke well howe theſe 5 numbers ſtand

stand: Then multiplie the first number by the second, as 30 by 1, which maketh but 30, and that number keepe for your diuisor: Then multiplie the other 3 numbers the one into the other: that is to wit 12 by 5, which maketh 60, lastly 60 by 100, which as you see here in our Tables ariseth to 6000, which 6000 you shall diuide by the product of the two first numbers, which here is 30. And as you see there is found 200 pence, which is the duetie that you ought to pay for the cariage of 5 C weight 100 miles, after the rate of 12 d a C, and agreeth with the conclusion of the double rule of 3.

Scholer. Sir I thanke you, it is euen so.

Master. Yet note this for a generalitie in this rule, loke what nature or denomination your midle number is of (which here are pence) and of the like denomination or nature is alwaies your quotient. Note.

Scholer. Well nowe and it please you by your patience, I will see how I can ende the question then next following of 30 bushels of wheat sowed, which in one yere yelded 360: how many thē wil 80 bushels yeld in 7 yere, sowing euerie yere of those 7 stil 80 bushels, and according to your reasons I set my numbers thus.

Bushels.	*Yere.*	*Bushels.*	*Bushels.*	*Yeres.*
30	1	360	80	7
			80	
			28800	
			7	
			201600	

Where I multiplie 30 by 1, and it maketh 30 my diuisor: then multiplying the other 3 numbers the one into the other, as here appeareth in my tables: they make 20160: which I diuide by 30: And my quotient is 6720 bushels my desire, for so much also it came to at two workings by the rule of 3.

Master. Yet one question more I will propound vnto you and so leaue this rule, till it please God hereafter, that I may make you worke it in broken numbers.

What comes the interest of 258 l for fiue moneths to after the rate of 8 l taken in the 100 l for 12 moneths.

Scholer. Sir this is yet within the compasse of some reasonable balance therefore to minister equitie in this case, I will see howe I can worke the same, which I set downe thus, praying you if I haue not done well to shew me mine errour.

l	moneths.	l	l.	moneths.
100	12	8	258	5

Master. Proceede, you haue done very wel.

Scholer Then I doubt not by the grace of God but to ende it: I multiplie 100 by 12, it yeeldeth 1200: and the 3 other numbers multiplied together, produceth 10320: which I diuide by 1200: and my quotient is 8 pounds. Then according as you haue taught me heretofore, I turne the 720 l that is left, into

into shillings: and diuiding it by my first number, my quotient is 12 s. So I answere that the lone of 258 l for 5 moneths, after the rate of 8 l in the 100 l for a yere, comes to 8 l, 12 s.

Master. You say true, I commend your diligence, nowe behold the maner of the second part of this rule.

Master. In the second part of this rule of 3 composed: the third number is like vnto the first. And the rule is to be wrought as thus: you shall nowe contrarie to the last rule multiplie the third number and the fourth together: and that product shall be your diuisor. Then multiplie the fift by the second, and the product thereof by the first: and that is the number that shall be diuided. For example I propound this question, for a prowfe of my last question of interest. A merchant hath receiued 8 l, 12 s, for interest of certaine mony for 5 moneths terme, which he receiued after the rate of 8 l in the 100 l, for a yere, The question is nowe howe much monie was deliuered to raise this interest: Behold therefore the manner, howe the question is sette forth.

The second part of the rule of 3 compound.

The prose of the last question.

l.	moneths.	l.	moneths.	l.	s.
100	12	8	5	8	12

Scholer. Sir I perceiue it verie well: and according to the doctrine which you prescribed for the working thereof: if it please you now

it is set downe, I thinke I can followe the worke.

Master. May stay a while, and afore you worke marke well howe I deliuer a reason for the perfect vnderstanding of this rule, which is thus: If 8 l. in 12 moneths doe yéld me 100 l. to take 8 l. 12 s. for 5 moneths, must néedes yéld a great deale more.

Note.

So vpon the knowledge that I haue in this Art, the first part of this rule is answerable to the rule of 3 forward: And this latter part accordeth to the rule of 3 backeward.

Scholer. Sir I yéld you most heartie thankes for these your last instructions, they haue giuen me great light into these two rules, whereby I may the better by deliberation conceiue howe to vse them hereafter, when occasion shall require.

Master. You say wel, goe to nowe if you will, and trie your cunning in the question: But this note take with you by the way, in as much as here is mention made of shillings: turne all your money as you worke into shillings, for your more ease in worke.

Note.

Scholer. If it please you to behole me a little, I will quickely ende it: for I haue but my first: my second: and my last number to be multiplied together for my diuidend: And my third into my fourth for my diuisor.

100 l.

£	Moneths.	£	Moneths.	£	s
100	12	8	5	8	12
20		20		20	
2000		160		172	
12		5			
4000		800			
2000					
24000					
172					
48000					
168000					
24000		4128000			
4128000		8	00		

Which 4128000 I diuide by 800, and my quotient is 5160 shillings, which in pounds yéldeth 258, my desire.

Master. I will here for this time in whole numbers ende this rule, and I wil instruct you in the rules of Felowship. You may at your conuenient leisure for your exercise worke the same by the rule of 3 at twice. And for your aide and incouragement therein, I set downe here a proper howe to applie it.

Moneths. A	£	s	Pound. B	£
5	8	12	8	100
12	4128 s.		4128 s.	258 £.

The Rule of Fellowship.

BVt nowe wil I shew you of the rule of Felowship or Companie, which hath sundrie operations, according to the diuerse number of the companie. This rule is sometime without difference of time, and some times there is in it difference of time. First I will speake of that without difference of time, of which let this be an example.

Foure Merchaunts of one companie made a banke of money diuerslie, for the first laid in 30 l, the second 50 l, the third 60 l, and the fourth 100 l, which stocke they occupied so long, till it was encreased to 3000 l. Nowe I demaund of you, what should each man receiue at the parting of this money.

Scholer. I perceiue that this rule is like the other, but yet there is a difference, which I perceiue not.

Master. Then wil I shew it to you. First by Addition you shall bring all the particular summes of the Merchaunts into one summe, which shall be the first summe in your working by the Golden rule, and the whole sum of the gaines by that stocke shall be the second summe. Nowe for the third summe, you shall set

set the portion of each man one after an other, and then worke by the Golden rule, and the fourth sum wil shew you each mans gaines: as in example.

The parcels of those foure Merchaunts make in one summe 240 l: set that in the first place, the gaines in the second, & the first mans portion of stocke in the 3 place thus.

240 — 3000.
30 —

Now multiplie the second by the third, and it will be 90000, which you shall diuide by 240, and there will appeare 375 l, thus.

240 — 3000
30 — 375

And that is the gaines for the first man.

Nowe for the second man, set the 50 l, that he brought, in the third place, and worke as before: and his part wil be 625 l, as this figure sheweth.

240 — 3000
50 — 625

Likewaies for the third man set his money which was 60 l, and his part of gaines will be 750 l, as here appeareth.

240 — 3000
90 — 750

And so for the fourth man, if you set his sum which is 100 l, his gaines will be 1250 l, as the worke wil declare.

240 — 3000
100 — 1250

Scholer. This I perceiue: but is there

any way to examine whether I haue well done or no?

Master. For the triall hereof adde together all their 4 portions, and if their Addition make the whole summe of the gaines, then is the worke well done.

Scholer. That will I trie by and by, the foure parcels are these, which added together, make 3000, which is the iust summe of money that they gained, whereby I knowe the worke is well done.

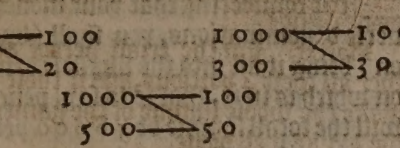
	375
	625
	750
	1250
	3000

Master. Wel, nowe an other example wil I put to you, not of gaines, but of losse: for one reason serueth for both.

If three merchaunts in one shippe and of one fellowship, had bought marchandise, so that the first had laide out 200 l, the seconde 300 l, and the third 500 l, and it chaunced by tempest that they did cast ouer board into the sea marchandise of the value of 100 l, howe much should each man beare in this losse?

Scholer. If I shall doe in this as you did in the other question, then must I ioyne their three portions together, 200, 300, & 500, which maketh 1000. Then say I, if 1000 lese 100, then shall 200 lose 20, & 300 shall lose 30, and 500, shall lose 50, as by these

these three figures it doth appeare plaine.



Master. Wel, sith nowe you haue done these, I will propound a question of more importance, which shall make you not onely the abler to vnderstand this rule, but also it will greatly aide you in the next rule of Fellowship with time, if such neede be that your money be of diuerse denominations.

For this may not be forgotten in all such questions: if the number be of diuerse kindes, you must by Reduction bring it into one kind, that is to say, to the least value that is named in the question. And likewaies shall you doe, if the time be of diuerse kinds, as some yeres, some moneths, weekes & daies, you shal make al moneths, weeks or days according as ỹ least name of time in ỹ question is: As for example. Note.

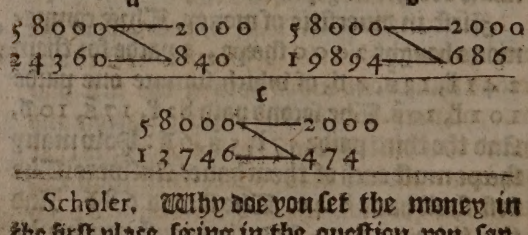
First in diuersitie of money. Three companions bought 2000 sheepe, & paide for them 241 l, 13 s, 4 d, of which summe one paide 101 l, 10 s. The second paid 82 l, 17 s, 10 d, And the third paide 57 l, 5 s, 6 d: How many sheepe must each of them haue? Answere: The first shall haue 840. The second 686. And the thirde 474. And that must you worke Question of sheepe.

Solution. thus.

First considering that your money is of diuerse denominations, you shall (by Reduction) bring it all into the smallest denomination which is in it, that is to say, pence, and so will the totall summe be 58000 pence.

Now, if you turne each mans money into pence also, the first mans sum will be 24360 pence: The second mans sum 19894 pence. And the third mans money will be 13746 pence.

Nowe to knowe howe many sheepe euerie man shall haue, let the whole sum of money that is 58000 pence be set in the first place, & in the second place set the number of sheepe, and then orderly in the third place set each mans money, and then multiplying the third and the second summes together, and diuiding that that amounteth by the first, there will appeare the number of sheepe that each man ought to haue: as these three figures do shewe.



Scholer. Why doe you set the money in the first place, seeing in the question you say,

2000

2000 shéepe cost 58000 d, and not thus, 58000 d, cost 2000 shéepe.

Master. You remember, I taught you at the beginning of this Golden rule, that the first and third numbers must be of one name, and of like things: and euermore the number that the question is asked of, must be set in the third place. Nowe is the question plainely this: If foure men bought 2000 shéepe for 58000 pence, howe many shéepe shall each man haue.

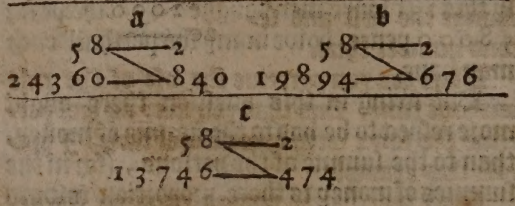
But séeing in this question there ought more respect to be had to the summe of money, than to the summe of the persons, (for in the summes of money is there proportion toward the shéepe, and not in the number of persons) therefore must we turne the question thus.

If 58000 pence bought 2000 shéep, how many did 24360 buie? Againe, how many did 19894 d buie? and howe many bought 13746 pence?

Scholer. I perceiue it reasonable, and so shall I doe in all like questions.

Master. Euen so. But for easinesse of the worke marke this: Whensoeuer the first and second numbers haue ciphers in the first places, you may both in the multiplication and in the diuision leaue out those ciphers, so that you leaue out like many out of both summes, as in this question the first number 58000 hath thrée ciphers, and so hath the second, that is Note.

is 2000: therefore cast away their ciphers, and so will the first number be 58, and the second 2: set them in their places, and worke according to the rule, and you shal perceiue that it will be all one, sauing that this is the shorter and easier way, as these three figures doe shewe.

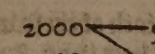


And this you see is both easier, and also the more certaine way to knowe the aunswere to this question.

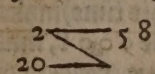
Scholer. Trueth it is as you say: but sir, me seemeth I might aske a further question here, not onely how many sheepe each man should haue, but also what euerie sheepe cost.

Master. That question doeth not onely belong to this rule, but may also be discussed by Diuision, especially if the questions number be one onely: as thus. Diuide the total sum 58000 pence by 2000 (or 58 by 2, omitting the ciphers) and the quotient will be 29 pence, that is 2 s, 5 d, howbeit, by this rule you may doe it, and best when the number of the question doth exceede 1: as if I should aske this

this question, 2000 sheep cost 58000 d, howe much did 20 cost? Then shall I set my figure thus.



And doing after the rule, there wil amount 580 pence, that is 2 l, 8 s, 4 d: the price of one score: But if you will vse that easie way that I did teach you, you may change the first and second number thus.



Thus doe you perceiue the vse of the rule without time, And that you may as wel perceiue the same with diuersitie of time, I propose this example. The rule of fellowship with time.

Foure merchants made a common stocke, which at the yeares ende was increased to 35145 l. Nowe to knowe what shalbe each mans portion of gaines, you must know each mans stocke and time of continuance. Question of a banke

The first man of these foure laide in 669 l, which he did take from the stocke againe, at the end of 10 moneths. The second man laid in 810 l for 8 moneths. The third laide in 900 l for 7 moneths. And the fourth laide in 1040 l for 12 moneths.

This question shal you examine as you did ỳ other before, sauing that whereas in the third place of ỳ figure you did set ech mãs sum alone, here you shal set the same being multiplied by the number of their time, & likewise in the first place of the figure you shal set ỳ number which Note. A general rule.

amoun-

mounteth of their whole summes so multiplied by their time, and added into one whole summe as thus.

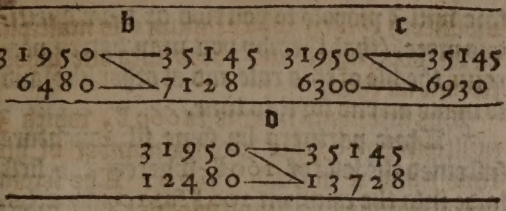
The first mans summe is 669 l, which I multiplie by 10 (that was the number of his time) and it maketh 6690. The second mans summe 810 l multiplied by 8, (which was his time) maketh 6480. The third mans summe 900 l, multiplied by 7 (for that was his time, yeldeth 6300. The fourth mans summe was 1040 l, and his time 12, multiplie the one by the other, & it wil be 12480.

These foure summes thus multiplied by their time, must be set orderly in the thirde place of the figure: and in the first place must be set the whole summe of all foure, which is 31950, and the gaine must be in the second place, which is 35145. Nowe to ende the question, I say first: If 31950 did get 35145, what did 6690 get? Aunswere, 7359 l, as by this figure appeareth.

		a
31950	35145	
6690	7359	

Likewise the second man had to his part 7128 l, the third must haue 6930 l. And the fourth man shall haue for his part 13728 l, as these figures doe partly declare.

Scho-



Scholer. This I like verie well: but what prooфe is there of this worke?

Master. The same that I taught you for the other, to adde all the portions together, and if they agree to the whole summe, then is your worke well done.

Scholer. That wil I proue in this example. The foure parcels are these, which if I adde together, there wil amount 35145, and that was the whole sum: whereby I perceiue the worke is well done.

	7359
	7128
	6930
	13728
	35145

Master. If it fall out otherwise, be sure it is not well.

Scholer. Then doe I vnderstand this worke also verie well, but what haue I nowe to learne.

Master. There are many other excellent parts behinde, of which I will not as nowe, make mention, because that without the knowledge of Fractions they cannot be duely taught, and much lesse vnderstanded. Therfore

fore will I propose to you two or three questions more, (that thereby you may better perceiue the vse of this rule and al other like) and so make an end for this time.

* Three partners by some ill aduenture sustained the losse of 160 l, whereof the first laide into the common 200 l for 10 moneths. The second laide in 350 l, and the third 100 l, but for howe long the 2 latter is vnknowen. But breaking of their partenership, the first found himselfe a loser 80 l, the second 56, and the third 24 l. The question is for howe long time was the money of the 2 latter in companie.

For the solution hereof and of such other like, you must also multiplie the first mans 200 l that he put into the stocke by his time of continuance, which was 10 moneths, and it maketh 2000: wherefore now I affirme if his money that lost 80 l, multiplied by his time, make 2000: what shall his money make that lost 56 l, and his that lost 24 l, which 2 numbers I commit to the triall of the rule of 3 at 2 workings thus: If 80 giue 2000, what giueth 56? And againe, if 80 giue 2000, what giueth 24?

80 — 2000
56 — 1400

80 — 2000
24 — 600

To conclude, if you now diuide 1400 the second mans proportion by 350, which was his

his stocke that he laid into companie, you shal finde in your quotient 4 moneths, and for so long time did the second man put his money into the common stocke.

Lastly if you diuide the third mans newe laying in, which was 600 by 100, which was his stocke that he put into companie: the quotient declareth his time of continuance, which was 6 moneths. And thus is the question resolued.

Scholer. Sir I haue attentiuely beheld your working, and the more we trauell herein, the more me thinke I am in loue with this excellent art.

Master. Then what say you to this question? ‡

There is in a Cathedrall Church 20 Canons, and 30 Vicars, those may spende by yere 2600 l, but euerie Canon must haue to his part 5 times so much as euery Vicar hath: howe much is euerie mans portion say you? Question of Canons

Scholer. I pray you make the aunswere your selfe also, so shall I perceiue best the meanes to answere to such other like.

Master. In this question you must doe as in those before saide, that haue diuersitie of time, for here is diuersitie of portions: Therfore shall you multiplie the number of the persons by their difference of portion: (as you did in the other by time) Then must you multiplie the 20 (which is the number of Canons) by

by 5, (for that is the number of their portion) so will it be 100: Then 30 (that is the number of Vicars) by 1, (that is the number of their proportion) and it will be 30: put those two summes together, and they make 130: then say thus: If 130 spend 2600 l, what may 100 spend? The rule sheweth 2000 l.

Againe for Vicars: If 130 spend 2600 pound, what may 30 spend? Answere 600 l, as these figures shewe.

130	2600		130	2600
100	2000		30	600

But if euerie Canon should haue so often times 4 l, as the Vicar should haue 3 l, then should I multiplie 20 by 4, (that were 80) and 30 by 3 (that were 90) and then both were 170. Then should the figures be set thus.

	l	s	d			l	s	d
170	2600				170	2600		
80	1223,	10,	7,		90	1376,	9,	5

But this sort is too hard for you, by reason of the Fractions, therefore I will let it rest to that place.

And by this rule you see what the 20 Canons may spend, which summe if you diuide by 20, you shall see each Canons portion: and so of the Vicars, if you diuide their summe by 30, the quotient will declare euerie Vicars portion.

The

The second Dialogue.

The accounting by Counters.

Master.

NOw that you haue learned the common kindes of Arithmetike with the pen, you shall see the same Arte in Counters: which feate doeth not onely serue for them that cannot write and reade, but also for them that can doe both, but haue not at sometime their pen or tables readie with them.

This sort is in two formes commonly: The one by lines, and the other without lines. In that that hath lines, the lines doe stande for the order of places: and in that that hath no lines, there must be set in their stead so many Counters as shal néede, for each line one, and they shall supplie the stead of the lines.

Scholer. By examples I should better perceiue your meaning.

Master. For example of the lines, loe here you see fiue lines, which stand for fiue places, so

—I 0 0 0 0 0—
—I 0 0 0 0—
* I 0 0 0—
—I 0 0—
—I 0—
—I—

that the neathermost standeth for the first place, and the next aboue it for the second, and so vpward, till you come to the highest, which is the first line, and standeth for the first place. Nowe what is the value of euerie place or line you may perceiue by the figures which I haue set on them, which is according as you learned before in Numeration of Figures by the pen: for, the first place is the place of vnits or ones, and euerie counter set in that line betokeneth but one: and the second line is the place of 10, for euerie counter there standeth for 10. The third line the place of hundreds, the fourth of thousands, and so forth.

Scholer. Sir, I doe perceiue that the same order is here of lines, as was in the other figures by places, so that you shall not neede longer to stand about Numeration, except there be any other difference.

Master. If you doe vnderstand it, then how will you set 1543?

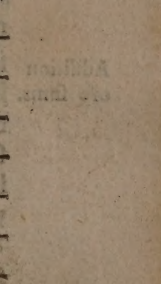
Scholer. Thus as I suppose.

Master. You haue set the places truely, but your figures be not meete for this vse: for the meetest figure in this behalfe, is the figure of a Counter round, as you see here, where I haue expressed that same summe.

Scholer. So that you haue not one figure for 2, nor 3, nor 4, and so forth, but as many Digits as you haue, so many Counters you set in the lowest line: and for euerie 10 you set one in the second line: and so of other. But I knowe not by what reason you set that one Counter for 500 betweene two lines.

Master. You shall remember this, that whensoeuer you neede to set downe 5,50, or 500, or 5000, or so forth any number whose Numerator is 5, you shall set one counter for it in the next space aboue the line that it hath his denomination of: as in this example of that 500, because the numerator is 5, it must be set in a voide space: and because the denominator is hundred, I knowe that his place is the voide space next aboue hundreds, that is to say, aboue the third line.

And further you shall marke, that in all working by this sort, if you shall set downe any summe betweene 4 and 10, for the first part of that number you shall set downe 5, and then so many Counters more, as there rest numbers aboue 5. And this is true both of Digits and Articles. And for example I will set downe this summe 287965, which sum if you marke well, you neede none other examples for to learne the Nume-



ration of this forme.

But this shall you marke, that as you did in the other kindes of Arithmetike, set a prick in the places of thousands, in this worke you shall set a Starre, as you see before.

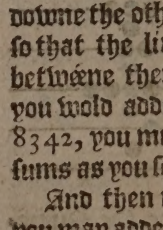
Scholer. Then I perceiue Numeration: but I pray you howe shall I doe in this Art to adde two summes or more together?

Addition.

Master.

THe easiest way in this Art, is to adde but two summes at once together: howbeit, you may adde more, as I will tell you anon. Therefore when you will adde two summes, you shall first set downe one of them, it forceth not which, and then by it drawe a line crosse the other lines. And afterward set downe the other summe, so that the line may be betweene them: as if you wold adde 2659 to 8342, you must set your sums as you see here.

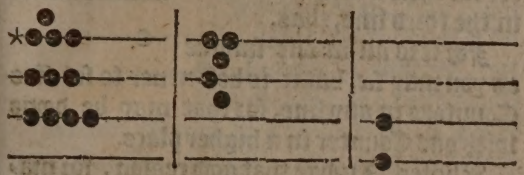
Addition of 2 sums.



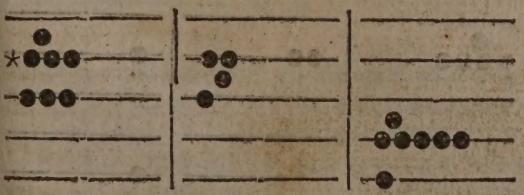
And then if you list, you may adde the one to the other in the same place:

place: or else you may adde them both together in a newe place: which way because it is most plainest, I will shewe you first.

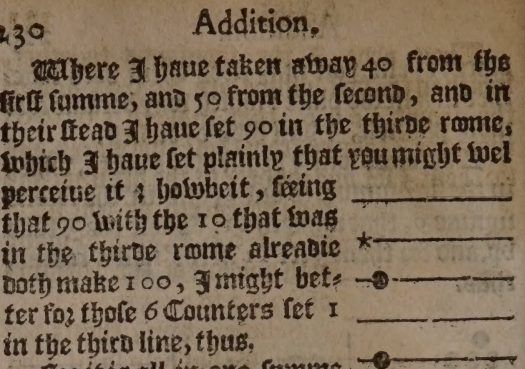
Therefore will I begin at the vnits, which in the first summe is but 2, and in the second summe 9, that maketh 11. Those doe I take vp, and for them I set 11 in the newe roume, thus.



Then doe I take vp all the Articles vnder a hundred, which in the first summe are 40, and in the second summe 50, that maketh 90: or you may say better, that in the first summe there are 4 articles of 10, and in the second summe 5, which maketh 9, but then take hēd that you set them in their right lines as you see here.



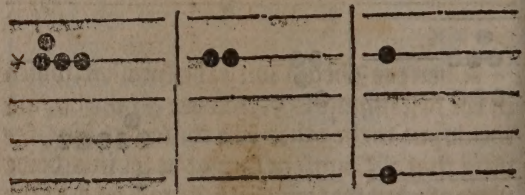
Where I haue taken away 40 from the first summe, and 50 from the second, and in their stead I haue set 90 in the thirde rome, which I haue set plainly that you might wel perceiue it: howbeit, seeing that 90 with the 10 that was in the thirde rome alreadie doth make 100, I might better for those 6 Counters set 1 in the third line, thus.



For it is all in one summe as you may see, but it is best neuer to set fiue Counters in any line, for that may be done with one Counter in a higher place.

Scholer. I iudge that good reason, for many are vnnedefull where one will serue.

Master. Well, then will I adde forth of hundreds: I finde 3 in the first summe, and 6 in the second, which maketh 900, them doe I take vp, and set in the third rome, where is 100 alreadie, to which I put 900, and it wil be 1000, therefore I set one counter in the fourth line for them all, as you see here.

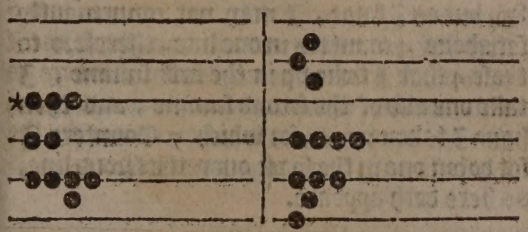


Then

Then adde I the thousands together, which in the first summe are 8000, and in the second 2000, that maketh 10000: them doe I take vp from those two places, and for them I set one counter in the fift line, and then appeareth as you see to be 11001, for so many doth amount of the Addition of 8342 to 2659.

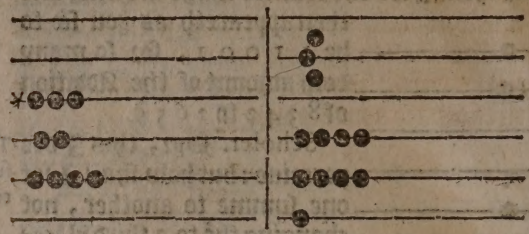
Scholer. Yea, this I doe perceiue: but how shall I set one summe to another, not changing thē to a third place? To adde summes together.

Master. Marke well howe I doe it: I will adde together 65436 and 3245, which first I set downe thus.



Then doe I begin with the smallest, which in the first summe is 5, that doe I take vp, and would put to the other 5 in the second sum, sauing that 2 Counters cannot be set in a voide place of 5 but for them both I must set 1 in the

second line, which is the place of 10: therfore I take vp the fiue of the first summe, and the 5 of the second, and for them I set 1 in the second line, as you see here.

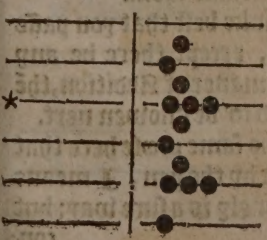


Then do I likewise take vp the 4 counters of the first sum and second line (which make 40) and adde them to the 4 counters of the same line in the second summe, and it maketh 80, but as I saide, I may not conueniently set aboue 4 counters in one line, therefore to those 4 that I toke vp in the first summe, I take one also of the second summe, and then haue I taken vp 50, for which 5 Counters I set down one in the space ouer the second line, as here doth appeare.

And then is there 80, as well with those 4 counters, as if you had set downe the other 4 also.

Nowe doe I take the 200 in the first sum, and adde them to the 400 in the second sum, and it maketh 600, therefore I take vp the 2 Counters in the first summe, and 3 of them in the second summe, and for them 5, I set 1 in the space aboue, thus.

Then I take the 3000 in the first sum, vnto which there are none in the second sum agréeing, therefore I doe onely remoue those three Counters from the first summe into the second, as here doth appeare.

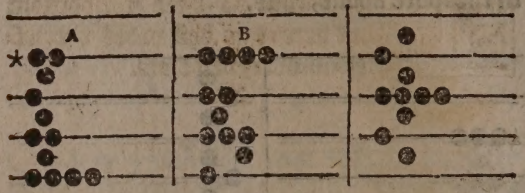


And so you sée the whole summe that amounteth of the Addition of 65436 with 3245, to be 68681.

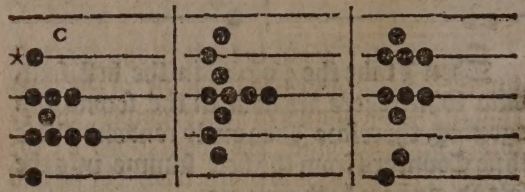
And if you haue marked

marked these two examples well, you nede no further instruction in Addition of 2 onely summes: but if you haue more than two sums to adde, you may adde them thus.

First adde two of them, and then adde the third & the fourth, or more if there be so many: as if I would adde 2679 with 4286 and 1391. First I adde the two first sums thus.



And then I adde the third thereto thus.



And so of more, if you haue them.

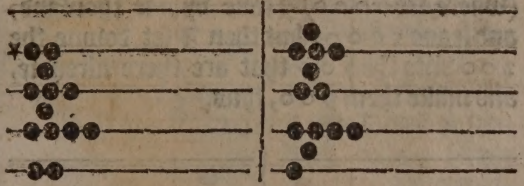
Scholer. Now I thinke best that you passe forth to Subtraction, except there be any waies to examine this maner of Addition, the I thinke that were good to be knowen next.

A proofe. Master. There is the same proofe here that is in the other Addition by the pen, I meane Subtraction, for that onely is a sure way: but con-

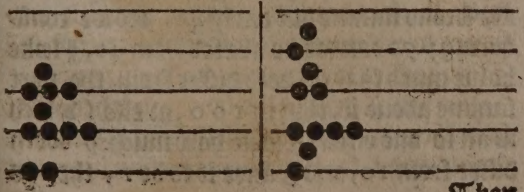
considering that Subtraction must be first knowen, I wil first teach you the Art of Subtraction, and that by this example.

Subtraction.

I Would subtract 2892 out of 8746. Whese summes must I set downe as I did in Addition: but here it is best to set the lesser number, first thus.

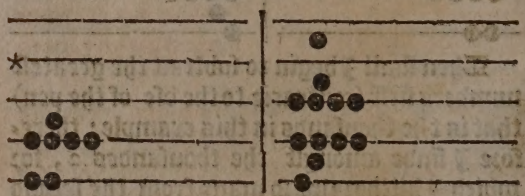


Then shall I begin to subtract the greatest numbers first (contrarie to the use of the pen) that is the thousands in this example: therefore I finde amongst the thousandes 2, for which I withdrawe so many from the second summe (where are 8) and so remaineth there 6, as this example sheweth.



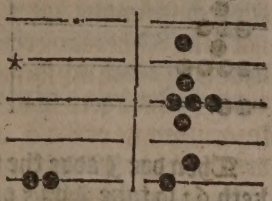
Then

Then doe I likewaies with the hundreds, of which in the first summe I finde 8, and in the second summe but 7, out of which I can not take 8, therefore this must I doe: I must loke how much my summe differeth from 10, which I finde here to be 2, then must I bate for my summe of 800, one thousand, and set downe the excesse of hundreds, that is to say, 2, for so much as 1000 is more than I shold take vp. Therefore from the first summe I take that 800, and from the second summe (where are 6000) I take vp one thousand, and leaue 5000, but then I set downe the 200 vnto the 700 that are there alreadie, and make them 900, thus.



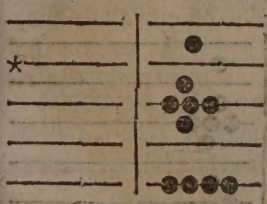
Then come I to the Articles of tennes, where in the first summe I finde 90, and in the second summe but onely 40. Nowe considering ỹ 90 cannot be abated from 40, I loke howe much that 90 doth differ from the next summe aboue it, that is 100, or else (which is all to one effect) I loke how much 9 doeth differ from 10, and I finde it to be 1, then in the

the stead of that 90, I doe take from the second summe 100: but considering that is 10 too much, I set down 1 in the next line beneath for it, as you see here.



Saving that here I haue set one counter in the space, in stead of 5, in the next line.

And thus haue I subtract al, saue two which I must bate from 6 in the second sum, and there will remaine 4, thus.

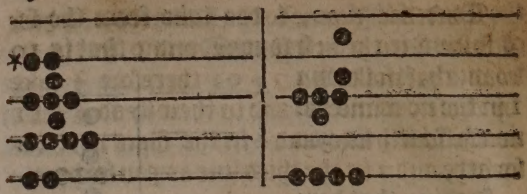


So that if I subtract 2892 from 8746, the remainer will be 5854.

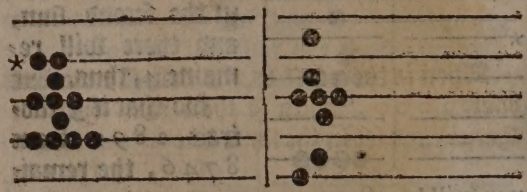
And that this is truely wrought, you may proue by Addition: for if you adde to this remainer the same summe that you did subtract, then will the former summe 8746 amount againe.

Scholer. That will I proue: and first I set the summe that was subtracted, which was 2892, and then the remainer 5854, thus.

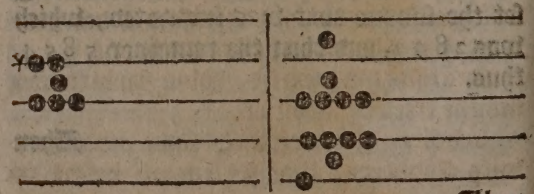
Then



Then doe I adde the first 2 to 4, which maketh 6: so take I vp 5 of those counters, and in their stead I set 1 in the space, and 1 in lowest line, as here appeareth.



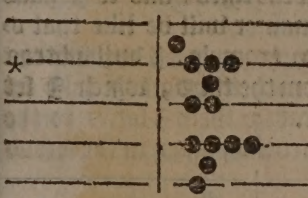
Then doe I adde the 90 next aboue to the 50, and it maketh 140, therefore I take vp those 6 counters, and for them I set 1 to the hundreds in the third line, and 4 in the second line thus.



Then

Then doe I come to the hundreds, of which I finde 8 in the first summe, and 9 in the second, that maketh 1700: therefore I take vp those 9 counters, and in their stead I set 1 in the fourth line, and 1 in the space next beneath, and 2 in the third line as you see here.

Then is there left in the first summe but onely 2000, which I shall take vp from thence, and set in the same line in the second sum, to the one that is there alreadie: and then will the whole summe appeare, as you may well see, to be 8746, which was the first grosse summe, and therefore I doe perceiue that I had well subtracted before.



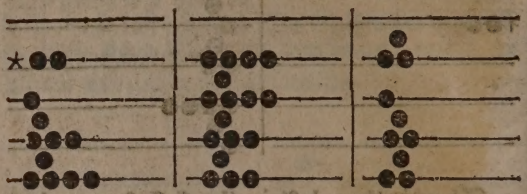
And thus may you see, howe Subtraction may be tried by Addition.

Scholer. I perceiue the same order here with Counters, that I learned before in si-

figures.

Master. Then let me see howe you can trie Addition by Subtraction.

Scholer. First I will set forth this example of Addition, where I haue added 2189, to 4988. And the whole summe appeareth to be 7177.

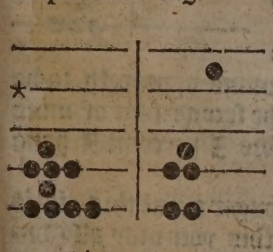


Nowe to trie whether that summe be wel added or no, I will subtract one of the first two summes from the third, and if I haue well done, the remainer will be like that other summe, as for example. I will subtract the first summe from the third, which I set thus in their order.

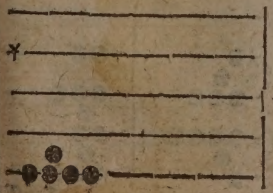
Then doe I subtract 2000 of the first summe

summe from the second summe, and then remaineth there 5000, thus.

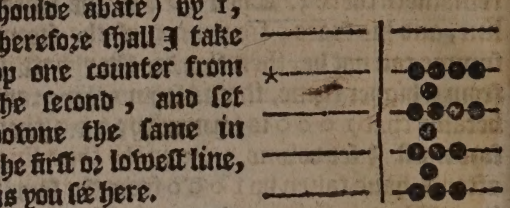
Then in the thirde line I subtract ỹ 100 of the first sum from the second sum where is onely 100 also: & then in the third line resteth nothing, as you may see in the example following.



Then in the second line with his space ouer him, I finde 80, which I should subtract from the other sum: then seeing there are but onely 70, I must take it out of some higher sum, which is here onely 5000: therefore I take vp 5000: and seeing that is too much by 4920, I set downe so many in the second roome, which with the 70 being there alreadie, doe make 4990, and then the summes doe stand thus.



Yet remaineth there in the first summe, 9 to be abated from the second summe, wherein that place of vnits doth appeare onely 7: then must I bate a higher sum, that is to say 10, but séeing that 10 is more than 9 (which I shoulde abate) by 1, therefore shall I take vp one counter from the second, and set downe the same in the first or lowest line, as you sée here.

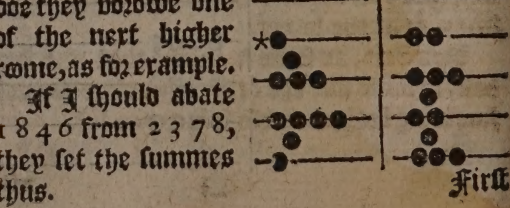


And so haue I ended this worke, and the summe appeareth to be the same which was the second sum of mine Addition, and therefore I perceiue I haue well done.

Another way of Addition.

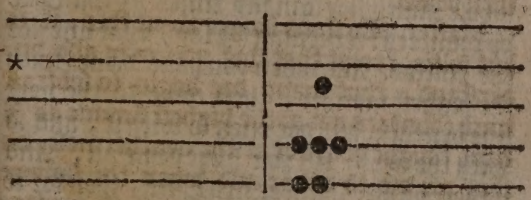
Master. To stand longer about this, it is but follie, except that this you may also vnderstand, that many doe begin to subtract with counters not at the highest summe as I haue taught you, but at the neathermost, as they doe vse to adde: and when the summe to be abated in any line appeareth greater than the other, then doe they borowe one of the next higher rome, as for example.

If I should abate 1846 from 2378, they set the summes thus.



First

First they take 6, which is the lower line, and his space, from 8 in the same rowmes in the second summe, and yet there remaineth 2 Counters in the lowest line. Then in the second line must 4 be subtracted from 7, and so remaineth there 3. Then 800 in the thirde line, and his space, from 200 of the second summe can not be, therefore doe they bate it from a higher rowme, that is from 1000: and because that 1000 is too much by 200, therefore must I set downe 200 in the third line, after I haue taken vp 1000 from the fourth line. Then is there yet 1000 in the fourth line of the first summe, which if I withdraw from the second summe, then doeth all the figures stand in order, thus.



So that (as you sée) it differeth not greatly whether you begin Subtraction at the higher lines, or at the lower.

Howbeit, as some men like that one way best, so some like the other: therefore you now knowing both, may vse which you list.

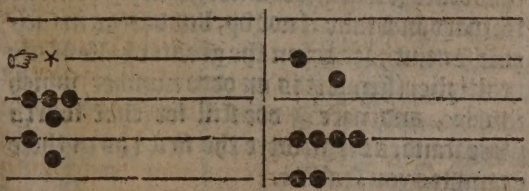
Multiplication.

But nowe touching Multiplication: you shall set your numbers in two rowes (as you did in those other kinds) but so that the multiplier be set in the first rowe, then shall you begin with the highest numbers of the second rowe, and multiplie them first, after this sort. Take the ouermost line in your first worcking, as it were the lowest line, setting on it some moueable marke (as you list) and loke howe many counters be in him, take them vp, and for them set downe the whole multiplier so many times as you toke vp counters: reckening (I say) that line for the Vnites. And when you haue done with the highest number, then come to the next line beneath, and doe euen so with it, and so with the next, till you haue done all. And if there be any number in a space, then for it shall you take the multiplier 5 times: and then must you recken that line for the vnites, which is next beneath that space. Or else after a shorter way, you shall take onely halfe the multiplier, but then shal you take the line next aboue that space for the line of vnites. But in such worcking, if by chance your multiplier be an odde number, so that you can not take

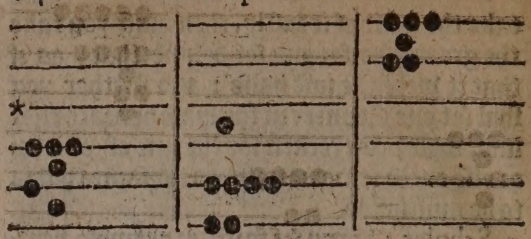
take the halfe of it iustly, then must you take the greater halfe, and set downe that, as if that it were the iust halfe: and further you shal set one Counter in the space beneath that line, which you recken for the line of vnites, or else onely remoue forward the same that is to be multiplied.

Scholer. If you set forth an example hereto, I thinke I shall perceiue you.

Master. Take this example: I would multiplie 1542 by 365, therefore I set the numbers thus.

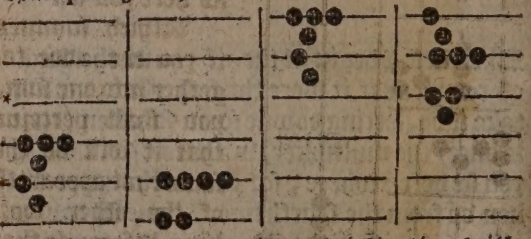


Then first I begin at the 1000 in the highest rome, as if it were the first place, and I take it vp, setting downe for it so often (that is once) the multiplier, which is 365, thus as you see here: where, for the one counter taken vp from the fourth line, I haue set downe other 6, which make the summe of the multiplier, reckening that fourth line as if it were the first, which thing I haue marked by the hand set at the beginning of the same.



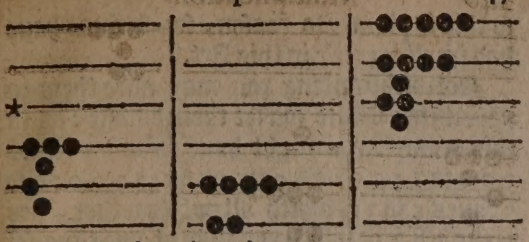
Scholer. I perceiue this well, for in deede this sum that you haue set downe is 365000: for so much doth amount of 1000, multiplied by 365.

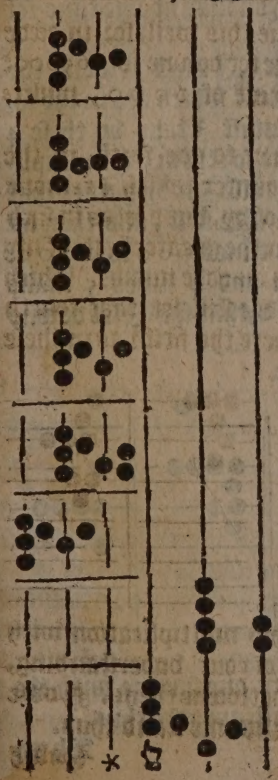
Master. Well, then to goe forth, in the next space I find one counter, which I remoue forward, but take it not vp, but doe (as in such case I must) set down the greater halfe of my multiplier (seeing it is an odde number) which is 182, and here I doe still let that fourth place stand, as if it were the first: as in these examples you shall see.



Where I haue set this multiplication with other, but for the ease of your vnderstanding, I haue set a litle line betweene them. Nowe should they both in one summe stand thus.

Howe





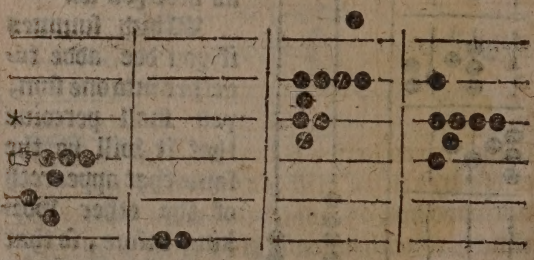
Howbeit, an other fourme to multiplie such Counters in space, is this: First to remoue the finger to the next line beneath that space, and then to take vp that Counter, and to set down the multiplier fiue times: as here you see. Another forme of Multiplication.

Which summes if you doe adde together into one sum, you shall perceiue that it will be the same that appeareth of the other working before, so that both sortes are to one intent: but as the other is shorter,

so this is plainer to reason, for such as haue had small exercise in this Art.

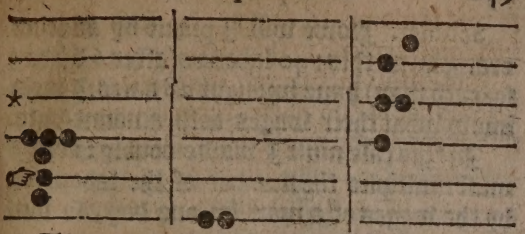
Notwithstanding you may adde them in your minde before you set them downe: as in this example you might haue saide, 5 times 300 is 1500, and 5 times 60 is 300: also 5 times 5 is 25, which all put together, doe make 1825, which you may at one time set downe if you list.

But now to goe forth, I must remoue the hand to the next counters which are in the second line, and there must I take vp those 4 counters setting downe for them my multiplier 4 times seuerally, or else I may gather that whole summe in my minde first, and then set it downe: as to say, 4 times 300 is 1200: 4 times 60 are 240: and 4 times 5 make 20, that is in all 1460, that shal I set downe also, as here you see.

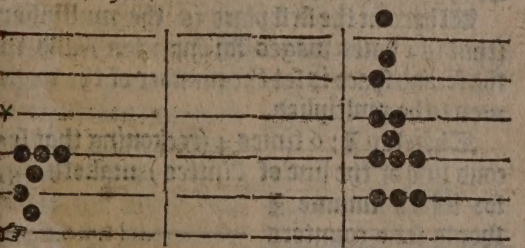


Which if I ioyne in one summe with the former numbers, it will appeare thus.

Then



Then to ende this Multiplication, I remoue the finger to the lowest line, where are onely 2, them do I take vp, and in their stead doe I set downe twice 365, that is 730, for which I set one in the space aboue the thirde line for 500, and 2 more in the third line with that one that is there alreadie, and the rest in their order, and so haue I ended the whole summe, thus.



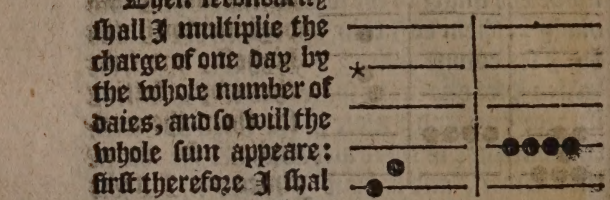
Wherby you see, ỹ 1542 (which is the number of yeres sith Christ his incarnation) being multiplied by 365 (which is ỹ number of days in one yere) doth amount vnto 562830, which declareth ỹ number of days sith Christs incarnation vnto the end of 1542 yeares, (beside 385 daies and 12 houres for leape yeares.) The sum of the daies sith Christs incarn.

Scho-

Scholer. Nowe will I proue by another example, as this: 40 labozers (after 6 d ỷ day foz each man) haue wzought 28 daies: I wold know what their wages doth amount vnto.

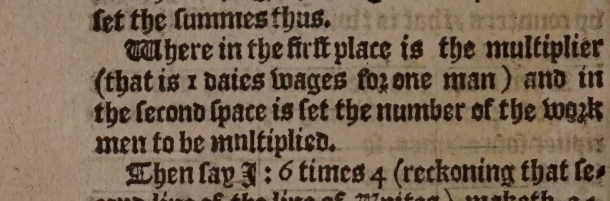
In this case must I wozke doubly: first I must multiplie the number of the labourers by the wages of a man foz one day, so will the charge of one day amount.

Then secondarily shall I multiplie the charge of one day by the whole number of daies, and so will the whole sum appeare: first therefoze I shal set the summes thus.



Where in the first place is the multiplier (that is 1 daies wages foz one man) and in the second space is set the number of the wozk men to be multiplied.

Then say I: 6 times 4 (reckoning that second line of the line of Unites) maketh 24, foz which summe I should set 2 counters in the third line, and 4 in the second, therfoze doe I set 2 in the third line, and let the 4 stand still in the second line thus.



So appeareth the whole daies wages to

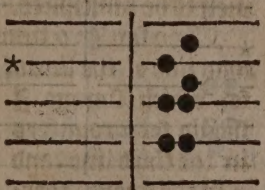
be

be 240 d, that is 20 s.

Then doe I multiplie againe the same summe by the number of daies, and first I set the numbers thus. Then because there are counters in

diuers lines, I shal beginne with the highest, and take them vp, setting for them the multiplier so many times as I take vp counters, that is twise, then will the sum stand thus.

Then come I to the second line, and take vp those 4 counters, setting for them the multiplier foure times, so wil the whole summe appeare thus.



So is the whole wages of 40 workemen for 28 daies (after 6 d ech day for a man) 6720 d, that is 560 s, or 28 pound.

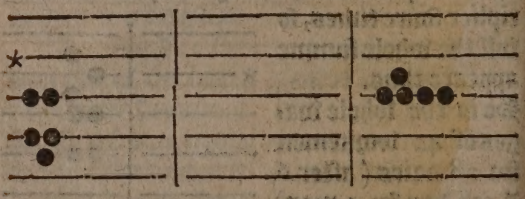
M. Now if you wold proue Multiplication, the surest way is by Diuision, therfore will I ouerpasse it, till I haue taught you the art of Diuision, which you shall worke thus.

Diui-

Diuision.

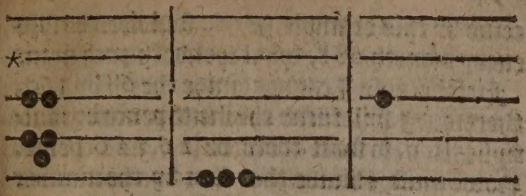
FIrst set downe the diuisor, for feare of forgetting, and then set the number that shalbe diuided, at the right side, so far from the diuisor, that the quotient may be set betweene them: as for example.

If 225 sheepe cost 45 l, what did euerie sheepe cost? To knowe this, I should diuide the whole summe, that is 45 l, by 225, but that cannot be: therefore must I first reduce that 45 l into a lesser denomination, as into shillings, then I multiplie 45 by 20, and it is 900: that summe shal I diuide by the number of sheepe, which is 225, these two numbers therefore I set thus.

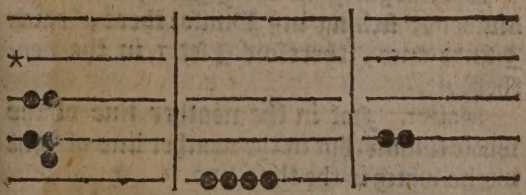


Then begin I at the highest line of the diuident, and seeke howe often I may haue the diuisor therein, and that may I doe foure times: then say I foure times 2 are 8, which if I take from 9, there resteth but 1, thus.

And



And because I found the diuisor 4 times in the diuident, I haue set as you see, 4 in the midle rowme, which is the place of the quotient: but nowe must I take the rest of the diuisor as often out of the remainer, therefore come I to the second line of the diuisor, saying: 2 times 4 make 8, take 8 from 10, and there resteth 2, thus.



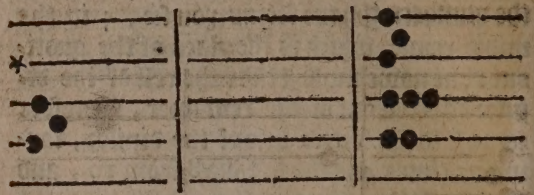
Then come I to the lowest number which is 5, and multiplie it 4 times, so is it 20, that take I from 20, & there remaineth nothing, so that I see my quotient to be 4, which are in dalewe shillings, for so was the diuident: and thereby I knowe that if 225 shéepe did cost 45 ℔, euerie shéepe cost 4 s.

Scholer. This can I doe, as you shall perceiue

Example of wages.

ceiue by this example. If 160 souldiers do spēd euery moneth 68 l, what spendeth each man?

Firſt becauſe I cannot diuide the 68 by 160, therefore I will turne the l into pence by multiplication, ſo ſhall there be 16320 pence. Nowe muſt I diuide this ſum by the number of ſouldiers, therfore I ſet them in order thus.

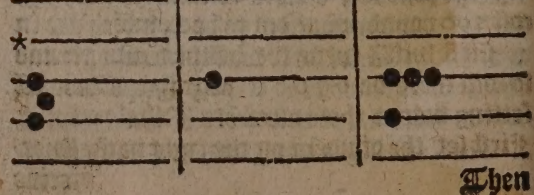


Then begin I at the higheſt place of the diuident, ſeeking my Diuiſor there, which I finde once, therefore I ſet 1 in the neather line.

Maſter. Not in the neather line of the whole ſumme, but in the neather line of that worke which is the third line.

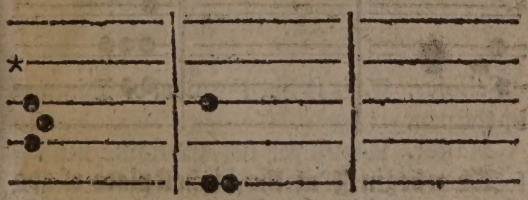
Scholer. So ſtandeth it with reaſon.

Maſter. Then thus doe they ſtand.



Then

Then seeke I againe the rest, how often I may find my diuisor: and I see that in the 300 I might finde 100 three times, but then the 60 will not be so often found in 20, therefore I take 2 for my quotient: then take I 100 twice from 300, and there resteth 100, out of which with the 20 (that maketh 120) I may take 60 also twice and then standeth the numbers thus.



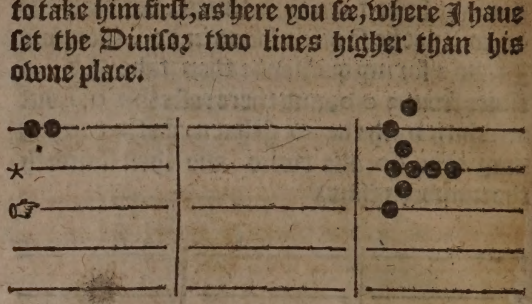
Where I haue set the quotient 2 in the lowest line: So is euerie souldiers portion 102 ℔, that is 8 ℥, 6 ℔.

Master. But yet because you shall iustly perceiue the reason of Diuision, it shall be good that you doe set your diuisor still against those numbers from which you doe take it, as by this example I will declare.

If the purchase of 200 acres of ground did cost 290 pound, what did one acre cost? Example of purchase.

First will I turne the pounds into pence, so will there be 69600 pence. Then in setting downe these numbers I shal do thus. First set the diuident on the right hand as it ought,

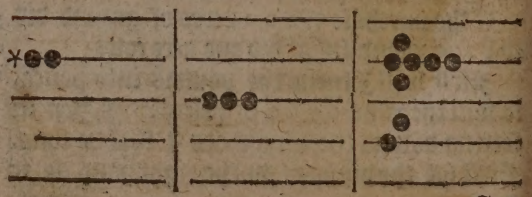
ought, and then the diuisor on the left hande against those numbers from which I intende to take him first, as here you see, where I haue set the Diuisor two lines higher than his owne place.



Scholer. This is like the order of Diuision by the pen.

Master. Truth you say, and nowe must I set the quotient of this worke in the third line, for that is the line of vnits in respect to the diuisor in this worke.

Then I seeke howe often the diuisor may be found in the diuident, and that I finde 3 times, then set I 3 in the third line for the quotient, and take away that 60000 from the diuident, and further I set the diuisor one line lower, as you see here.



And

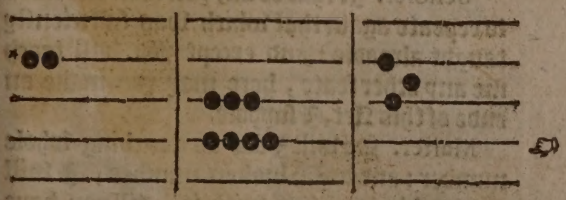
And then seeke I howe often the diuisor will be taken from the number against it, which will be 4 times, and 1 remaining.

Scholer. But what if it chaunce that when the diuisor is so removed, it cannot be once taken out of the diuident against it?

Master. Then must the diuisor be set in an other line lower.

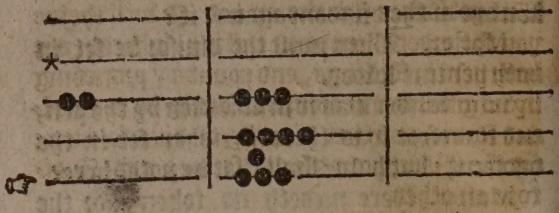
Scholer. So was it in Diuision by the pen, and therefore was there a cipher set in the quotient: but howe shall that be noted here?

Master. Here needeth no token, for the lines doe represent the places: only loke that you set your quotient in that place which standeth for vnits in respect of the diuisor: but nowe to returne to the example. I find the diuisor 4 times in the diuident, and 1 remaining, for 4 times 2 make 8, which I take from 9, and there resteth 1, as this figure following sheweth: and in the midle space for the quotient I set 4 in the second line, which is in this worke the place of vnits.



Then remoue I the diuisor to the next

lower line, and seeke how often I may haue it in the diuident, which I may do here 8 times iust, and nothing remaine, as in this forme.



Where you may see that the whole quotient is 348 d, that is 29 s, whereby I knowe that so much cost the purchase of one acre.

Scholer. Nowe resteth the prowes of Multiplication, and also of Diuision.

Master. Their best prowes are each one by the other: for Multiplication is prowed by Diuision, and Diuision by Multiplication, as in the worke by the pen you learned.

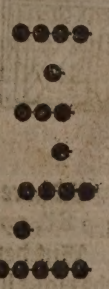
Scholer. If that be all, you shall not neede to repeate again that which was sufficiently taught alreadie: and except you will teach me any other feate, here may you make an ende of this Art, I suppose.

Master. So will I doe as touching whole number: and as for broken number, I will not trouble your wit with it, till you haue practised this so well, that you be full perfect,

so

so that you need not to doubt in any point that I haue taught you, and then may I boldely instruct you in the Art of Fractions or Broken number: wherein I will also shewe you the reasons of all that you haue now learned. But yet before I make an end, I will shewe you the order of common casting, wherein are both pence, shillings, and pounds, proceeding by no grounded reason, but onely by a receiued forme, and that diuersly of diuerse men: for the Marchants vse one forme, and Auditors an other.

Marchants vse.



BVt first for Marchants forme, mark this example here, in which I haue expressed this summe 198 l, 19 s, 11 d. So that you may see that the lowest line serueth for pence, the next aboue for shillings, the third for pounds, and the fourth for scores of pounds.

And further you may see that the space betweene d and s may receiue but one counter (as all other spaces likewise doe) and that one standeth in that place for 6 d.

Likewise betweene the shillings and the pounds, one counter standeth for 10 s.

And betweene the poundes and 20 l, one counter standeth for 10 l.

But beside those you may see at the left side of shillings, that one counter standeth alone, and betokeneth 5 s.

So against the pounds, that one counter standeth for 5 l. And against the 20 pounds, the one counter standeth for 5 score pounds, that is 100 pound, so that euerie side counter is 5 times so much as one of them against which he standeth.

Auditors account.

NOwe for the account of Auditors, take this example.

Auditors account.

Where I haue expressed the same summe 198 l, 19 s, 11 d.

But here you see the pence stand towards the right hand, and the other increasing orderly toward the left hand.

Againe you may see that Auditors will make 2 lines (yea and more) for pence, shillings, and all other values, if their summes extend thereto. Also you see that they set one counter at the right ende of each rowe, which so set there, standeth for 5 of that rowme: and

on

on the left corner of the rowe it standeth for 10 of the same rowe.

But nowif you would adde eyther subtract after any of both those sorts, if you marke the order of the other feate which I taught you, you may easilie doe the same here without much teaching: for in Addition you must first set downe one summe, and to the same set the other orderly, and in like maner if you haue many: but in Subtraction you must set downe first the greatest summe, and from it must you abate the other, euerie denomination from his due place.

Scholer. I doe not doubt but with a litle practise I shall attaine these both: but howe shall I multiplie & diuide after these formes?

Master. You cannot duely doe any of both by these sorts, therefore in such case you must resort to your other artes.

Scholer. They that vse such accounts that it excede 200 in one summe, they set not 5 at the left hand of the scores of pounds, but they set all the hundreds in an other farther row, and 500 at the left hande thereof, and the thousands they set in a farther rowe yet, and at the left side thereof they set the 5000, and in the space ouer they set the 10000, & in a higher rowe 20000, which al I haue expressed in this example, which is 97869l, 12s, 9d, ob, q. Ninetie seuen thousande, eight hundred, three score and nine pounde,

twelue shillings & nine pence halfepenie farthing, for I had not tolde you before, where, neither howe you shoulde set downe farthings, which (as you see here) must be set in a void space sideling beneath the pence: for farthing one counter, for ob. 2 counters: for ob. farthing, 3 counters: & more there cannot be: for 4 farthings make 1 d, which must be set in his due place.

And if you desire the same summe after Auditors manner: Loe here it is.

But in this thing you shall take this for sufficient, and the rest you shall obserue as you may see by the working of each sort: for the diuerse wits of men haue inuented diuers and sundrie wayes, almost innumerable.

The

The second part of Arithmetike touching Fractions, briefely set foorth.

Scholer.

ALbeit I perceiue your manifold businesse doeth so occupie, or rather oppresse you, that you can not as yet completelie ende that treatise of Fractions Arithmeticall, which you haue prepared, wherein not onely sundrie worckes of Geometrie, Musicke, and Astronomie be largely set forth, but also diuerse conclusions and naturall worckes, touching mixtures of metals, and compositions of medicines, with other strange examples, yet in the meane season I can not stay my earnest desire, but importunely craue of you some briefe preparation towarde the vse of Fractions, whereby at the least I may be able to vnderstand the common worckes of them, and the vulgar vse of those rules, which without them cannot well be wrought.

Master. If my leisure were as great as my will is good, you should not needs to vse any importunate crauing, for the attaining of that thing, wherby I may be perswaded that

I shal any waies profite the common wealth, or helpe the honest studies of any good members in the same: wherefore, while mine attendance will permit me to walke and talke, I am well willing to helpe you as I may.

What a Fractiō is. Therefore first to begin with explication of this name Fraction, what take you it to be?

Scholer. Marie sir, I thinke a Fraction (as I haue heard it often named) to be a broken number, that is to say, to be no whole number, but a part of a number.

Master. A Fraction in dede is a broken number, and so consequently, the part of an other number: but that must be vnderstanded of such an other number, as cannot be diuided into any other parts than Fractions: for although I may take the third part of 60, or the fourth part of it, and so of other parts diuersly, yet these parts be not properly, nor ought not to be called Fractions, because they may be expressed by whole numbers: for the third part of it is 20: the fourth part is 15: the twelfth part is 5, and so forth of other parts, which all be whole numbers.

What a Fraction is properly. Wherefore properly a Fraction expresseth the parts or part onely of an vnit, that is to say, that the number which is the whole or entire summe of any Fraction, may not be greater than one: and therefore it followeth, that no one Fraction alone can be so great,

that

that it shall make 1, as by examples I wil declare as sone as I haue taught you to knowe the forme howe a Fraction is expressed or representeth in writing.

Numeration.

But first to begin with the expressing of a Fraction, which is the numeration of it, you must vnderstand that a Fraction is represented by 2 numbers, set one ouer the other, and a line drawen betweene them as thus, $\frac{1}{3}$. $\frac{3}{4}$. $\frac{2}{5}$. $\frac{10}{17}$. which foure Fractions you must pronounce thus: $\frac{1}{3}$ one third part: $\frac{3}{4}$ three quarters: $\frac{2}{5}$ two fift parts: $\frac{10}{17}$ ten seuentéene parts.

Scholer. I vnderstande the forme of their expression and pronounciation, but their meaning or valuation séemeth more obscure: yet I thinke that by the two first Fractions I vnderstand the valuation of the two latter Fractions, and so consequently of other.

Master. Value them then, that I may perceiue your taking of them.

Scholer. $\frac{2}{5}$ betokeneth two fift parts, that is to say, if one be diuided into 5 parts, that Fraction doth expresse two of those fift parts: $\frac{10}{17}$ doth signifie, that if one be diuided into xvij. parts,

parts, I must take ten of them. And this I gather of the two first examples: for $\frac{1}{3}$ that is one third part, doth easily declare, that if any one thing be diuided into three parts, I must take but one of them: so $\frac{3}{4}$ that is three quarters, doeth declare that one being diuided into foure quarters, I must take (for this Fraction) three of those quarters.

If there be no more difficultie in their Numeration, then I pray you goe forwarde to their Addition and Subtraction, and so to the other kindes of workes, for I vnderstand that the same kindes of workes be in Fractions, that be in whole numbers.

Master. There are the same kindes of workes in both, albeit the order of them is diuerse, as I will anon declare: but yet more in Numeration before we leaue it. You must vnderstand, that those two numbers which expresse a Fraction, haue seuerall names. The ouermost which is aboue the line, is called the Numerator, and the other beneath the line, is called the Denominator.

Numerator & Denominator.

Scholer. And what is the reason of their diuerse names? For in mine opinion both be Numerators, seeing both they doe expresse the numeration of the Fraction.

Master. You are deceiued: for one onely (which is the ouermost) doeth expresse the numeration: and the denominatour doeth declare the number of parts into which the vnit

is

is diuided, as in this example, when I say: Diuide a pound weight of gold betweene foure men, so that the first man shall haue $\frac{2}{15}$ the second $\frac{3}{15}$ the third $\frac{4}{15}$ and the fourth $\frac{6}{15}$.

Nowe doe you perceiue that by the Denominator (which is one in all foure Fractions) it is intended, that the pound waight should be diuided into so many parts, I meane 15, and by the foure seuerall numerators is limited the diuerse portion that each man should haue, that is, that when the whole is parted into 15, the first man shall haue 2 of those 15 parts: the second man three of them: the third man 4: and the fourth man 6. And so may you see the seuerall offices (as it were) of those two numbers, I meane of the Numerator & the Denominator.

And hereby you perceiue, that a man can haue no more parts of any thing than it was diuided into, neither yet aptly so many: so that it were vnaptly saide: You shal haue $\frac{15}{15}$, that is 15 parts of any thing, seeing it were better saide: You shall haue the whole thing.

Scholer. So doeth it appeare reasonably. for the labour is vaine to diuide any thing, and then to applie the Diuision to no vse. And much lesse reasonable were it to say $\frac{16}{15}$: for if the whole be diuided into 15 parts onely, it is not possible to take 16 of them, that is to say, more than altogether.

Master. This is true touching the proper

and

and apt vse of the name of a Fraction: yet improperly, and after a vulgar acceptation (for easinesse in worke) both those formes be called Fractions, because they be written like Fractions, although they be none in déde for $\frac{15}{15}$, and generally all such other: where the Numerator and Denominator be equall, are not Fractions: but the whole thing with all his parts. And so $\frac{15}{12}$ is not to be called a fraction, but a mixt number, of a whole number & a fraction: for it is as much, as $1\frac{3}{12}$, that is one whole one & 3 twelue parts, as shalbe declared in reduction. Therfore they do abuse ỹ names, that call them fractions, where ỹ Numerator is either equal or greater than ỹ denominator.

Scholer. But is there any nédefull cause why they should so abuse the name?

Master. There is cause why they shall sometimes, for easinesse in worke, write some numbers after that sort, like fractions: but they néded not to call them fractions, but as they be, whole numbers or mixt numbers (that is whole numbers with fractions) expressed like fractions.

Nowe must you vnderstande, that as no fraction properly can be greater than 1, so in smalnesse vnder one the nature of fractions doeth extend infinitly: as the nature of whole numbers is to increase aboue one infinitely, so that not only one may be diuided into infinite fractions or parts, but also euery fraction may

be

be diuided into infinite fractions or partes, which commonly be called fraction of fractions, and they be expressed diuersly: As for example. $\frac{1}{4}$ of $\frac{2}{3}$ of $\frac{1}{2}$, that is three quarters of two third parts, of one halfe part. Whereby is signified, that if one be diuided into two halfs, and the one halfe in to three parts, and two of those three parts be diuided ioyntly into foure quarters, this fraction of fractions doth represent three of those quarters.

Scholer. I pray you let me proue by an example in common money, whether I doe rightly vnderstand you or no. One Crowne, which I take for an vnit, doeth containe 60 pence, therefore the halfe of it is 30 pence: $\frac{2}{3}$ of that halfe is 20 pence, whereof $\frac{3}{4}$ is fiftéene pence, so then 15 pence is $\frac{3}{4}$ of $\frac{2}{3}$ of $\frac{1}{2}$ of a crown. And so 3 pence is $\frac{3}{4}$ of $\frac{2}{3}$ of $\frac{1}{2}$ of a shilling.

Master. You perceiue this well ynough: yet this note I giue you by the way that the forme of expressing these Fractions is voluntarie: and hath no other reason than the will of the Deuiser, which forme many followe: for some expresse them thus $\frac{3}{4}\,\frac{2}{3}\,\frac{1}{2}$ without any figure of distinction betwéene them, which forme many followe. Some other doe make lines betwéene euerie Fraction, and adde words of distinction, after this sort, $\frac{3}{4}$ of $\frac{2}{3}$, of $\frac{1}{2}$, which forme is most best.

Some

Some other expresse them thus in slope forme, to distinct them from seuerall Fractions of one whole number, for if they were set in one right line thus, $\frac{1}{4}\frac{2}{3}\frac{1}{2}$: then ought it to be pronounced, three quarters, and two third parts and an halfe, which maketh almost two whole vnites, lacking but one fift part. And so is it nothing agreeable with the other fraction of fractions, wherefore it is a great ouersight in certain learned men, which doe expresse them so confusedly with such seuerall fractions, that a man can not knowe the one from the other.

Therefore some men (as Stifelius) doe expresse without a line numbers of proportion, being applied to Addition or Subtraction: because they must be taken as two, where the line in Fractions maketh them to be taken for one: for of the Numerator and Denominator is made one number.

Three seuerall varieties.

Scholer. When I perceiue there be three seuerall varieties in fractions: First when one onely fraction is set for one number, as $\frac{4}{5}$, that is foure fifth parts. The second is, when there be set two or more seuerall fractions of one number, as $\frac{4}{9}\frac{2}{5}$, that is foure ninth parts, and two fift parts. The third sort is fractions of fractions, as $\frac{4}{9}$ of $\frac{2}{5}$, that is 4 ninth parts of two fifth parts.

Master. You haue saide well, if you vnder-

derstand well your owne words.

Scholer. If it shall please you, I will by an example in the parts of an old English Angell expresse my meaning.

Master. Let me heare you.

Scholer. The olde English Angel did containe 7 shillings 6 d, that is 90 d. Nowe $\frac{4}{5}$ of it, is 72 d. And of the same 90 pence, if I take $\frac{4}{9}$ and $\frac{2}{5}$, that is foure ninth parts, and 2 fifth parts, $\frac{4}{9}$ is 40, and $\frac{2}{5}$ is 36, which both make 76: but if I take $\frac{4}{9}$ of $\frac{2}{5}$, that is 4 ninth parts of two fifth parts, seeing $\frac{2}{5}$ is but 36, then $\frac{4}{9}$ of 36 will yeld but 16: for $\frac{1}{9}$ of 36, is but 4, and that taken 4 times maketh 16.

Master. This is plainely expressed, and truely, and hereby (I doubt not) but you doe perceiue, that as great a difference as is betweene 16 and 76, so much difference is betweene those two fractions $\frac{4}{9}$ and $\frac{2}{5}$: and $\frac{4}{9}$ of $\frac{2}{5}$.

And nowe that you vnderstand these varieties, I will proceede to the rest of the workes: first admonishing you, that there is an other order to be followed in fractions than there was in whole numbers, for in whole numbers this was the order: Numeration, Addition, Subtraction, Multiplication, Diuision, and Reduction, but in fractions (to followe the same aptnesse in proceeding from the easiest workes to the harder) we must vse this order of the workes: Numeration, Multiplication, Diuision, Reduction, Addition, and Subtraction.

ction.

Scholer. That Multiplication and Diuiſion ſhould goe together, and Subtraction to followe Addition, naturall order doeth perſwade: but why Multiplication ſhould be firſt in order here next to Numeration, and Reduction in the midle, I deſire to vnderſtand the reaſon.

Maſter. As in the Art of whole numbers order would reaſonably begin with the eaſieſt, and ſo goe forward by degrées to the hardeſt, euen ſo reaſon teacheth in fractions the like order. And conſidering that Addition or Subtraction of fractions can verie ſeldome be wrought without Multiplication and Reduction: and contrariewayes, Multiplication and Reduction may be wrought without this forme of Addition or Subtraction. Therfore was it orderly required, that Multiplication and Reduction ſhould goe before Addition and Subtraction. And the ſame reaſon ſerueth for the placing of Multiplication before Reduction.

Scholer. Then if Multiplication be the eaſieſt, I pray you declare the forme of it firſt by rule, and then by example.

Maſter. Your example is good.

Multi-

Multiplication.

THerefore when any two Fractions be proponed to be multiplied together, the Numerator of the one muſt be multiplied by the Numerator of the other: and the ſumme that amounteth thereof, muſt be ſet for a newe Numerator: likewiſe the Denominator of the one muſt be multiplied by ỹ denominator of the other, & that that amounteth, ſhall be ſet for the common denominator: and this newe third fraction expreſſeth the product of the multiplication of the two firſt fractions proponed, whereof take this example, $\frac{3}{5}$ multiplied by $\frac{5}{12}$, doth make $\frac{15}{60}$.

Scholer. I perceiue then, that 3 being the numerator of the firſt fraction, is multiplied by 5, being the numerator of the ſecond fraction, whereof amounteth 15, the numerator of the third fraction. And ſo likewiſe, 5 being denominator of the firſt fraction, is multiplied by 12 the denominator of the ſecond fraction, whereof amounteth 60 the newe denominator: ſo that I perceiue how the worke is donne, but I doe not perceiue howe $\frac{15}{60}$ is greater than $\frac{3}{5}$: For I ſhall not vſe my former maner of examination by the partes of ſome Coine, I ſee that $\frac{3}{5}$ of a Crowne is 36 ß, and $\frac{5}{12}$ of a Crowne, is 25 ß, whereof the one

multiplied by the other, doth make 900 ℔, which is 15 Crownes: but by your multiplication there amounteth $\frac{15}{60}$, which is but 15 ℔, and that is much lesse than any of both the first fractions.

Master. That difference is betweene multiplication in whole numbers, and multiplication in broken numbers, that in whole numbers the summe that amounteth, is greater than both the other whereof it came: but in fractions it is contrariewaies: for the sum that amounteth, is lesser than any of the other two fractions, whereof it came.

Scholer. I desire much to vnderstand the reason thereof.

Master. Although I purposed to reserue the reasons of workes Arithmeticall for the perfect booke of Arithmetike, yet I will shew you this, because of the strangenesse of the worke.

You see in whole numbers, that of two numbers being multiplied together, is made the third number: which third number doeth beare the same proportion to the number multiplied, that the multiplier doth beare to an vnite. And so in fractions, the third number which amounteth of multiplication, beareth the same proportion to each of the two first fractions, that the other of those two fractions doth beare to an vnite.

Scholer. Sir I vnderstand your wordes thus:

thus: when 40 is multiplied by 12, there doth amount 480, which 480 doeth containe 40 so many times in it, as twelue doth containe vnits, that is to say: twelue times. And so it appeareth that 480 doeth containe twelue so many times also, as 40 doeth containe vnites, that is 40 times. But nowe I see not how the third number in this example of Fractions can containe any of the two former (as it happened in whole numbers) seeing it is lesser than either of them.

Master. No maruaile if you cannot see that thing which is not possible to be seene of any man, how the third number in multiplication of Fractions should be greater than any of the two former fractions, but yet this may you see (which I saide) that the third number in Fractions so multiplied, doeth beare the same proportion to any of the two former fractions, that the other of those two fractions doth beare to an vnite, as in your example $\frac{3}{5}$ being multiplied by $\frac{5}{12}$, doth make $\frac{15}{60}$. Now say I, that $\frac{15}{60}$ doth beare the same proportion to $\frac{3}{5}$, that $\frac{5}{12}$ doth beare to an vnite, as you may in your owne forme of examination by coyne trie it. For in an olde Angell (which in times past was currant for 7 s, 6 d) are 180 halfe pence, which I set for the intire vnite, whose partes (according to the fractions aforesaid) are these, for $\frac{15}{60}$ set 45 ob. for $\frac{3}{5}$ take 180 ob. and for $\frac{5}{12}$ put 75 ob. Nowe doeth 45 beare the same pro-

portion to 108 that 75 doth beare to 180: for 45 is $\frac{5}{12}$ of 108, and so is 75 also $\frac{5}{12}$ of 180.

But these reasons may be better reserued till another time, when the knowledge of proportions in due order shall be taught. Yet in the meane season I will shewe you howe it commeth to passe that in Fractions the third summe must nedes be lesser than any of the other two,

Consider thus, that when a Fraction is proponed, as in ỹ former example $\frac{3}{4}$, if it be multiplied by more than 1, it wil make more thã one entire number. As if I multiplie $\frac{3}{4}$ by 5, that is to say, if I take it 5 times, it will make three entire vnits: example in a crown, $\frac{3}{4}$ of it maketh 3 s̃, which if I take fiue times, it will amount to 15 s̃, that is three intire Crownes: so if I take the same $\frac{3}{4}$ but twice, it will yeld 6 s̃, that is one entire Crowne and $\frac{1}{2}$. Nowe if I take it but once, it cannot be more than it was before, that is 5 s̃. And ☞ if I take it lesse than once, it cannot be so much as it was before. Then seing that a fraction is lesse than one, if I multiplie a fraction by another fraction, it followeth that I doe take that first fraction lesse than once, and therefore the summe that amounteth, must nedes be lesse than the first fraction.

Scholer. Sir, I thanke you much for this reason. And I trust I doe perceiue the thing, as by example of this same fraction $\frac{3}{4}$ I wil expresse.

preſſe. If I take $\frac{3}{3}$ of a crowne once, that is to ſay, if I multiplie $\frac{3}{3}$ by 1, it will be as it was before, but 3 s: ſo if I doe multiplie it by $\frac{1}{2}$, that is, if I take but halfe one time, then wil it be but halfe ſo much: likewiſe if I multiplie it by $\frac{1}{3}$, that is, if I take but the third part of once, it will yéld but 12 pence, that is the third part of the firſt fraction.

And ſo to make an ende. If I take it but the twelfth part of once, that is, if I doe multiplie it by $\frac{1}{12}$, it will yéld but the twelfth part of the firſt fraction, which is but thrée pence. And it followeth that if $\frac{1}{12}$ make 3 pence, then $\frac{5}{12}$ muſt néedes make fiue times ſo much, that is 15 pence, which was the ſumme that hath giuen the occaſion of all this doubt.

Maſter. Then I perceiue you haue ſufficient vnderſtanding in this ſort of Multiplication for this time, wherefore I wil omit that I might ſay more of Multiplication, till we come to Reduction, and will paſſe to the other workes, and firſt to Diuiſion, whoſe place followeth Multiplication, both by natural order, and alſo in eaſineſſe of worke.

Diuision.

WHensoeuer two Fractions be proponed, that the one should be diuided by the other, I must set downe first the Fraction that shall be diuided (which is called the Diuident) and then after it the other, which is the Diuisor. Then shal I multiplie the numerator of the diuident by the denominator of the diuisor, and that which amounteth, I must put for a newe Numerator. Againe, I shall multiplie the denominator of the diuident by the numerator of the diuisor, and the number that amounteth thereof, I must put for the newe denominator. And this third fraction is the quotient of the saide Diuision.

Scholer. This seemeth easie in forme, as by example, thus: If I would diuide $\frac{5}{8}$ by $\frac{6}{2}$, first I must multiplie 5 (being the numerator of the diuident) by 6, which is the denominator of the diuisor, and thereof riseth 30: then I multiplie 8 (being the denominator of the diuident) by 2, being the numerator in the diuisor, and so riseth 16, the which I must make in a third fraction thus $\frac{30}{16}$.

Master. Me seemeth you are quicker in vnderstanding nowe, than you were when I taught you the art of whole numbers: but that

that is no maruell, for the more knowledge that any man getteth, the readier shal he find his wit, and quicker in vnderstanding: but yet of two things I will admonish you, which you might haue obserued here for ease of worke and lightnesse of vnderstanding the nature of the Quotient.

Whensoeuer you diuide one Fraction by an other, either they be both equall together, eyther else the one is greater than the other: if they be equall, their quotient shall be such, that the numerator and the denominator of it shall be equall also. And if the two first fractions be vnequall, their quotient shall declare the same by the vnequalitie of the numerator and denominator, as in these examples following shall appeare.

First of equall Fractions: $\frac{1}{2}$ and $\frac{108}{216}$ be equall together: and if the one be diuided by the other, the quotient will be $\frac{108}{108}$, as you may perceiue by that rule aforesaide.

Nowe in the vnequall fractions, as $\frac{1}{2}$ and $\frac{3}{10}$ the quotient wil be $\frac{10}{27}$: where the Numerator is greater than the denominator.

Scholer. I see it is so, but I see not the reason why it should be so,

Master. The reason is this. When any Fraction is diuided by an other, the quotient declareth what proportion the diuident beareth to the diuisor. So $\frac{1}{2}$ diuided by $\frac{1}{4}$, maketh 2, which must be founded, not two, but twice: Note how to knowe the proportion betwene two numbers.

twice: declaring that $\frac{1}{2}$ is contained twice in $\frac{1}{1}$.

And note this, that the numerator in the quotient, representeth the diuident, and the ☞ denominator representeth the diuisor. And this is alwaies true, whether the greater fraction be diuided by the lesser, or the lesser by the greater. But this proportion will not be exactly knowne, till you haue learned the art of proportions: notwithstanding somewhat of it I will declare in the next rule of Reduction. But nowe for the easie remembrance of the quotient in Diuision, as sone as you haue set downe your two Fractions, the one against the other, then make a straight line for the quotient: and as sone as you haue multiplied the numerator of the diuident, by the denominator of the diuisor, set the number that amounteth, ouer the saide line, and then multiplie the other two numbers, and set their totall vnder the same line.

Scholer. I perceiue you would not haue me trust to memorie til I were better expert, least oftentimes I happen by misse remembrance to be abused. This example I take for that declaration.

If I would diuide $\frac{2}{3}$ by $\frac{3}{4}$, I must set the numbers one against the other, (as here doeth appeare) and then make an other line for the quotient in some good distance, where I may set the numbers of the quotient, as sone as any of them

2 3
— by —
3 4

them is multiplied: So then as sone as I haue multiplied 2 by 4, which maketh 8, I shall set that 8 ouer that line thus, 8 —. And then multiplie 3 by 3, which yeldeth 9: and that 9 must I set vnder the same line, & then wil the whole quotient appeare thus $\frac{8}{9}$. Whereby appeareth (as I remember your words) that $\frac{2}{3}$ is in proportion to $\frac{3}{4}$, as 8 is to 9: but howe may I perceiue that? Note.

Master. Although you shal better perceiue it by the rule of Reduction, yet this example may be declared in common coine, as in a common shilling of xij d, of which $\frac{2}{3}$ maketh 8 d, and $\frac{3}{4}$ doth make 9 d, and so you may easily see that their proportions doe agree. And if you had taken this example before, when you toke the example of $\frac{1}{2}$ and $\frac{2}{3}$, your Quotient would appeare (as this doeth) more easier to vnderstand, whereas that quotient being $\frac{30}{45}$, is not an easie proportion for you to perceiue, being yet litle acquainted with proportions: whereof to giue you some taste, I will enter to the rule of Reduction: in which also I will declare other workes, both of Multiplication and also of Diuision, which nowe I must for a time omit, as things that doe nede the helpe of Reduction.

Redu-

Reduction.

Fiue varieties of Reductiō.

THerefore will I nowe declare the diuersities of Reduction of Fractions, which commonly haue 5 varieties.

1 First, when there be sundrie fractions of one entire vnite, they must be reduced to one denomination, and also into one fraction.

2 Secondarily, when there be proponed fractions of fractions, they must be reduced likewise into one fraction, for otherwaies they cannot be brought into one denomination.

3 Thirdly, when an improper fraction is proponed, that is to say, a fraction in forme, which in deed is greater than an vnite, it must be reduced into apt forme expressing the vnite or vnites of it, & the proper fraction distinctly. And sometimes also it shall be nedeful to conuert such a mixt number of vnities, with fractions into the forme of a fraction, that is into an improper fraction, which 2 formes I esteem but as one, because they worke on one kinde of number.

4 Fourthly, there happeneth sometimes fractions to be written in great numbers, which might be written in lesser numbers, therefore is there a meane to reduce such great numbers into their smallest termes.

5 Fiftly

5 Fiftly, when any fraction betokeneth the parts of a whole thing, which hath by common partition certaine parts, but none of like Denomination with that fraction, then may you reduce the saide fraction into an other, whose denomination shall expresse the common parts of that whole thing.

Scholer. This distinction in doctrine delighteth me much, but more with hope than present fruit, for as yet I doe not vnderstand scarcely the varieties, and much lesse the practise and vse of their workes.

Master. Reduction is an orderly alteration of numbers out of one forme into another, which is neuer done orderly but for some needfull vse, as in euerie of the saide 5 seueral varieties I will distinctly declare.

The first sort of Reduction.

First therefore, when two or more seuerall fractions of any vnite be proponed, as for example $\frac{3}{12}$ and $\frac{4}{6}$: because it is hard to tell what proportion of the entire number those two fractions doe expresse, therefore was Reduction diuised, to be a meane whereby these seuerall fractions might be brought into one denomination and fraction.

How to reduce fractions of diuerse denominations into one denomination.

And in these fractions this is the arte for bringing them to one denomination.

Multiplie first the denominators together, and the totall therof you shal set twice downe vnder two seuerall lines for two newe denominators, or rather for one common denomi-

nator:

nator: Then multiplie the numerator of the first fraction, by the denominator of the second, and set the totall thereof for the numerator ouer the first line. Likewise multiply the numerator of the second fraction by the denominator of the first, and set that totall ouer the second line for the numerator of that fraction, and so are those two first fractions of seuerall denominations, brought to one denomination.

Scholer. If I vnderstand you, as I thinke I doe, my example shal declare the same. The fractions which you proponed were these, $\frac{3}{16}$ and $\frac{4}{6}$, whose denominators (being 16 and 6) I multiplie together, and there amounteth 96, which I set vnder two lines, thus. $\frac{96}{96}$:

Then I multiplie the numerator of the first fraction by the denominator of the second, saying: 3 into 6 maketh 18, that set I ouer the first line for a newe numerator, and it will be thus. $\frac{18}{96}$

Likewaies I multiplie the numerator of the second fraction by the denominator of the first, saying: 4 times 16 maketh 64, that I set for the second numerator, and the fraction will appeare thus. $\frac{64}{96}$

So that both fractions brought to one denomination, must stand thus: $\frac{18}{96}$ and $\frac{64}{96}$

Master. You haue done well.

Scholer. I beseech you, let me examine it after my accustomed forme, by common parts

of

of coine or other measure.

Master. Goe to.

Scholer. I haue a peece of golde which is accounted worth 8 shillings, and containeth 96 pence, whereof $\frac{1}{12}$ that is the xij part, is fiue pence, and $\frac{3}{12}$ is 18 pence, that is $\frac{18}{96}$. Again $\frac{1}{2}$ of the same peece of golde, is 16 d, so that $\frac{1}{2}$ maketh 64 d, that is $\frac{64}{96}$. And so I finde the summes to agree with the other before.

Master. So haue you nowe the Arte to bring such two fractions into one denomination. And if there be more than 2, then must you multiplie all the denominators together, and set the totall thereof so many times downe as there be fractions, and then to get for ech one a newe numerator. Multiplie the numerator of the first, by the denominator of the second, and the totall thereof multiplie by the denominator of the third, and so forth if there be more. Likewise multiplie the numerator of the second, by the denominator of the first, and the totall thereof by the denominator of the third. And in the same sort multiplie the numerator of the third into the denominator of the first: and the totall thereof into the denominator of the second, and so forth, if there were moe. So these 3 fractions $\frac{2}{5}\frac{3}{4}\frac{3}{5}$ doeth make by Reduction these other 3 fractions of one denomination $\frac{48}{80}\frac{45}{80}\frac{42}{80}$. All which you may bring into one fraction by adding the Numerators together, and putting that totall for the

the common numerator, reseruing still that same common denominator. And those 3 fractions make one improper fraction thus. $\frac{107}{60}$

Scholer. All this I perceiue, and also that this last fraction is more than an vnite, and therefore you did call it an Improper Fraction.

Master. There be certaine other formes of working in this Reduction, which I will briefely touch also, to giue you an occasion to exercise your wit therein.

The first varietie of this Reduction.

The first varietie is this. When you haue made and written downe your common denominator (as I haue taught before) then to get a numerator for the first, doe thus. Diuide the common denominator by the denominator of the first fraction, and the quotient multiplied by the numerator of the same, yeldeth a newe numerator for the first newe fraction. So likewise doe with the second and the third, and with all the residue if there be more.

Scholer. That will I proue in your last example of these 3 fractions $\frac{3}{5} : \frac{3}{4} : \frac{2}{3}$. When the denominators be multiplied, they make 60, for 5 into 4 maketh 20, and 20 by 3 yeldeth 60, that I set downe three times, thus. $\frac{60.60.60}{}$: then to haue a numerator for the first, I must diuide 60 by 5, (the denominator of the first) and the quotient is 12, which I must multiplie by 2 (the numerator of the first) and that maketh 24, and so haue I for the first fraction

fraction $\frac{44}{80}$.

Likewise for the second fraction: I diuide 60 by 4, and there commeth 15: which I multiplie by 3, and so haue I 45 and the second fraction $\frac{45}{80}$. Then for the third in like sort will come $\frac{40}{80}$. The second varietie.

Master. An other way is this. If it happen so that the lesser Denominator can by any multiplication make the greater, the note the multiplier, and by it multiplie the numerator ouer that lesser denominator, and for the lesser denominator put the greater, as thus in these two fractions, $\frac{3}{12}$ and $\frac{2}{3}$ the æ being the lesser denominator multiplied by 4, will make 12, which is the greater denominator: therefore by the same 4, I doe multiplie 2, which is the numerator ouer 3, and that maketh 8: vnder which I doe put 12 being the greater denominator, which is also made by multiplication of 4 into 3, & so haue I these 2 Fractions $\frac{31}{12}$, $\frac{8}{12}$: thus shortly reduced without altering the one fraction.

Scholer. This I vnderstand.

Master. Then marke this third way: If the denominators doe not happen so, ỹ one by multiplication may make ỹ other, then loke whether they both may be parts of any other one number, as in $\frac{5}{12}$ and $\frac{7}{18}$, although the lesser taken but twice, be to great to make 18, yet they both may be parts vnto 36: therefore loke how many times twelue is in 36, and The third varietie.

that

that quotient being multiplied by ȳ numerator ouer 12, ȳ total shall be put in stead of the numerator ouer 12, and for 12 put 36, thus, $\frac{15}{14}$. So likewise loke howe often is 18 in 36, and because it is twice, therefore by 2 multiplie 7 which is ouer 18, and it will be 14, set that for the numerator, and in stead of 18 put 36, and then shall your fractions reduced, stand thus, $\frac{15}{36}\ \frac{14}{36}$ in stead of $\frac{5}{12}$ and $\frac{7}{18}$.

And if you will prowe whether you haue wrought well or no, that may be prowed by Reduction of them againe to their former denominations, which arte shall be taught in the fourth kinde of Reduction, where greater termes of fractions be reduced into smaller in number, but no smaller in proportion. And if in such Reduction the same termes or numbers come again that were before, then is the worke good, else not.

Scholer. Sir, I heare your words, but I doe not vnderstand many of them, which it may please you to declare.

Master. With a good will, when conuenient place serueth, but that must be in the said fourth kinde of Reduction. In the meane season I will declare the second forme of Reduction, which teacheth howe to reduce fractions of fractions into one fraction, and so to one denomination.

When fractions of fractions be proponed, you shall multiplie the numerators of each into

into other, and set the totall for the newe numerator, and then multiplie all the denominators likewaies, and take their totall for the newe denominator, and so are they spædily reduced.

Reduction of fractiõs of fractiõs into one denomination.

Scholer. If that be all then I vnderstand it alreadie, as by this example I wil declare. These be the fractions, $\frac{3}{4}$ of $\frac{5}{7}$ of $\frac{7}{8}$ which I would reduce to one denomination.

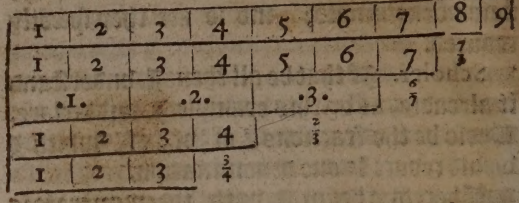
Therefore begin I with the numerators & multiplie them all together, saying: 3 into 2 maketh 6, and 6 by 6 maketh 36, which multiplied by 7, yeldeth 252, that I set ouer a line for the numerator, thus. 252

Then I multiplie the denominators, 4 by 3 maketh 12, & that by 7 bringeth 84, which multiplied by 9, yeldeth 756, the newe denominator. And so the whole reduced fraction is this, which is to hard a fraction for me to vnderstand yet. $\frac{252}{756}$

Master. You thinke so, and no maruel, but anon you shall learne to iudge it easilie, for this fraction is no more in deede than $\frac{1}{3}$, although it be in greater termes, and therefore more straunger and more obscure.

And this sufficeth for this Reduction, saue that I will shewe you by a figure of measure, the iust rate and reason of this kinde of fractions, and also the due vnderstanding of the Reduction.

The entire measure parted into 9.



Here you see the longest measure, (which standeth for the whole and entire quantitie) first parted into 9 Divisions, whereof 7 are severed by the second measure: & thereof againe are parted out 6. And that 6 being distinct into 3 parts, 2 of thē are parted by the fourth measure, of which fourth measure, being divided into 4 parts, the lowest measure doeth containe $\frac{1}{4}$, so that the same $\frac{1}{4}$ must be named, not $\frac{3}{4}$ of the whole measure, but in deede is $\frac{1}{3}$ of $\frac{2}{3}$ of $\frac{6}{7}$ of $\frac{7}{9}$.

Scholer. This example is so sensible, that I cannot chuse but see it. And furthermore, I see also that the same fraction is equall to $\frac{7}{9}$ of the entire measure, as the lines which run vp and downe doe expressely set forth. Also I see here, that $\frac{2}{3}$ of $\frac{6}{7}$ of $\frac{7}{9}$ is equall to $\frac{4}{9}$. And further yet that $\frac{6}{7}$ of $\frac{7}{9}$ is equall to $\frac{6}{9}$ or $\frac{2}{3}$.

Master. I am glad that you see it so wel, not doubting but you wil gather greater light of knowledge hereby.

But

But nowe it is time that we come to the third forme of Reduction, which teacheth of improper fractions, that is to say, mixt numbers of vnites and fractions, although they appeare like fractions, as this $\frac{26}{5}$, which doeth include 5 vnites wholly, and $\frac{1}{5}$ ouer. Wherefore firſt you ſhall knowe them, by that the numerator is greater than the denominator. The third forme of Reduction of improper fractions.

Scholer. In déede ſir that appeareth reaſonable, that if the numerator doe expreſſe more parts to be taken of any vnite than the denominator doth ſignifie that vnite to be diuided into, it muſt néedes followe that ſuch a fraction importeth more than the whole, that is to ſay, the whole with certaine partes ouer. But what Reduction is there in it?

Maſter. There be two ſeuerall kindes of Reduction, concerning ſuch fractions. Sometimes it ſhall be néedeful to conuert thoſe fractions into the vnites and the proper fraction that will remaine. And ſometimes contrarie waies, it ſhal be méet to reduce mixt numbers, that is vnites, written with fractions into the forme of one ſimple fraction, and ſo be there two wayes.

Scholer. What is the meane of the firſt way to turne improper fractions into vnites with their proper fractions?

Maſter. That is thus. Your numerator béeing greater than the denominator, muſt be diuided by the ſame denominator, and the

Reduction of improper fractions into vnits with their proper fractions.

quotient thereof expresseth the vnites, the remainer shall be put for the numerator of the fraction that resteth, and the denominator must be the same that was before.

Scholer. For example, I take $\frac{17}{5}$. And diuiding 17 by 5, the quotient wil be 3, and there will remaine 2.

Master. That must you write thus, $3\frac{2}{5}$ where (you see) I haue written 3, without any line, as entire numbers ought to be written, and the 2 that remained, I haue set ouer the former denominator with a line, as a proper fraction. And this number doeth signifie nowe 3 vnits, and $\frac{2}{5}$ of one.

Scholer. Then if I would by vnits here vnderstand crownes, so it were 3 crownes, and $\frac{2}{5}$, that is 2 s.

Master. Euen so, and therefore $\frac{17}{5}$ did signifie the same. But this happeneth sometimes, that when the Reduction is so wrought, there remaineth nothing. And then it is not a mixt number, but a simple entire number, represented like a fraction.

Reduction of whole numbers, either alone, or ioyned with fractions into fractions.

Scholer. As $\frac{12}{2}$ will make 3 iust, and $\frac{18}{3}$ will make euen 6. This I will remember. But nowe, what is the second forme of Reduction, that you spake of for these sorts of fractions?

Master. Whensoeuer you haue any of these two sortes of numbers, that is to say, whole numbers without fractions, or whole numbers with fractions, and you would turn them

them into the forme of a fraction, you must multiplie the whole number by that denominator, which you will haue to remaine still, and to the totall thereof adde the numerator which you haue alreadie, and all that shall you set for the newe numerator, keeping still the former denominator: as if you haue $6\frac{3}{4}$ which you would conuert into an improper fraction, you must multiplie 6 by 4, whereof commeth 24, and thereto adde the numerator which is 3, and so haue you 27 for the numerator, and 4 still for the denominator.

Scholer. Then is $\frac{27}{4}$ equall to $6\frac{3}{4}$.

Master. Euen iust, and so backward (as appeareth by the former reduction) $6\frac{3}{4}$ maketh $\frac{27}{4}$. And thus one of these Reductions may be the proofe of the others worke.

Scholer. This I perceiue, but nowe if you would turne whole numbers without fractions into any fraction, I see not how that may be done, because there is no denominator to make the multiplication by.

Master. That was well marked: but this you knowe, that no man entendeth to turne any whole number into a fraction, but hee hath in his minde that denominator by which the multiplication must be made: for the proofe whereof I set downe 7, which is a whole number. And if you will haue this number conuerted into any certaine fraction, will me to doe it. Note.

Scholer. I pray you reduce 7 into a fraction.

Master. Then you care not what the fraction be, so it be some fraction.

Scholer. No, I passe not for the sort of the fraction.

Master Then howe can you thinke that you require me to doe any thing certaine, when you leaue me to doe as I list? And seeing you stand at that stay, whether thinke you that I must first intende in minde what fraction I will make of it, before I can doe it indeede?

Scholer. Else you should doe ignorantly.

Master. Then will I limit my selfe (seeing you will not) to turne it into quarters. And therefore I multiplie 7 by 4 (which is the denominations of quarters) and there amounteth 28 to be set for the numerator, and the 4 must be set for the denominator, and the fraction will be thus, $\frac{28}{4}$.

Scholer. Indeede I perceiue this to be reasonable, for without much triall I vnderstand that $\frac{28}{4}$ of any thing doth make 7. And so then if I would turne 8 into fift parts, it wil make $\frac{40}{5}$ which is all one with 8, for 8 crownes turned into fift parts (that is, into shillings) will make 40 shillings, that is $\frac{40}{5}$ of a crowne.

Master. Seeing you vnderstand now these three kindes of Reduction, I willdeclare vnto you the fourth kinde, that is when fractions

be

be written in greater termes than they nede, howe they may be brought to lesser termes.

Scholer. To write any thing in greater termes than needeth, seemeth to be a fault, and so this rule seemeth to amend that fault. The fourth kinde of Reductiō.

Master. It were a fault to doe any thing without nede which after must be required: but in this case it is not so: neither did I say absolutely (as you doe) that it needeth not to expresse those fractions in so great termes, but that the fractions doe not nede, I meane for their value to be understanded: but yet it may be needefull for the ease of those workes whereto they be applied, as for example: In the first kinde of Reduction this was your owne example: $\frac{3}{16}$ and $\frac{4}{5}$, which when you would reduce, you were faine to turne them first into one denomination, and so appeared they thus, $\frac{18}{80}$, and $\frac{64}{80}$, where the fractions (for their owne understanding) neded not to be turned out of smaller termes into greater, but yet the easinesse of working neded it.

Scholer. Sir, I understand now, not onely the difference of this ned (for the fractions might better be understanded as fractions severall, each in his value, when they were in lesser termes, although they could not so well be reduced) but also I understand what you meane by greater termes and lesser termes, whereof before I was in doubt, for I see you Termes of fractions.

call the numerator and denominator, the termes of the fraction.

Master. I am glad you vnderstand it so wel. Nowe then when you would value any fractions (because that may best be done when the termes are smallest) you shal reduce them to the smallest that you can, which thing you may doe thus: Diuide the greatest of any such two termes by the lesser, and if any thing remaine by that remainer, diuide the last diuisor: and if any thing remaine nowe, by that diuide the last diuisor (which was before the remainer of the first Diuision) and so continue still, till nothing doe remaine in the Diuision: and then marke your last diuisor, for it is the number that will easilie reduce your fraction, if you diuide both the numerator and the denominator by the same number, and put for the numerator the quotient of his Diuision, and for the denominator also his quotient, that riseth by his Diuision.

Reduction of fractiõs into their smallest termes or least value

Scholer. I take for example $\frac{18}{96}$, and because 96 is the greatest number, I diuide it by 18, and the quotient is 5, and there resteth 6: what shall I doe with this quotient?

Master. Nothing in this worke, but nowe seeing there remaineth somewhat, by that remainer must you diuide the last diuisor.

Scholer. If I shall diuide 18 (which was the last diuisor) by 6, that was the remainer, so is the quotient 3, and nothing resteth.

Master.

Master. As for the quotient I omit him yet: but because there doth remaine nothing, therefore is 6 (which was your last diuisor) that number by which you may reduce the fraction proponed.

Scholer. Then as you taught me, I must diuide the numerator 18 by 6, and the quotient is 3, which I must put for the numerator ouer a line, thus. $\frac{3}{}$ And then by the saide 6, must I diuide also the denominator 96, and the quotient will be 16, which I must take for the denominator, and so is the fraction $\frac{3}{16}$. And so me thinketh this rule doth proue the worke of the first Reduction.

Master. That is true, if the first Reduction were made of fractions in their least termes, and else not, without some helpe, as the second number in that place wil declare.

Scholer. The second number was $\frac{2}{3}$, which was turned into $\frac{64}{96}$ by that rule. Nowe if I shal by this rule reduce it again into the least termes, I must diuide 96 by 64, and there remaineth nothing, wherefore I must take that 32 for the diuisor, to reduce the said fraction. Then doe I diuide 64 by 32, and the quotient is 2, which I set for my numerator. Againe, I diuide 96 by 32, and the quotient wil be 3, and so haue I but $\frac{2}{3}$.

Master. Muse not at the matter, for you haue done well ynough: but you thinke you haue

haue not the fraction that you loked for, that is $\frac{4}{5}$, yet haue you one equall to it, as by the parts of a shilling you may proue.

Scholer. Truth it is, for each of them will bring forth 8 pence, so that $\frac{8}{12}$ and $\frac{4}{6}$ and $\frac{2}{3}$, bee all three equall. And nowe I perceiue that because $\frac{2}{3}$ was not written in the least termes that it might be, therefore this Reduction brought forth not it, but that other which is written in the least termes. Nowe vnderstand I this rule well. But is there any other way to worke this Reduction?

Another way for to worke Reduction.

Master. Yes, but first note this, that if you finde no such diuisor, to reduce the fraction till you come to 1, because 1 doeth make no diuision, therefore that fraction is alreadie in his least termes, as by $\frac{71}{100}$ you may proue, & so of $\frac{81}{98}$ and many other like.

Note that to mediate any thing is to diuide the same by 2.

* But nowe for your better aide to finde the due proportion in least termes, with more ease for a young learner, you shall mediate or take the halfe of the numerator, and also the denominator as long as you may vpon a line, alwaies parting them with a right downe dash of your pen as you worke, which may easilie be done, if the numbers be euen: as 2, 4, 6, 8, or 10: But if they be odde (though it be but in one of them) then must you abbreuiate them by 3, 5, 7, or 9, &c.

And because examples doe most instruct, I haue here set downe the maner of 2 or 3, whose

whose last number at the ende of the line, sheweth the least terme or valuation of that fraction.

As for example, I would reduce $\frac{288}{576}$ into his least terme or value: whereupon I set forth $\frac{288}{576}$, with a long line drawen frō it thus $\frac{288}{576}$— And because both the numerator and the denominator ende in euen numbers, I see this may be abbreuiated by 2, or 4, or 6, &c. therefore on the other side of the right downe dash toward the right hand, I first take the halfe of the numerator: saying the halfe of 2 is 1, the halfe of 8 is 4, and againe the halfe of 8 is 4, which 144 in nowe a newe numerator, and therefore I part it with a right downe dash as before.

Then doe I also take the halfe of 576 in saying the halfe of 5 is 2, and the halfe of 17 is 8, and the halfe of 16 is 8, and so haue I 288 for a newe denominator.

Then beginning againe: saying the halfe of 144 is 71, and the halfe of 288 is 144: thus continuing the mediation or diuision by 2, vntill you come to the last worke as appeareth here in the example, where the same is reduced to $\frac{1}{2}$, which is equall to $\frac{288}{576}$.

288	144	72	36	18	9	3	1
576	288	144	72	36	18	6	2
	28	14	7	1			
	112	56	28	4			

Ab-

Abbreuiated first by 5, then by 293.

1465	293	1
4395	879	3

Scholer. Sir I thanke you much, this is verie easie, and good for a young learner. ‡

Master. So it is, but yet notwithstanding, if you can without that diuision by memorie espie the greatest number that may diuide exactly both termes of your fraction proponed, then neede you not to vse that diuision, as in this fraction $\frac{60}{72}$ I see that 12 is the greatest number that can diuide them both: and therefore without any worke, by memorie onely, I turne that into $\frac{5}{6}$, but this abilitie in knowledge is gotten by exercise.

Yet one other way of easie Reduction in this kinde there is, when your fraction hath any ciphers in the first places of both termes, then may you by casting away the ciphers, make a briefe Reduction, as thus $\frac{300}{400}$, here take away the ciphers, and it will be $\frac{3}{4}$, which is the same in value with $\frac{300}{400}$.

Scholer. And so if I haue $\frac{400}{500}$, it will be $\frac{4}{5}$.

Master. You are deceiued, for you take away more ciphers from the Numerator, than you doe take from the denominator, which you may not doe.

Scholer. I confesse my fault, which came of too much hast, I was more gladder of the rule

rule than wise in vsing it: but nowe I vnderstand it I trust. The fift kinde of Reduction

Master. Then may I goe in hande with the fift or last kinde of Reduction, which teacheth howe to turne any fraction proponed into any other denomination that you list: or into any parts of common coines, waights, or measures, or such like.

For declaration whereof, first you shall marke whether your fraction be a simple fraction, either else a fraction of sundrie parts, I meane of more termes than 2. And if your fraction be a fraction of fractions, or otherwaies compound, you must reduce it to one simple fraction. And then marke well the denomination of that other fraction, into which you would turne this, for by that denominator you must multiplie the numerator of your first fraction, and the totall product thereof shall you diuide by the denominator of your first fraction, and that quotient shall be the numerator to the denominator proponed: as for example, I haue this fraction $\frac{1}{2}$, which I would turne into tenth parts, therefore I multiplie this 10 by 3, that is the numerator of my fraction, and there riseth 30, which I diuide by 5, and the quotient is 6, which must be the numerator to 10, and so $\frac{3}{5}$ will be $\frac{6}{10}$.

Scholer. This is easie ynough to doe.

Master. Then shall you see an other example of the same fraction that is not so easier

as

as if I would turne $\frac{3}{5}$ into viij. partes, prowe you that worke.

Scholer. I must multiplie 8 by 3, and there amounteth 24, which I diuide by 5, and the quotient is 4, then is the newe fraction $\frac{4}{5}$.

Master. And see you nothing doubtfull in this worke?

Scholer. I see that when 24 was diuided by 5, there remained 4, which I did not passe of, because ye speake nothing of any remainer, but onely of the quotient.

Master. By likelihode you remember what I said to you in Diuision of whole numbers, that you should not passe of the remainer there, but onely note it as a summe that could not be diuided without knowledge of fractions. Wherefore nowe marke this, that in al diuisions of whole numbers, when there is any remainer, you shall set it ouer a line as a numerator, and set the diuisor for the denominator, and that fraction doth make the diuision complete, and is part of the quotient: as if I would diuide 48 by 5, the quotient wil be $9\frac{3}{5}$: so in your former worke when 24 was diuided by 5, the quotient should be $4\frac{4}{5}$, and so the newe fraction should be thus: $\frac{4}{5}$ and $\frac{4}{5}$ of 8 that is $\frac{4}{5}$ of the entire number, and $\frac{4}{5}$ of $\frac{1}{8}$ part of any thing, which you may prowe by example of some coine.

Scholer. Then I take a crowne, whole is 3 s. Now if I would prowe whether ỳ 3 s be $\frac{4}{5}$ and

$\frac{4}{5}$ and $\frac{4}{5}$ of $\frac{1}{8}$ I shall haue a combrous worke to do.

Master. In deede for whole pence, your example is a litle troublesome: yet turning the crowne into halfe pence, it is easie ynough.

Scholer. That will I trie.

* First I see that $\frac{3}{5}$ of a crowne is 3 s: which is 36 d, or 72 halfe pence. Nowe if I can find that this fraction $\frac{4}{5}$ and $\frac{4}{5}$ of $\frac{1}{8}$ be equall vnto 3 s: then am I fully aunswered.

Because I cannot take $\frac{4}{5}$ of a crowne, I turne the crowne into halfe pence, as you willed me, which makes 120, which I diuide by 8, my quotient is 15, which taken 4 times, makes 60 ob. Nowe resteth me to haue $\frac{4}{5}$ of the $\frac{1}{8}$ part of a crowne, whereof $\frac{1}{8}$ part is 15 ob. that 15 being parted into 5 parts, the quotient is 3, which taken 4 times maketh 12 ob. which with my 60 before, amounteth to 72, which are then equall to $\frac{3}{5}$ my desire.

Master. I commend you for your diligence, you might haue wrought it thus either: $\frac{4}{5}$ being abbreuied as before I taught is $\frac{1}{2}$. Nowe the halfe of a crowne is 2 s, 6 d. Then $\frac{4}{5}$ of $\frac{1}{8}$ is a fraction of fractions, which if you doe reduce into one entire fraction as before you haue learned, in saying 5 times 8 is 40, for a newe denominator and once 4 is 4, for a new numerator it maketh $\frac{4}{40}$, and abbreuiated also make but $\frac{1}{10}$. Now the tenth part of a crowne is 6 d, which put to 2 s, 6 d make also 3 s your desire. ‡

But

But nowe one example more for this rule, and then shall we ende it. If I haue $\frac{7}{15}$ of a Soueraigne (accounting the Soueraigne 20 shillings) howe many shillings is that $\frac{7}{15}$?

Scholer. I must multiplie 7 by 20, and that maketh 140, which I shall diuide by 15, and the quotient will be $9\frac{5}{15}$: or else in lesser termes, $9\frac{1}{3}$.

Master. That is 9 s, and one third parte of a shilling, that is 4 d, as by this same rule you may proue. And this for this time shall suffice for Reduction, saue that I must nowe repeate a litle touching Multiplication and Diuision, and so goe forward.

Multiplication.

IN Multiplication it happeneth sometime that there be whole numbers to be multiplied with fractions: And may be in two sorts, for either the whole number is seuerall from the fraction, and is the multiplier, or else, the whole number is ioyned with one, or both of the fractions, and so maketh a mixt number thereof. If it be in the first sort, then needeth there no Reduction, but onely multiplie the numerator of the fraction by that whole number, and the totall thereof set for the newe numerator.

Reduction of whole numbers into fractions.

Scholer.

Scholer. I vnderstand you thus. If I haue $\frac{6}{23}$ to be multiplied by 16, then must I multiplie that 16 with 6, which is the Numerator, whereof commeth 96, and that must I set for the new Numerator, keeping still 23 for the Denominator, and so the Fraction will be $\frac{96}{23}$ that is $4\frac{4}{23}$.

Master. And in this sort of worke you may abridge the labour, thus. If it happen the Denominator to be such a number, as may euenly be deuided by the saide whole number proposed, then deuide it thereby, and set the quotient of that diuision for the former denominator: but reserue still the Numerator, and so is the Multiplication ended.

Scholer. Then I faine this example, $\frac{7}{20}$ to be multiplied by 5. And because 5 will iustly diuide 20, therefore I take the quotient of that diuision which is 4, & set in stead of 20, & so the Fraction wil be $\frac{7}{4}$, that is $1\frac{3}{4}$.

Master. Which is all one with $\frac{35}{20}$, that would haue followed of the other sort of worke.

Scholer. I perceiue it very well.

How to multiplie mixt numbers.

Master. Now then for the other sort where the number is mixt, take this way: first to reduce the saide whole number, and fraction into one fraction Improper (as I shewed you in Reduction) and then multiplie them together, as if they were proper fractions.

Scholer. $13\frac{3}{5}$ being set to be multiplyed by $\frac{1}{4}$, first I must reduce the mixt number, as

$13\frac{3}{5}$ / $\frac{68}{5}$

appeareth in the margent, by multiplying 13 by 5, and that maketh 65, whereto I must adde the Numerator 3, & so the fraction will be $\frac{68}{5}$, which two fractions now I shal multiply after the accustomed forme, & it wilbe $\frac{340}{40}$.

Master. You haue done well: and so may you see, that although most part of the formes of Multiplication may be wrought without Reduction, yet some cannot, as namely mixed numbers.

And yet one note more will I tell you of Multiplication, before we leaue it: That is, whensoeuer you would multiply any Fraction by 2, which commonly is called Duplation, you may doe it not onely by doubling the Numerator, but also by parting the Denominator into halfe, if it be euen.

Scholer. Then if I woulde double $\frac{5}{12}$, I may chose whether I will make it, $\frac{10}{12}$, or else $\frac{5}{6}$. And in deede I see that all is one, but that the diuiding of the Denominator seemeth the better way to make smaller tearmes of the Fraction, & so they shal need the lesse reductiõ.

Master. It is so: & nowe I shall not neede to tell you that Multiplication is proued by Diuision, and Diuision likewayes by multiplication, but the like workes that I shewed you in Multiplication, will I shewe you in Diuision also.

Diuision.

Diuision.

Diuision to diuide a whole nũber by a Fraction.

When any whole number shal be diuided by a fraction, you must multiply the saide whole number with the Denominator of the fraction, and set the totall thereof for the newe Numerator, and for the Denominator, set the Numerator of the fraction.

Scholer. Then 20 diuided by $\frac{3}{4}$ $\frac{20}{1}$ by $\frac{3}{4}$ wil make $\frac{80}{3}$, as here appeareth: $\frac{80}{3}$

To diuide a Fraction by a whole number.

Master. Euen so. But if you would diuide the Fraction by the whole number, then multiply the Denominator by the same whole number, & set the total for the Denominator, without changing the Numerator.

Scholer. Then to diuide $\frac{20}{23}$ by 4, it will be $\frac{20}{92}$. As here appeareth $\frac{20}{23}$ by $\frac{4}{1}$. in this example. $\frac{20}{92}$.

Another brief way.

Master. You say wel. And by the same example you giue me occasion to remember an other briefe way to do the same: for if you had diuided the said Numerator by 4, & set the quotient for the Numerator, keeping stil the olde Denominator, it would haue bẽn not only as wel don, but also in a fractiõ of lesser termes.

Scholer. I gesse it to be euen so, by a lyke worke that you taught me in Multiplication. And for proofe thereof $\frac{20}{23}$ being the diuident, & 4 the Diuisor, I diuide the Numerator 20

by 4, and the quotient is 5, which I set for 20 ouer 23, thus $\frac{5}{23}$. And I see that it is all one with $\frac{20}{92}$, as by diuiding or abreuing both these termes by 4, and so reducing them to their least Denomination, I may easily proue: as appeareth in this example; $\frac{20}{92}|\frac{5}{23}$

Master. You conceiue it wel. And if there be mixt numbers (either one or both) you must first reduce that mixt number into an improper fraction. And then work as you haue learned.

Scholer. That was sufficiently taught in Multiplication. Therefore I pray you go forward to some other thing.

Master. Then take this note yet for diuision. If the Denominatours be like, then diuide the Numeratours as if it were in whole numbers, and the quotient whether it be fraction, whole number, or mixt, is a good quotient for that Diuision. And generally if one of the Numerators may iustly deuide the other, by that quotient multiplie the Denominator of the lesser Numerator, and set it that both amount in the rowme of the same denominator, and then for a Numerator to it, set the denominator of the other fraction.

Scholer. Then if I would diuide $\frac{3}{4}$ by $\frac{12}{17}$ I see that 3 will diuide 12, and the quotient wil be 4, by which I must multiplie the other 4 that is the Denominator vnder 3, and then it is 16, which I set for the Denominator 4, ouer

ouer it in stede of the 3, I must set 17, the other Denominator, and so is it thus, $\frac{17}{12}$.

Master. And so is $\frac{17}{18}$ in stede of $\frac{11}{18}$, which would haue risen by the common worke by $\frac{11}{17}$ worke: as here appeareth: $\frac{11}{18}$

And now for Mediation (which is to diuide by 2) marke this: If the numerator be euen, set the halfe of it in his place without the Diuisor, and so haue you done: and if the Numerator be not euen, then double the Denominator.

Scholer. That is if I would mediate $\frac{6}{11}$ I may make the quotient $\frac{3}{11}$ And if I would mediate $\frac{7}{11}$ I must make it $\frac{7}{22}$.

Master. Now trust I that you haue sufficient knowledge in Reduction, Multiplicatiō, and Diuision: and therefore wil I go in hand with Addition & Subtraction, which now wil appeare easie ynough.

Addition.

To adde Fractions of one denominatiō.

WHensoeuer you haue any fractions to be added, you must consider whether they be of one Denomination or not. And if they be of one denomination, then adde the Numerators togither, & set that that amounteth, for the Numerator ouer the common Denominator, & so you haue done. The reason is, because that such differ little in Ad-

dition or Subtraction from the work of vulgare denominations, where ỹ Denominators be no numbers: as 3 ð & 5 ð make 8 ð, where the denomination is not altered. But & if the fractions be not of one denomination, or any of them be mixt of whole numbers & fractiõs, then must you first reduce them to one denominatiõ, & after adde them. And if they be many, then adde first two of them, & to the summe ỹ doth amount of the Addition, ad the thirde, & then the fourth, & so forth, if you haue so many.

To add fractions of diuerse denominations.

Scholer. This semeth easie ynough, now ỹ I haue alreadie learned to multiply & to reduce, without which two, I could neuer haue wrought this. And therefore now I se good reason, why you did place Multiplication and Reduction before Addition.

M. It is wel cõsidered, but yet refuse not to erpresse your vnderstãding of it, by an exãple.

Scholer. Then would I adde first $\frac{7}{18}$ with $\frac{5}{18}$, & because the Denominators are like (and so needeth no Reduction) I adde 7 to 5 which maketh 12, & then is my summe $\frac{12}{18}$, that is in smaller numbers being abbreuiated $\frac{2}{3}$.

And if I haue many numbers to be added, as here $\frac{3}{8}$ $\frac{4}{5}$ $\frac{9}{10}$, (first I must reduce them, because they haue diuerse denominators) into one denomination, & then wil they be thus: $\frac{150}{400}$ $\frac{320}{400}$ $\frac{360}{400}$ or in lesse tearmes $\frac{15}{40}$ $\frac{32}{40}$ $\frac{36}{40}$, which by Addition do make $\frac{83}{40}$ that is $2\frac{3}{40}$.

Master. Now may we go to Subtraction.

Sub-

Subtraction.

Subtraction hath the same preceptes that Addition had, for if the Denominators be like, then must you subtract the one Numerator from the other, & the rest is to be set ouer the common denominator, & so your Subtraction is ended: but & if you haue many fractions to be subtracted out of many, then must you reduce them to one denomination, & into two seuerall fractions, that is, al that must be subtracted into one fraction, & the residue into an other Fraction, and then worke as I said before.

Scholer. For the first example I take $\frac{11}{12}$ to be subtracted out of $\frac{17}{12}$, and the rest will be $\frac{2}{12}$ or $\frac{1}{6}$.

For an other example I take $\frac{3}{4}$ to be subtracted out of $\frac{7}{8}$, which I must reduce, and it will be thus, $\frac{24}{32}$ and $\frac{28}{32}$.

Then doe I subtract 24 out of 28, and there resteth 4, which I set ouer the common denominator for a Remainer, thus, $\frac{4}{32}$, that is $\frac{1}{8}$.

Now for the third exãple, I take $\frac{3}{4}$ & $\frac{5}{6}$ to be subtracted from $\frac{7}{8}$ & $\frac{9}{10}$. And because their denominators be diuers, I do reduce them into one denomination thus: $\frac{1440}{1920}$ $\frac{1600}{1920}$ $\frac{1680}{1920}$ $\frac{1728}{1920}$.

Then do I adde the two first, & they make $\frac{3040}{1920}$. Also I adde the two last, and they yeelde $\frac{3408}{1920}$. Then do I subtract 3040 out of 3408, & there resteth 368, so is the remainer $\frac{368}{1920}$ that is in smaller Tearmes, $\frac{23}{120}$. And thus haue I done with Subtraction, except you haue any more to teach me.

Master. Proue one example more of two Fractions of diuers denominations.

Scoler. I take these two Fractions $\frac{7}{8}$ to be subtracted from $\frac{9}{24}$, which being reduced, will stand thus, $\frac{168}{192}$ and $\frac{72}{192}$. Now would I subtract 168 out of 72, but I cannot.

Master. Then may you perceiue that you mistoke the Fractiõs: for you can neuer subtract the greater out of the lesser, although you may adde, multiplie or diuide the greater with the lesser. And albeit that $\frac{7}{8}$ hath both his tearmes lesser than $\frac{9}{24}$, yet is $\frac{9}{24}$ the lesser Fraction: for generally if you multiply the Numerators and denominators of two fractions crossewaies, that fraction is the greatest, of whose numerator commeth the greatest summe, as in this example: 7 multiplied by 24, maketh 168: and 9 being multiplyed by 8, yeldeth but 72 therefore is the first Fraction $\frac{7}{8}$ the greatest of these two, so can you not subtract it out of a lesser Fraction.

The greatest of two fractions.

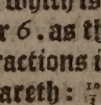
But and you should subtract a fraction out of a whole number what would you do?

Scholer. Marrie I would reduce the whole number

number into a Fraction of the same Denomination that my Fraction is, and then worke by Subtraction.

Master. So may you doe, but it is easier much, if your fraction be a proper fraction, ỹ is to say, lesse then an Unite, to take an Unite from the whole number, and then turne it into an Improper Fraction, and so worke your subtraction. As if I would subtract $\frac{2}{3}$ from 4, I may take one from 4, and turne it into $\frac{3}{3}$. from which if I abate $\frac{2}{3}$ there wil remaine $3\frac{2}{3}$. And if the first fraction be an Improper fraction, then may I take so many Unites from the whole number, that they may make an improper fraction greater then that first, and then worke by Subtraction: As if there be proponed $\frac{10}{3}$ to be subtracted from 6, because $\frac{10}{3}$ is more than 3, and not so much as 4, I must take 4 from 6, and turne them into thirds thus, $\frac{12}{3}$ then abate $\frac{10}{3}$, and there resteth $\frac{2}{3}$, so the whole remainer is $2\frac{2}{3}$. Or else you may at your pleasure take $3\frac{1}{3}$ which is $\frac{10}{3}$ from 6. whole: Then set one under 6. as thus $\frac{6}{1}$: And then to reduce those 2 fractions into one Denomination as here appeareth: $\frac{10}{3}$ from $\frac{6}{1}$.

Then $\frac{10}{3}$ from $\frac{18}{3}$ resteth $\frac{8}{3}$: which maketh $2\frac{2}{3}$ your desire:



And thus wil I make an end of the workes of common fractions for this time, not doubting, but you can apply them both unto the

rules of Progression, & also vnto the Golden rule, without any other teaching than you haue learned before, which might séeme tedious to repeat, saue that in some special diuersities, which be peculiar to fractiõs, I cannot ouerpasse, but instruct you somwhat by̆ way.

The golden rule.

THerefore as touching the Golden rule for the placing of the 3 numbers proponed in ỹ question, whereby to finde the thirde, & for the forme of their worke, with other like notes, I referre you to that which you haue alreadie learned.

But this easie forme of working by fractions shal you note, that if your thrée numbers be fractions, for an apt worke & certain, multiply the Numerator of the first number in the question, by the Denominator of the seconde: And all that againe multiplie by the denominator of the third number, and the total thereof shal you keepe for to be the Deuisor. Then multiplie the Denominator of the first number by the Numerator of the second, and the whole thereof by the Numerator of the third, and the total thereof shal be your diuident. Nowe diuide this diuident by the Deuisor which you found out before, and that number

shall

shall be the fourth number of the question, which you seeke for: As in this exãple. If $\frac{3}{4}$ of a yard of Veluet cost $\frac{2}{3}$ of a Soueraigne (estemed at twentie shillinges,) what shall $\frac{5}{8}$ cost?

Scholer. If it please you to let me make the aunswere, I would first place these three numbers, as I learned in whole numbers thus: $\frac{3}{4} \frac{2}{3} \frac{5}{8}$ A question of Veluet.

And then according to your newe rule, I must multiply 3 being Numerator in the first number, by 3 the Denominatour of the second, and thereof commeth 9, which I multiplie againe by 6, the Denominatour of the third number, & so haue I 54, which I keepe for the diuisor, then multiplie I 4, the denominator of the first, by 2, the Numerator of the second, and there riseth 8, which againe I multiplie by 5, the Numerator of the thirde, & it maketh 40: then must I diuide 40, by 54, and it wil be $\frac{40}{54}$ that is $\frac{20}{27}$ in lesser tearmes, and then the figure $\frac{3}{4} \frac{2}{3} \frac{5}{8} \frac{20}{27}$ wil stand thus:

But what that is in money, I can not tell except I shall worke it by Reduction, as you taught me.

Master. It forceth not now, you may reduce it when you list, but it were disorderlie done here to mingle diuerse workes together, where we do not seek the value of the thing in cõmon money, but in an apt nũber, which you haue wel done. And therefore will I yet shew you

you an other like way of easinesse in worke, how you may change your 3 Fractions into 3 whole numbers, by which you shall worke as if the question were proponed in whole numbers. The first number you shall finde as I taught you: nowe to finde the diuisor of the seconde number, take the Numerator for the second fraction: and for the third number take that that riseth of Multiplication of the Denominator of the first, by the Numerator of the thirde, and then worke your question.

A question of siluer.

Scholer. For example hereof, I put this question, If $\frac{11}{12}$ of 1 ℔ waight of siluer, be worth $\frac{11}{4}$ of a Soueraigne, what is $\frac{1}{3}$ of 1 ℔ weight worth? For the aunswere, first I place the Fractions in order thus: $\frac{11}{12} \quad \frac{11}{4}$ $\frac{1}{3}$

Then to turne these fractions into whole numbers, I multiply 11 which is the Numerator of the first, by 4 (the Denominator of the seconde) and there commeth 44, which I multiplie by 2 the Denominator of the third, and so amounteth 88, which I set for the Deuisor in the first place. Then in the seconde place I set 12, which is the Numerator in the seconde Fraction, and in the third place I set the summe that amounteth of 12, being the Denominator in the first number, multiplyed by 1, being Numerator in the third number, and so the figure 88 12 12 wil stand as here you see.

Then

Then to worke it forth, I multiplie 12 by 12, and there amounteth 144, which I diuide by 88, and the quotient will be $1\frac{56}{88}$, or in lesser termes, $1\frac{7}{11}$ and then the figure will stand thus. $\frac{1}{2}$ — $1\frac{7}{11}$

Maister. These two formes now you vnderstand wel ynough. And as for any other at this time I will not repeate, onely this shall you marke for the proofe of this rule, whether your worke be well wrought or no. Multiply the first number by the fourth, and note what amounteth: then multiplie the second by the third, and marke what amounteth also. Now if those two numbers so amounting be equal, then is your worke well done, else you haue erred. And this shall suffise for the former rule, but in the backer rule, this shal you note for ease of worke, that you multiply the Numerator of the first by the Numerator of the second, and the whole thereof by the Denominator of the thirde, and that amounteth thereof, shalbe the Diuident. Then multiplie the Denominator of the first by the Denominator of the seconde, and that whole by the Numerator of the thirde, and that riseth thereof shal be the Diuisor. Example of this: I did lend my friend $\frac{3}{4}$ of a Portegulse vij. monethes, vppon promise that he should doe as much for me againe: and when I should borrow of him, he coulde lend me but $\frac{5}{11}$ of a Portegulse, now I demaund how long time must

The proofe of the golden rule.

The Backer rule.

A question of loane.

I

I kepe his money in iust recompence of my lone, accompting 13 monethes in the yeare?

Scholer. The first number must be the first money borowed, that is $\frac{3}{4}$ of the Portegues: the second number the 7 monethes, that is $\frac{7}{12}$ of a yeare: and the thirde number the money that was lent in recompence, that is $\frac{5}{12}$ of a Portegues: then I set the numbers thus: $\frac{1}{4} \frac{7}{12} \frac{5}{12}$

Then (as you taught mée) I multiplie three (being Numerator in the first number) by 7 the Numerator of the second number, & it maketh 21, which I multiply by 12, the denominator of the third, & so haue I 252 for the diuident: then I multiply 4 the denominator of the first, by 13 the denominator of the second, and it yeeldeth 52 which I multiplie againe by 5, the Numerator of the third, and it wil make 260, that is the diuisor. Then must I diuide 252, by 260, so it wil be in the smalest fraction, $\frac{63}{65}$ of a yere.

Master. And thus doe you sée some ease in working, better than to multiplie and diuide tediously so many Fractions. Another question yet will I propose, to the intent you may sée thereby the reason of the statute of assise of bread and Ale, which in all statute bookes in French, Latine and English, is much corrupted for want of knowledge in this art: for the right understanding whereof I propounde this question.

Statute of Assise of bread and Ale.

When

When the price of a quarter of Wheat is 2 s, the farthing white lofe shall wey 68 s, then I demaund, what shall such a Lofe wey, when a quarter of wheate is sold for 3 s? Question.

Scholer. The Question must be wrought as it is proued in whole numbers, and not in Fractions.

Master. You seme to say reasonably, how be it, in that Statute of Assise, the rate is made by the proportion of partes in a pounde weight Troie, else coulde it not be a statute of any long continuance, seeing the shillings doe change often, as all other moneys doe: but this Statute being well vnderstanded, is a continuall rule for euer, as I will anon declare by a newe table of Assise, conuerting the shillings into vnces and partes of vnces. Therefore here by a shilling you must vnderstand $\frac{1}{20}$ of a pound weight, and so by pence $\frac{1}{240}$ of an vnce, wherefore although ye might worke this question proponed by whole number well enough, for that time when the statute was made, yet to applie it to our time, & to make it serue for all times generally it is best to work it by fractions, setting for 2 shilling $\frac{2}{20}$: and for 68 shillings, $\frac{68}{20}$: and so for three shillings $\frac{3}{20}$, and then will the figure of the question stand thus: $\frac{2}{20}$ — $\frac{68}{20}$ / $\frac{3}{20}$ —

In which question because all the denominators be like, you shal worke only with the numerators.

Scholer.

Scholer. Then I shal multiplie 68 by 2, whereof commeth 136, which if I diuide by 3, the quotient wil be 45 1/3: but howe shall I make a Fraction of that to stand with the other?

Master. Haue you so sone forgotten what was taught you so lately? This is his forme.

Scholer. I remember it now and then it signifieth 45 twentie parts, & 45 1/3 the thirde deale of one twentie part. 20

Master. So is it, and that maketh in shillings, 45 s 4 d: whereby you may note one great error in the Statute bookes, which haue constantly 48 s in that Assise. And by this rule, if you examine the Statute, you shall finde many summes false, wherefore for the true vnderstanding of that statute & such like as I haue made mention of it, & somewhat recogniſed it, so do I wish that all gentlemen & other students of the lawes, woulde not neglect this arte of Arithmetike as vnnedefull to their studies. Wherefore, to encourage them thereto, and to gratifie both them & all other in generall, I will exhibite a Table of that part of the statute in two columnes, and in a third columne I will ad the correction of those errors which haue crept into it.

Here followeth the Table.

The

The price of a quarter of wheate.		The weight of a farthing white lofe by the ſtatute bookes.			The correction by iuſt Aſſiſe.		
s	d	l	s	d	l	s	d
1	0	6	16	0	6	16	0
1	6	4	10	8	4	10	8
2	0	3	8	0	3	8	0
2	6	2	14	4 1/3	2	14	4 2/3
3	0	2	8	0	2	5	4
3	6	2	2	0	1	18	10 2/7
4	0	1	16	0	1	14	0
4	6	1	10	0	1	10	2 2/3
5	0	1	8	2 1/2	1	7	2 2/3
5	6	1	4	8 1/4	1	4	8 8/11
6	0	1	2	8	1	2	8
6	6	0	19	11	1	0	11 1/13
7	0	0	19	1	0	19	5 1/7
7	6	0	18	1 1/2	0	18	1 3/5
8	0	0	17	0	0	17	0
8	6	0	16	0	0	16	0
9	0	0	15	0 1/4	0	15	1 1/3
9	6	0	14	4 3/4	0	14	3 15/19
10	0	0	13	7 1/2	0	13	7 1/3
10	6	0	12	11 1/4	0	12	11 3/7
11	0	0	12	4 1/4	0	12	4 4/11
11	6	0	11	10	0	11	9 21/23
12	0	0	11	4	0	11	4

In the common bokes there is no further rate of Assise made, than vnto 12 s the quarter of wheate: but in an ancient copie of 200 yeres olde (which I haue) there is added the rate of assise vnto 20 s the quarter, but yet was that assise also either wrong cast at the first penning, or else corrupt sith that time, for lacke of iust knowledge in the rule of proportion, which I will adde here also, to gratifie such as be desirous to vnderstād truth exactly.

The price of a quarter of wheate.		The weight of a farthing white lofe by the statute bookes.		The correction of the errors.	
s	d	s	d	s	d
12	6	11	0	10	10 14/35
13	0	15	0 1/2	10	5 7/33
13	6	10	1 1/2	10	0 8/9
14	0	9	7	9	8 9/7
14	6	9	2 1/3	9	4 16/39
15	0	9	1 1/2	9	0 4/5
15	6	9	1 1/4	8	9 9/31
16	0	9	0	8	6 0
16	6	8	6	8	2 10/15
17	0	8	3	8	0 0
17	6	7	10	7	10 2/7
18	0	7	6	7	6 2/3
18	6	7	3	7	4 8/17
19	0	7	2	7	1 17/19
19	6	5	10	6	11 9/13
20	0	5	6	6	9

These two tables I haue set seuerall, because no man should thinke ỹ I would either adde or take away from any lawe those parts which might of right seeme either superfluous, eyther diminute, but yet I may not be so curious as to neglect manifest errors, which is not onely my part, but euerie good subiects dutie with sobrietie to correct. And for auoyding of offence I haue rather done it in this priuate booke, rather than in any booke of the statutes selfe, trusting that all men will take it in good part.

Scholer. I would wish so, but I dare not hope so sith neuer good man that would reform errors, could escape the venemous tongues of enuious detractors, which because they either cannot or list not to doe any good themselues, doe delight to barke at the doinges of other, but I beseech you to stay nothing for their peruerse behauiour.

Master. I consider many things that some may obiect, whereunto I am not vnprouided of iust answeres, but I will not seeme so hastie to make the answers before I heare their obiections, but as I trust that men are of a better nature, and more gratefull nowe than some hath beene in time passed, as I haue done in the statute of Assise for bread in rate of s, so wil I set forth the like table in pounds and ounces, and the parts thereof, that it may be easily applied to all times: but I meane not

by this to alter any word of the statute (being so good an ordinance, and of so great continuance) but onely to make it as a kinde of exposition and declaration of the saide statute, trusting that thereby the statute may be better vnderstoode, and consequently better put in execution. And here you shall note, that I haue accounted the shillings after the rate of lx. s̄, to the pound weight, because I esteeme it the most apt rate for our time. Wherefore if in the first colume you finde the price of wheate directly against it in the seconde columne, you may finde the weight of the farthing white loafe, in this our time: and if you double the number (as I haue done in the third columne) then haue you the weight of the halfe penie white lofe, and so in the fourth columne is set the weight of the penie white loafe. It nedeth not to tel you ỹ, that the sight doth testifie, howe that euerie colume is parted into 3 smaller pillers, whereof the first columne hath these 3 titles, pounds, shillings, and pence: the other 3 columnes haue eth of them these 3 titles, pounds, vnces, and penie weights. And as in the first columne xij d̄ make a s̄, and 20 s̄ make a pound, so in the other iij columnes xx pence weight maketh an vnce, and xij vnces doe make a pound.

A pound weight.

The

The price of a quarter of Wheate.

£	s	d
0	3	0
0	4	6
0	6	0
0	7	6
0	9	0
0	10	5
0	12	0
0	13	6
0	15	0
0	16	6
0	18	0
0	19	6
1	0	0
1	2	6
1	4	0
1	5	6
1	7	0
1	8	6
1	10	0
1	11	6
1	13	0
1	14	6
1	16	0
1	17	6
1	19	0
2	0	6

The waight of the Farthing white Loafe.

Pounds	Ounces	Penie waight
6	9	12
4	6	8
3	4	16
2	8	12 4/7
2	3	4
1	11	6 2/7
1	8	8
1	6	2 2/3
1	4	6 2/5
1	2	16 8/11
1	1	12
1	0	11 1/13
1	0	4 4/7
0	10	18 3/5
0	10	4
0	9	12
0	9	1 1/3
0	8	11 15/19
0	8	3 1/5
0	7	15 3/7
0	7	8 4/11
0	7	1 21/23
0	6	17
0	6	10 14/25
0	6	5 7/13
0	6	0 8/9

The waight of the halfe penie white loafe.

Pounds	Ounces.	Penie waight.
13	7	4
9	0	16
6	9	12
5	5	5 1/3
4	6	8
3	10	12 4/7
3	4	16
3	0	5 1/3
2	8	12 4/5
2	5	13 5/11
2	3	4
2	1	2 2/13
2	0	9 3/5
1	8	19 1/5
1	8	8
1	7	4
1	6	2 2/3
1	5	3 11/19
1	4	6 2/5
1	3	10 6/7
1	2	16 8/11
1	2	3 19/23
1	1	14
1	1	1 3/25
1	0	11 1/13
1	0	1 7/9

The waight of the penie white Loafe.

Pounds	Ounces.	Penie waight.
27	2	8
18	1	12
13	7	4
10	10	11 1/5
9	0	16
7	9	5 1/7
6	9	12
6	0	10 2/3
5	5	5 3/5
4	11	6 10/11
4	6	8
4	2	4 4/13
4	0	19 1/7
3	5	18 2/5
3	4	16
3	2	8
3	0	5 1/3
2	10	7 3/19
2	8	12 4/5
2	7	1 5/7
2	5	13 5/11
2	4	7 13/25
2	3	8
2	2	2 6/25
2	1	2 2/13
2	0	3 5/9

The price of a Quarter of Wheate.

£	s	d
	3	0
	4	6
	6	0
	7	6
	9	0
	10	6
	12	0
	13	6
	15	0
	16	6
	18	0
	19	6
1	0	0
1	2	6
1	4	0
1	5	6
1	7	0
1	8	6
1	10	0
1	11	6
1	13	0
1	14	6
1	16	0
1	17	6
1	19	0
2	0	6

The waight of a farthing white loafe in sterling money by assise.

£	s	d
2	5	4
1	10	2 2/3
1	2	8
0	18	1 3/5
0	15	1 1/3
0	12	11 3/7
0	11	4
0	10	0 8/9
0	9	0 4/5
0	8	2 10/11
0	7	6 2/3
0	6	11 9/13
	6	9 3/5
	6	0 8/15
	5	8
	5	4
	5	0 4/9
	4	9 5/19
	4	6 2/5
	4	3 17/21
	4	1 5/11
	3	11 7/25
	3	9 2/3
	3	7 13/25
	3	5 11/13
	3	4 8/27

The waight of a farthing white loafe in Troy waight by assise.

Pounds	Ounces.	Penie waight.
2	3	4
1	6	2 2/3
1	1	12
0	10	17 3/5
0	9	1 1/3
0	7	15 3/7
	6	16
	6	0 8/9
	5	8 4/5
	4	18 10/11
	4	10 2/3
	4	3 7/13
	4	1 3/5
	3	12 8/13
	3	8
	3	4
	3	0 4/9
	2	17 6/19
	2	14 2/5
	2	11 17/21
	2	9 1/3
	2	7 7/33
	2	5 2/3
	2	3 13/25
	2	1 11/13
	2	0 8/27

The waight of a halfe penie white loafe in Troy waight by assise.

Pounds	Ounces.	Penie waight.
4	6	8
3	0	5 1/3
2	3	4
1	9	1 5/3
1	6	2 2/3
1	3	10 6/7
1	1	12
1	0	1 7/9
	10	17 3/5
	9	17 9/11
	9	1 1/3
	8	7 1/13
	8	3 1/5
	7	5 1/13
	6	16
	6	8
	6	0 8/9
	5	14 10/19
	5	8 4/5
	5	3 33/21
	4	19
	4	14 14/33
	4	11 1/3
	4	7 1/25
	4	3 16/27
	4	0

The waight of a penie white Loafe in Troy waight by assise. ‡

Pounds	Ounces	Penie waight.
9	0	16
6	0	10 2/3
4	6	8
3	7	10 2/3
3	0	5 1/3
2	7	1 5/7
2	3	4
2	0	3 5/9
1	9	15 1/5
1	7	15 7/11
1	6	3 2/3
1	4	14 2/13
1	4	6 2/5
1	2	10 2/13
1	1	12
1	0	16
1	0	1 7/9
11		9 2/89
10		17 3/5
10		7 5/21
9		18
9		9 5/23
9		2 2/3
8		14 2/25
8		7 5/13
8		1 3/27

Place this Table betweene 324 and 325.

Entle Reader touching the vnderstanding of this Table, wherein according to our time Master Recorde alloweth 60 pence to the ounce, and 3 l. or 60 shillings to the pound. And thereupon after the rate of 60 shillings to the pounde Troy doth he frame or produce this his Table, beginning at 3 shillings the quarter till he comes to 40 shillings 6 pence the quarter, And this his proportion for that he hath not set downe any one example to continue the worke, hath beene harde for many to conceiue or comprehend, and therefore the onely chiefe cause why I haue written this digression for the better vnderstanding of him therein. The first thing therefore that is sought for in this Table, as in the other aforesaide, is a Maxime grounded vpon the statute, which is this. When the quarter of Wheate is sold for 2 shillings, then the farthing white Loafe shall way 68 shillings, where by a shilling is meant $\frac{1}{20}$ of a pound, and by a penie $\frac{1}{240}$ of an ounce. Nowe therefore for a generall rule to finde what waight the farthing white loafe shal way at 3 shillings the quarter, till you come to 40 shillings 6 pence the quarter, is thus to be wrought. Comming to the first ground, and working by the Backer Rule, say: If 2 shillings the quarter giue or allowe the farthing white loafe to way 68 shillings, what waight ought the farthing white loafe to way at 3 shillings the quarter? Worke and you shall finde 45 shillings, 4 pence, as before in the correction of the first Table is noted. Then for the second worke say by the Rule of 3 direct, If 20 pence giue 1 ounce, what giueth 45 shillings 4 pence? Multiplie and diuide, and you shall finde 544 ounces, which 544 ounces being multiplied by 3, for 3 pound or 60 shillings yeldeth 1632 ounces, which diuided by 20, produceth 81 ounces and $\frac{12}{20}$, or rather $\frac{3}{5}$ of an ounce equall vnto 12 penie waight, which is halfe an ounce, and 2 penie waight, and so maketh in all 6 pound, $9\frac{1}{2}$ ounces, and 2 penie waight. Nowe the next way to continue this Table, to knowe the waight of the halfe penie white loafe is thus. Multiplie 1632 ounces by 2, and it bringeth forth 3264 ounces, and diuided by 20, it yeldeth 163 ounces, and $\frac{12}{20}$, which is equall to 13 pound, 7 ounces, and 4 penie waight, as Master Recorde his Table noteth. Thirdly for the waight of the penie white loafe multiplie 1632 ounces by 4, and diuide by 20, and after by 12, as afore. And you shall finde 27 pounde $\frac{1}{2}$ ounce, and 3 penie waight, &c. This method or else by doubling the farthing white loafe for the waight of the halfe penie white loafe, and so doubling the halfe penie white loafe for the waight of the penie white loafe is the order to continue this Table to the ende thereof.

Thus much for the vnderstanding of this Table here placed by Master Recorde a man endued with rare knowledge in Arithmeticall and Geometricall proportions touching the statute of Coynage, and the standarde thereof, as appeareth in his Epistle of this Booke dedicated to king Edwarde the first. Insinuating vnto his Highnesse that the standard of the Coyne is much altered from the 14 yere of king Edwarde the third, (when this statute and assise was confirmed) to the standard in this our time. For it appeareth that in king Edward the thirds time, when this assise of bread and drinke was established, that a sterling penie round without clipping, did then way 32 Cornes of Wheate, drie and taken out of the midle of the Eare, and 20 of these pence made an ounce, and 12 ounces made a pound Troy. And so from the waight of a penie to 20 shillings sterling, which then wayed 12 ounces, toke bread his waight and proportion: and nowe finding 60 pence in an ounce. That onely cause I perceiue for zeale to a common wealth moued him to set downe this table in this priuate booke, meaning not thereby to alter any word of the statute, being so good an ordinance and of so long continuance, but as a kinde of exposition by the way, that thereby the statute may be better vnderstoode, and so consequently better put in execution: which assise of his is three times greater than the statute nowe alloweth. Therefore also to gratifie such as are desirous of knowledge (according to these prices of a quarter of wheate) I haue added to this Author three other newe Tables, and reduced their prices into their iust proportion of sterling money, and also reduced the money into knowen waight Troy according to the statute. And thereafter according to proportion in my three other Tables, haue I noted the iust waight, that a farthing, halfe penie, and penie white loafe ought to way by the statute. ‡

Scholer. Sir, I doe thanke you most heartily for this, not onely in mine owne name and in the name of all Students, but also in the name of the whole Commons, to whom the restitution of this Assise (I trust) shall bring restitution of the weight in bread, which long time hath beene abused. And if you know any like things more, wherein you would vouchsafe to declare the errors, and set forth the trueth, you cannot but obtaine great thankes of all good hearted men that loue the common wealth.

Master. I haue sundrie things to declare, but I haue reserued them for a priuate booke by it selfe, yet notwithstanding because the statute of the rate of measuring of ground is so common that it toucheth all men, and yet no more common than needefull, but so much corrupt, that it is too farre out of all good rate, not onely in the English bookes of Statutes commonly printed, but also in the Latine bookes, and in the French also, for I haue read of each sort, and conferred them diligently, I will giue you a Table for the restitution of those errors, as may suffice for this present time. And first wil I propose one question to you touching the vse of that Statute, whereby you may perceiue the order howe to examine the whole statute, and euerie parcel thereof, and the question is this. A question of measure of ground.

When the Acre of ground doeth containe

foure perches in breadth, then must it containe 40 perches in length: then doe I demaund of you, howe much shall the length of an Acre be, when there is in the breadth of it 13 perches: but before you shall aunswere to this question, I will declare vnto you an other Statute, which is the ground of the former statute. And that statute is this. It is ordained that 3 Barlie cornes, drie & round, shall make vp the measure of an inch: 12 inches shall make a foote, and 3 foote shall make a yarde (the common English bookes haue an elne) fiue yards and a halfe shall make a perch, and 40 perches in length, and 4 in breadth, shall make an Acre. This is that statute: whereby ye may perceiue, that the intent of the statute is, that one Acre should containe 160 square perches. Nowe let me heare your answere to the question.

A statute of measures.

An Acre.

Scholer. As I perceiue by the wordes of that Statute, a perche to be $\frac{1}{160}$ of an Acre, so could I make those numbers all in fractions, and so worke the question: but seeing I may doe it also in whole numbers I take that forme for the most easie, therefore thus I set the question in forme. Then doe I multiplie 40 by 4, and it maketh 160, which I diuide by 13, and the quotient is $12\frac{4}{13}$.

4 \ 40
13

Master. Nowe turne that $\frac{4}{13}$ into the common parts of a perche, as they be named in the

the former statute: howbeit, it shall be best to take one of the least parts in Denomination for auoyding of much labour, as foote, whereof the perche containeth $16\frac{1}{2}$.

Scholer. Then to turne $\frac{4}{13}$ into foot, I multiplie $16\frac{1}{2}$ by 4, and it maketh 66, which I must diuide by 13, and the quotient is $5\frac{1}{13}$.

Master. So I finde that if the acre holde in breadth viij perches, it shall containe in length 12 perches, 5 foote, and $\frac{1}{13}$ of a foote, which is not fully an ynch, for the ynch is $\frac{1}{12}$ of a foote. But here all the Statute bookes in Latine and English (that I haue seene) doe note it to be 13 perches, 5 foote and 1 ynch: which maketh aboue 13 perches to many in the acre, so that I would haue thought the error to haue crept into the printed bookes by the great negligence that Printers in our time doe vse, saue that in written Copies of great antiquitie, I doe finde the same. Yet haue I one French Copie, which hath 12 perches $\frac{1}{2}$, and one foote, and that misseth very little of the truth. Note this errour.

Scholer. Then I see it is true that I haue often heard say, that the truest copies of the statutes be the French copies.

Master. That is often true, but not generally, as I haue by conference tried diuersly: but in this statute the french booke is most corrupt in all other places lightly.

But nowe to performe my promise, I wil

set forth the Table for measuring of an Acre of ground onely by such parts as the statute doth mention, because at this time I doe of purpose write it for the better vnderstanding of the statute, & hereafter with other things I intend to set forth this same more at large.

In this Table following, I haue not done as in the other statute before compared by restitution with the faults crept into the statute, but onely haue written that true measure, which the equitie of the statute doeth pretende. For it were vile to iudge of so noble Princes & worthie Counsellers, as haue authorised and set forth this statute, that they would make one Acre in any forme greater than an other, but euerie one to be iust and equall with each other, which is the ground also of my worke, and hereby may all men perceiue howe needefull Arithmetike is to the students of the Lawe. But now I thinke best to make an end of these matters for this present time, sith the Table hath in it none obscuritie, that I should neede to declare.

The breadth.	The length of the acre.		The breadth.	The length of the acre.	
Perches.	Perches.	Feete.	Perche.	Perch.	Feete.
10	16	0	28	5	11 11/14
11	14	9	29	5	8 31/58
12	13	5 1/2	30	5	5 1/2
13	12	5 1/13	31	5	2 41/62
14	11	7 1/14	32	5	0
15	10	11	33	4	14
16	10	0	34	4	11 11/17
17	9	6 27/34	35	4	9
18	8	14 1/3	36	4	7 1/3
19	8	6 18/19	37	4	5 13/37
20	8	0	38	4	3 9/19
21	7	10 5/14	39	4	9/13
22	7	4 1/2	40	4	0
23	6	15 18/23	41	3	1 63/82
24	6	11	42	3	13 6/41
25	6	6 3/5	43	3	11 77/86
26	6	2 7/13	44	3	10 1/2
27	5	15 5/28	45	3	9 1/5

Scholer. In dede Sir, I vnderstande the Table (as I thinke) by those other which you set forth before. For in the first Columne is set the perches of the breadth of any Acre, and then in the 2 Columnes following appeareth howe mane perches and howe many fote the same Acre must haue for his length.

Master. You take it well: howbeit to speake exactly of breadth and length, the first Columne doth sometime betoken the breadth, and sometime the length: for properly the longest side of any square doth limit his length, and the shorter side doth betoken the breadth, yet it is no great abuse in such Tables, where a man cannot well change the title, to let the name remaine, although the proportions of the numbers doe change: for skill by the first Columne, is expressed the measure of the one side, and by the two other pillers in one Columne, is set forth the measure of ỹ other side. And this shall be sufficient nowe for the vse of the Golden Rule.

The rule of Felowship.

Nowe somewhat will I touch certaine other rules, which for their seuerall names may seeme diuerse rules & distinct from this, but in dede they are but branches of it: yet because

cause they haue not onely seuerall workings in appearance, but also pleasant in vse, I wil giue you a taste of each of them. As for the rule of Felowship, both single and double, with time and without time, I shall neede to say litle more than I haue alreadie saide in teaching the workes of whole numbers, yet an example or two will we haue to refreth the remembrance of the same, and to declare certaine proper vses and applications of it, as this for one.

A question of vnequall societie.

Foure men got a bootie or prise in time of warre, the prise is in value of money 8190 l, and because the men be not of like degree, therefore their shares may not be equall, but the chiefest person will haue of the bootie the third part, and the tenth part ouer: the second will haue a quarter and the tenth part ouer: the third will haue the sixt part: and so there is left for the fourth man a verie small portion but such is his lot (whether he be pleased or wroth) he must be content with one xx. part of the pray. Nowe I demaund of you, what shall euerie man haue to his share?

Scholer. You must be faine to answere to your owne question, else is it not like to be aunswered at this time.

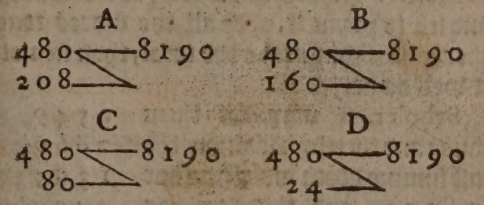
Master. The forme to vnderstand the solution of this question, and all such like, is this: Reduce all the denominators into one number by Multiplication, except that any of them

them be parts of some other of them, for all such parts you may ouerpasse, and take for them all those numbers, whose partes they be: as in this example the shares be these, $\frac{1}{3}\ \frac{1}{10}\ \frac{1}{4}\ \frac{1}{10}\ \frac{1}{5}\ \frac{1}{20}$ if I multiplie all the denominators together, beginning with 3, and so goe on vnto 20, it will make 144000: but considering that 3 is a part of 6, I shall omit that 3, and likewaies 10, which is a part of 20, I may ouerpasse also, & then is there but 3 denominators to multiplie, that is 4, 6, and 20, which make 480, which summe I take for my worke, because all the denominators will be found in it. Then I take such parts of it, as the question importeth, that is for the first man $\frac{1}{3}\ \frac{1}{10}$, the $\frac{1}{3}$ is 160, the $\frac{1}{10}$ is 48: which I put in one summe for the first mans share, and it maketh 208. Then for the second mans share, I take $\frac{1}{4}$, which is 120, and $\frac{1}{10}$ which is 48, and that maketh in the whole 168. Now for the third man which must haue $\frac{1}{5}$, I take 80. And for the fourth man there remaineth but 24, which is $\frac{1}{20}$ of the whole sum: so that if the whole pray had bene but 480 ℔, then were the question answered: but because the summe was of greater value, by this meanes nowe shall I knowe the partition of it. I must set my numbers by the order of the Golden rule, putting in the first place the number that I found by multiplying the denominators, and in the second place the summe of

the

the bottie. And loke what proportion is betweene the first number and that second, the same proportion shall be betweene the partes of that first number and the partes of the second, comparing each to his like. Therefore I must put in the third place, one of the parts or shares, and then worke by the former rule of proportion or Golden rule. And because I haue 4 seuerall parts of the first number, by which I would finde out 4 like partes of the second number, therefore must I make 4 seuerall figures. The reason of this rule.

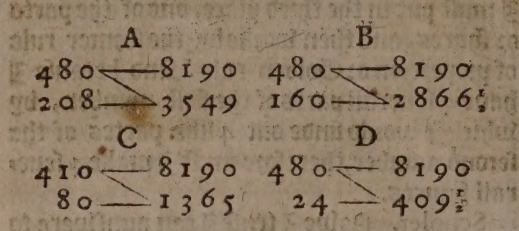
Scholer. Nowe I trust I can aunswere to your question, as by your fauour I wil proue.



And to trie it, I set the 4 figures thus, marked with A, B, C, D, to shewe their order. And then in each of them I multiplie the second number by the third, and diuide their totall by the first, and so amounteth the fourth summe which I seeke for, for if I doe multiplie 8190 by 280, it maketh 1703520, which being diuided by 480, maketh in the quotient 3549 for the first mans portion.

And

And so working with the other three figures, I finde for the second man 2866½, and for the third man 1365: and then for the fourth man 409½. And so is euerie mans share set forth in the figure here annexed.



And thus I thinke I haue done wel.

Master. If you misdoubt your working, and list to proue it, adde all the shares together, and if they make the total, then seemeth it well done.

The proue by Addition.

Scholer. I may set them thus: and then by Addition the iust summe doeth amount, that is 8190, and therefore (as you say) it seemeth to be well wrought.

3549	
2866½	
1365	
409½	
8190	

The iust proofe.

But I beseech you, is there any doubt in this tryall, that you vse that worde, Seemeth?

Master. You may easilie coniecture, that if you did assigne the first mans share to the last, and so change all the rest, that one had anothers share, yet would the Addition appeare all

all one, and therefore is not the proofe exact.

But if you will make a iust proofe for the first mans part, take $\frac{1}{3}$ and $\frac{1}{10}$ of the whole sum, and if it agree with the number in the figure, then it is well done. And so doe for the second, third, and fourth summes, and this proofe faileth not. Nowe will I propound certaine other questions which haue beene set forth by certaine learned men, albeit not without some ouersight, which questions I protest heartily I doe not repeate to depraue those good men, whose labours and studies I much praise and greatly delight in, but onely according to my profession, to seeke out trueth in al things, and to remoue all occasions of error, as much as in me lyeth: and for that cause I will onely name the questions without hurting the Authors name. The first question is this.

A question of building.

Foure men did builde a house, which cost them 3000 crownes, their shares were such, that one man should pay $\frac{1}{2}$ of the summe, and 6 crownes ouer: the second should pay $\frac{1}{3}$, and 12 crownes ouer: the third man must lay out $\frac{2}{3}$, abating 8 crownes, and the fourth man should pay $\frac{1}{4}$, and 20 crownes more, can you answere to this question?

Scholer. No in good faith sir, and that you knowe best of any man, for I knowe no more than you haue taught me.

Master. Then I dare say you cannot doe it

An impossible question.

it, neither yet the best learned man that euer did propose it, for the question is impossible: for declaration whereof I will be bolde to vse first the representation of the numbers in their aptest forme, (although I haue not yet taught that maner of worke) because it may appeare plainely that the question is not possible, for here I haue set the partes, and added them, and they make the whole summe and $\frac{1}{4}$ and 30 more. Nowe howe is it possible to diuide truely eyther gaines either charges so, that the particulars shall be more than the totall.

$\frac{1}{2}$	+ 6
$\frac{1}{3}$	+ 12
$\frac{2}{3}$	— 8
$\frac{1}{4}$	+ 20
$1\frac{1}{4}$	+ 30

Scholer. It is against the forme of proofe by Addition of parts.

Master. You say truth. And because you shall perceiue it the better, I will trie it after the vulgare forme, as in this figure you see where the $\frac{1}{2}$ with 6 ouer is 1506: for the totall is as you heard before 3000: the $\frac{1}{3}$ and the 12 more, is 1012: the $\frac{2}{3}$ wold be 2000, but then abating 8, it is but 1992, and then last of all, the $\frac{1}{4}$ is 750, and the 20 more maketh 770: which all beeing added in one summe, doe make 5280, where the totall summe should be but 3000, which summe of 3000 if you diuide by $\frac{1}{3}$, so shall you

1506
1012
1992
770
5280

you haue $\frac{3}{4}$ of it, that is 2250, and thereto adde 30 more, then will those 3 summes make 5280: whereby you may see how this forme as well as the other, 3000 doeth declare that the particulars in 2250 that question would make more than 30 the whole summe by $\frac{3}{4}$, and 30 more: 5280 and therefore can that question not be accepted as a possible thing, but yet do certaine learned men propounde such questions, and aunswere to them. Therefore somwhat to say to their excuse, rather of their good meaning than for their doing, I will anon declare what may be saide for their defence: but in the meane season I will propounde the question as it may be wrought by good possibilitie. As if foure men buyld a house together, and it cost them 3000 crownes, and then for the partition they agree thus: that as often as the first man doth pay 6 crownes, so often the seconde man shall pay 4, the third man 8, and the fourth man 3. Or else thus: that the first man shall pay double so much as the fourth, and the seconde man shall pay $\frac{2}{3}$ of the first mans charge: the thirde man shall pay double so much as the second: (and these two wayes are to one ende) but further for their agreement it is appointed also, that the first man shall giue 6 crownes ouerplus, and the seconde 12, and the fourth shall giue 20, but the thirde man shall giue no ouer-

plus, but shall haue 8 crownes abated of his charge. Now is the question possible to be assoyled, and this is the way to do it. Marke the proportion of the seuerall charges, and set out small numbers in that rate, by which you may reduce the worke to the Golden rule, as here in the first forme, the numbers are alredie named, 6, 4, 8, 3: & in the seconde forme, although they be not plainly named, yet they may be the same numbers: for 6 is double to 3, and 4 is $\frac{2}{3}$ of 6: and againe 8 is double to 4. Now adde these togither, & they make 21, which 21 must be set in the first number in the Golden rule: for if it with the ouerplus of eache mans charge woulde make the totall summe of the charges, then were those seuerall summes the charges of eche man, besides his ouerplus: but now it is not so.

The rule. But yet this is true, (so excellent are conclusions arithmeticall,) that loke what proportion each of these seuerall summes doeth beare to 21, the same proportion doth the iust charges of euery man (beside his ouerplus) beare to the totall of the charges, the ouerplus being deducted: wherefore this may you note, that before you doe apply the total of the charges to the Golden rule: you must deduct the ouerplus which is 6, 12, and 20, that is in the whole 38: but then 8 must be restored for the abatement of the third man, and then remaineth to be deducted 30, Take 30 there-

fore

fore out of 3000, and there wil reſt 2970, which I muſt ſet in the Golden rule for the ſeconde ſumme: and for the thirde ſumme I muſt put each of the ſmall numbers before mentioned, which although they be not the ſeuerall charges, yet they repreſent them in proportion. And ſo making for euery mans charge a ſeuerall queſtion, the figures will be 4, which I marke with foure letters, A, B. C. D. thus.

A

21 — 2970
6 — 848 4/7

B

21 — 2970
4 — 565 5/7

C

21 — 2970
8 — 1131 3/7

D

21 — 2970
3 — 424 2/7

Where I haue ſet for briefneſſe the ſumme of euerie mans charge in the fourth place preſuppoſing that you can tell howe to trye out that fourth ſumme by ſo many examples as yée haue had.

Scholer. As I truſt that I vnderſtande this fourme, ſo I deſire much to know what may be ſaide for them that miſtoke this queſtion.

Maſter. You ſéeme ſo deſirous to knowe this errour, that you haue forgotten to examin whether this worke be without.

Scholer. Yée ſéemeth this worke to be wel done, becauſe the Addition of the 4 ſeuerall

numbers doeth make the totall summe of 2970, which was to be diuided into such foure parts.

Master. But then haue you forgotten that the first man must pay 6 crownes more besides this share, and the seconde man twelue crownes more: the third man 8 crowns lesse: and the fourth man 20 crownes more, for without these, your first total of 3000 crowns wil not be made.

Scholer. Then must I adde to the first mans summe 6 more, and it will be $854\frac{4}{7}$: and to the second summe I must adde 12, and it will be $577\frac{5}{7}$: from the thirde summe I must abate 8, and then will the summe be $1123\frac{3}{7}$: then adding vnto the fourth summe 20, it will be $444\frac{3}{7}$: and these 4 summes will make 3000, which is the whole charge, as in this example it may appeare, where first I gather the $\frac{12}{7}$, that maketh 2, and so procéde I in the Addition to the ende.

	$854\frac{4}{7}$
	$577\frac{5}{7}$
	$1123\frac{3}{7}$
	$444\frac{3}{7}$
	3000

Master. Now haue you well done, and this worke in the same summes is brought of other learned men for the true solution of the question as it was first proponed, which as (I saide) was impossible: and nowe examine it by these seuerall summes, and see whether it do agrée with the summes in the question proponed.

The first man must pay ½ and 6 ouer the totall summe: how thinke you, is 845½ the halfe and 6 more of 3000?

Scholer. No that it is not, for it would be 1506: and for the second man 1012: and for the thirde man 1992: and for the fourth man 770, whereof not one summe agreeth to this worke. But I maruail that so wise men could be so much ouerséene.

Master. It is commonly séene, that when men will receiue things from elder writers, & wil not examine the thing, they séeme rather willing to erre with their auncients for companie, than to be bold to examine their works or writings, which scrupulositie hath ingendered infinite errors in all kindes of knowledge, and in al ciuil administration, & in euerie kind of arte: but these learned men did not meane any other thing by this question, than to find such numbers as should beare ỹ same proportion togither, as those numbers in the question proponed did beare one to another: which thing you shall perceiue more plainly by another question of theirs, that is this.

A man lying vpon his death bed, bequeatheth his goods (which were worth 3600 Crownes) in this sort. Because his wife was great with child, and he yet vncertaine whether the child were a male or female, he made his bequest cõditionally, that if his wife bare a daughter, then should the wife haue halfe his

A question of a testament.

goods, and the daughter $\frac{1}{3}$, but if shée were deliuered of a sonne, then that sonne should haue $\frac{1}{2}$ of the goods, & his wife but $\frac{1}{3}$. Now it chanced her to bring forth both a sonne & a daughter, the question is: How shall they part the goods agreeable to the testatour his will.

Scholer. If some cunning Lawyers had this matter in scanning, they woulde determine this Testament to be quite voide, and so the man to die intestate, because the testament was made insufficient, sith this condition was not expressed in it, and also it might haue chaunced that shee should haue brought forth neither sonne nor daughter, as often hath béene seene: so is the will vnsufficient in that point also.

Master. Such scanners should séeme too cunning, and yet not so cunning as cruel: for the minde of the Testatour is to be taken fauourably, for the aide of the legatories, when there riseth such doubts. But let vs trye this worke, not by force of law, but by proportion Geometricall, séeing the testatour did minde to prouide for each sort of them.

Scholer. If the sonne shal haue $\frac{1}{2}$ by force of the Testament, so must the mother haue $\frac{1}{3}$. Againe, because she hath a daughter also, therfore ought she to haue $\frac{1}{2}$ & the daughter $\frac{1}{3}$: that is both wayes $\frac{1}{2}:\frac{1}{3}$, and $\frac{1}{2}:\frac{1}{3}$, which commeth to the whole goods, and $\frac{2}{3}$ more. Wherefore it séemeth also impossible.

Master.

Ma. In this matter the mind of the Testatour is so to be vnderstande, that such proportion should be betweene the portion of the wife & the sonne, as is betweene $\frac{1}{3}$ & $\frac{1}{2}$, that is, the sonne must haue $\frac{1}{2}$ for $\frac{1}{3}$ to his mother. so shal he haue 3 to 2, that is as much as his mother, & half as much more: & the mother must haue ỹ like rate in cõparison to her daughter. Then must I find out 3 numbers in such proportiõ, that the first may be as much as the second, & half as much more (that is) in proportion sesquialtera, & the second to the third in the same proportion, such numbers be 6, 9, 4.

Scholer. I pray you sir, how shal I find out those numbers?

Master. That wil I gladly tel you.

To finde three numbers in any proportiõ.

Whatsoeuer the proportion be of any three numbers, multiplie the Termes of that proportion togither & the number that amounteth, shalbe the middle number of the 3: then multiply that middle number by the lesser terme, and diuide that totall by the greater, & the least number of the 3 wil amount. So if you multiplie that middle number by ỹ greater extreame, & diuide that totall by the lesser extreame, then will the greatest number of that progression amount.

To finde the proportion betweene numbers.

Scholer. Then in this example, to find the proportion of $\frac{1}{2}$ to $\frac{1}{3}$, I must diuide (as you taught me in Diuision) $\frac{1}{2}$ by $\frac{1}{3}$, & the quotient will be $\frac{3}{2}$, that is $1\frac{1}{2}$, whereby I perceiue that

the proportion in this question is, as 3 to 2. Therefore (as you taught me euen nowe) I multiply 3 by 2, and the summe is 6, which must be the middle number: then I multiply the middle number 6 by 2, which is the least tearme, and the summe is 12, that do I diuide by 3. being the greater Terme, & the quotient is 4, so is 4 the least nũber of the 3. Then I multiplie 6 by 3, whereof commeth 18, and that I diuide by 2 and so haue I 9, which is the greatest number of the 3.

Master. Another way yet may you find the third number in any progression, if you haue 2 of them: for if the middle number be one of them which you haue, then multiply it by it selfe (as in this example 6 by 6 maketh 36) & that totall diuide by the other number which you haue, and the thirde number will be the quotient.

Scholer. Then if I diuide 36 (which commeth of 6 multiplyed by it self) by 4, ỳ quotient wil be 9, & if I diuide 36 by 9, the quotient wil be 4. But what if I know the first number and the thirde, and would haue the middle number?

Master. Multiply the two numbers together, & in their total you must seeke the roote of that number, & it shal be the middle number: but because as yet you haue not learned how to extract rootes, therefore vse the first forme which I haue taught you, till I teache you to extract

extract rootes. And now go forward with the answere to the same question.

Scholer. I perceiue then that the sonne must not haue $\frac{1}{2}$ of the goods, neither the mother $\frac{1}{3}$, nor yet the daughter $\frac{1}{4}$ but yet must the goods be diuided in such proportion, that the sonne shal haue 9 crowns for 6 to his mother: and the mother shal haue 6 crownes for euery 4 to the daughter. Then I apply it to the golden rule in three eräples thus: where the first number is the Addition of those three numbers 9, 6, 4, & the third is one of them seuerally: the seconde is the totall of the goods in the testament: & then by ỹ worke of the Golden rule I finde out the fourth number in euery worke, that is for the sonne $1705\frac{5}{19}$: for the mother $1136\frac{16}{19}$: and for the daughter $757\frac{17}{19}$, which three summes added together do make the summe of the whole goods, as may be seene by this example.

19 — 9 — 3600

19 — 6 — 3600

19 — 4 — 3600

1705 | 5/19
1136 | 16/19
757 | 17/19
3600

And this (me thinketh) I doe perceiue, that because in this case there is a necessarie remedie diuised against an vrgent inconuenience, therfore those learned mẽ thought they might vse the like libertie in that other question.

Master. Your gesse is good, but they had so good reason for them in the one, as they haue

in the other: as in another example of theirs, it may better appeare, that is this:

Another question of a Testament.

A man left vnto his three sonnes 7851 crownes to be parted in this sort, that the first sonne should haue $\frac{1}{2}$, the second sonne $\frac{1}{3}$, and the third sonne $\frac{1}{4}$, which is not possible, for $\frac{1}{2}$, & $\frac{1}{3}$, and $\frac{1}{4}$, doth make $\frac{13}{12}$: or $1\frac{1}{12}$, that is $1\frac{1}{12}$, so is it more than the whole: but reduce these fractions into one denomination, the least that they wil come to, and they will be $\frac{6}{12}, \frac{4}{12}, \frac{3}{12}$, & so may you part the goods in such proportion as these 3 Numeratours beare together: that is, the first to haue 6 for euery 4 to the second: and the seconde to haue 4 as often as the third hath 3: and so their portions will be for the first, $2623\frac{7}{13}$: for the seconde $2415\frac{9}{13}$: & for the thirde $1811\frac{10}{13}$, and those three shares added together, will make the totall summe of the whole goods, as you may easily see in this example.

$$\begin{array}{r} 3623\frac{7}{13} \\ 2415\frac{9}{13} \\ 1811\frac{10}{13} \\ \hline 7851 \end{array}$$

Another question is there proponed thus:

There is 450 Crownes to be diuided betweene 3 men, so that the first man must haue $\frac{1}{2}$ & $\frac{1}{3}$, the second man $\frac{1}{3}$ & $\frac{1}{4}$, the third man shall haue $\frac{1}{4}$ and $\frac{1}{5}$.

Scholer. I maruaile that any man should be so ouerseene to propounde that question as a thing possible, sith $\frac{1}{2}\frac{1}{3}, \frac{1}{3}\frac{1}{4}, \frac{1}{4}\frac{1}{5}$, do make $1\frac{45}{70}$ that is almost double the whole summe.

But I perceiue it might be thus proponed,

ned, that as often as the first man did receiue 50 Crownes, so often the seconde man should receiue 35, and the thirde man 27, for $\frac{1}{2}$ & $\frac{1}{3}$, is equall to $\frac{50}{100}$, and so is $\frac{1}{3}$ & $\frac{1}{4}$ equal to $\frac{35}{100}$, and $\frac{1}{4}$ & $\frac{1}{5}$, is $\frac{27}{100}$, and so working the question, the 3 figures wil appeare in this forme: whereby the first mans portion is founde to be $200\frac{50}{538}$: the seconde man parte is $140\frac{35}{538}$: the thirde mans share is $108\frac{27}{538}$: which in the whole doeth make 450 crownes that was the whole summe to bee diuided betweene them.

112 — 450
50 — $200\frac{50}{538}$

112 — 450
35 — $140\frac{35}{538}$

112 — 450
27 — $108\frac{27}{538}$

Master. And thus you are (I thinke) sufficiently instructed in the rule of Felowship.

The Rule of Alligation.

NOw will I go in hand with the Rule of Alligation, which hath his name, for that by it there are diuerse parcels of sundrie prices, & sundrie quantities alligate bounde or mixed tother, whereby also it might be wel caled the rule of mixture, & it hath great vse

The rule of mixture

vse in composition of medicines, and also in mixtures of metals, and some vse it hath in medicines of wines, but I wish it were lesse vsed therein than it is now adayes. The order of the rule is this: When any summes are proponed to be mixed, set them in order one ouer another, and the common number whereunto you will reduce them, set on the left hand, then marke what summes be lesser than that common number, & which be greater, and with a draught of your penne, euermore linke 2 numbers together, so that one be lesser than the common number, and the other greater than he, for two greater or two smaller can not well be linked togither, and the reason is this, that one greater and one smaller may be so mixed, that they will make the meane or common number verie wel, but 2 lesser can neuer make so many as the common number, being taken orderly: no more can two summes greater than the meane, neuer make ỹ meane in due order, as it shal appeare better to you hereafter. And as it is of necessitie to linke euery smaller (once at the least) with one greater, and euery greater with one smaller: so it is at libertie to linke them oftner than once, and so may there be to one question many solutions. When you haue so linked them, then marke how much eche of the lesser numbers is smaller than the meane or common number, & that difference set

The reason of this rule.

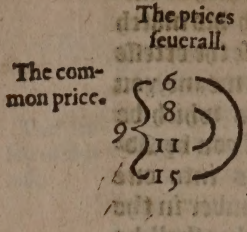
set against the greater numbers which be linked with those smaller, ech with his match still on the right hand, and likewise the exceſſe of the greater numbers aboue the meane, you shall set before the lesser numbers which be combined with them. Then shall you by Addition bring all these differences into one summe, which shall be the first number in the Golden rule: and the seconde number shall be the whole masse that you wil haue of al those particulars: the third summe shal be ech the difference by it selfe, and then by them shall be founde the fourth number, declaring the iuſt portion of euery particular in that mixture. As nowe by these examples I will make it playne.

There are foure sortes of Wine of seueral prices, one of 6 d a gallon, another of 8 d, the thirde of 11 d, and the fourth of 15 d the gallon, of all these Wines would I haue a mixture made to the summe of fiftie gallons, and so that the price of ech gallon may be 9 d. Now demaund I how much muſt be taken of euery sort of wine? A queſtiō of mixing of wine.

Scholer. If it shall please you to worke the first example, that I may marke the applying of it to the rule, then I truſt I shall be able not onely to doe the like, but also to see reason in the order of the worke.

Maſter. Marke then this forme & the plating of euery kinde of number in it,

The



Here (you see) I haue set downe the seuerall prices which be 6, 8, 11, 15, and haue linked together 6 with 15, and 8 with 11. The common price 9, I haue set on the left side: And the difference betweene it, and euery particular price, I haue set on the right hand not against the summe whose difference it is, but against the summe that it is linked withall: so the difference of 15 aboue 9, is 6, which I haue set not against 15, but against 6, that is linked with 15, and the difference betweene 6 and 9 (that is 3) I haue set against 15. So likewise the difference betweene 8 & 9 is but 1, that haue I set against 11, & the difference of 11 aboue 9 (which is 2) I haue set against 8. Then adde I all those 4 differences, and they make 12, which I set for the first number in the Golden rule: the seconde number I make 50, which is the summe of gallons that I woulde haue, and the thirde summe is euerie particular difference. Now if

if you worke by the Gode rule, you shall finde the number of Gallons that shall be taken of eache sort of Wine: For the better distinction whereof, I haue set these letters A, B, C, D, both against the numbers for which the workes doe serue, and ouer the workes also, which seuerally serue for eache of them. And nowe if you list to examine the trueth of these workes, adde those foure summes together, and they will make fiftie, that is the totall which I would haue, as by this example you may easily perceiue. And for to prooue howe the prices do agrée, doe this. Multiply this totall summe 50, by the common price 9, and it will make 450: then kéep that summe by it selfe, and afterwarde multiply euery seuerall summe of Gallons, by the price belonging to the same Gallons, and if that summe doe agrée with this which you haue kepte first, then is your worke well done. As here, 25 is the number of Gallons of 6 pence price, multiply then 25 by 6, and it maketh 150, which you shall set downe: then multiply $8\frac{2}{8}$ by 8, which is the price for the number of Gallons, & it will make $66\frac{4}{8}$: so againe $4\frac{1}{8}$ multiplyed by 11, doth make $45\frac{5}{8}$. And last of all $12\frac{3}{8}$ multiplied by 15, maketh $187\frac{3}{8}$. And these added togither doe

The profe of this rule.

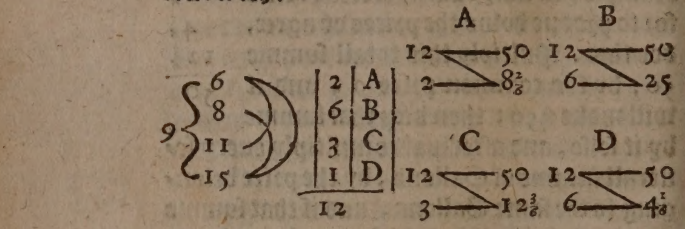
25—	
$8\frac{2}{8}$	
$4\frac{1}{8}$	
$12\frac{3}{8}$	
50	
150	
$66\frac{4}{8}$	
$45\frac{5}{8}$	
$187\frac{3}{8}$	
450	

do make 450, as in the example annexed you may see: wherefore seeing it doth agree with ye former summe of 50, multiplyed by 9, I may iustly affirme this worke to be good & wel don.

The variation of this question.

And now to proue how you can do the like, I propound the same question, onely willing you to vse some other forme of combining or linking the summes.

Scholer. That shall I proue with your fauour, and therefore I combine 8 with 15, and 6 with 11, and then the forme wil be as followeth:



whereby amounteth the same summe in total of the differences, as did before: and yet now the differences be altered, as the combination is chaunged, whereof I vnderstande the reason by your former worke. And therefore here appeareth no strange thing, but that nowe I must haue $8\frac{2}{8}$ gallons of fyre pence, and 25 gallons of 8 ỽ, and 12 gallons and $\frac{3}{8}$ of 11 ỽ, and so consequently 4 gallons & $\frac{1}{8}$ of 15 ỽ, so that multiplying $8\frac{2}{8}$ by 6, it maketh 50, &

50	
200	
$137\frac{1}{8}$	
$62\frac{3}{8}$	
450	

then

then 25 multiplied by 8, maketh 200: likewise $12\frac{3}{8}$ multiplied by 11, yéelded $137\frac{3}{8}$, & $4\frac{1}{8}$ multiplyed by 15, maketh $62\frac{3}{8}$, which 4 sums added in one, wil yéeld in the total 450, which agréeth with the multiplication of 50 (being the totall summe of gallons) by 9 the common or meane price.

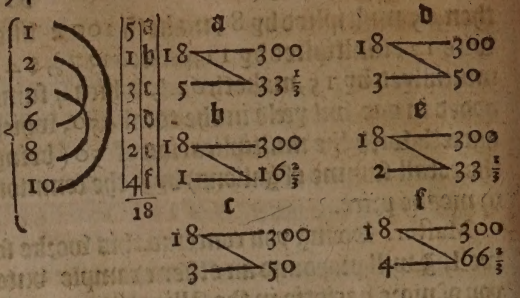
Master. Séeing you conceiue this worke so well, I will propound an other example vnto you of more varietie in the Alligations or combinings, as thus.

A merchant being minded to make a bargaine for spices in a mixt masse, that is to say, of Cloues, Nutmegges, Saffron, Pepper, Ginger, and Almondes, the Cloues being at 6 s a pound, the Nutmegges at 8 s, Saffron at 10 s, Pepper at 3 s, Ginger at 2 s, and Almonds at 1 s. A question of spices.

Nowe would he haue of each sort some, to the value of 300 l in the whole, & each pound one with another to beare in price 5 s, howe much shall he haue of each sort.

Scholer. That will I trie thus.

First I set downe those fiue seuerall prices, and at the left hand I set the common price 5 s. Then I linke them thus, 1 with 10, 2 with 6, and 3 with 8, as in the example following.



Master. I had minded to haue combined them in more varietie, but I am content to see your owne worke first, and then more varieties in combination may followe anone.

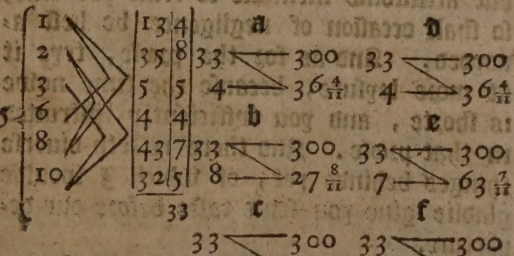
Scholer. Then to continue as I began, I seeke the difference betweene 1 and 5 (which is 4) and that I set against 10: then against 1 I set 5, which is the excesse of 10 aboue 5: so I gather the difference betweene 2 and 5, which is 3, and that I set against 6, because it is combined with 2: and likewise the difference of 6 aboue 5 (which is 1) I set against 2. Then take I the difference of 3 from 5, which is 2, and that I set against 8, and before that 3 I set the difference of 8 aboue 5, which is 3. Then gather I all these differences by Addition, and they make 18, which I set for my first number in the Golden rule, and so appeareth by those workes, that of Almonds I must take 83 l. 1/3, of Ginger 16 l. 2/3, of Pepper 50 pound, of Cloues 50 pound, of Nutmegges

megges 33 pound $\frac{1}{3}$, and of Saffron 66 pounde $\frac{2}{3}$. Then for tryall hereof, I multiplie euerie parcell by his seuerall price, as $83\frac{1}{3}$, which is the summe of Almondes, I multiplie by 1, which is their price.

	$83\frac{1}{3}$
	$33\frac{1}{3}$
	150
	300
	$266\frac{2}{3}$
	$666\frac{2}{3}$
	1500

Also $16\frac{2}{3}$ the summe of Ginger I multiplie by 2, which is the price of it. And so each other in his kinde, as this table annexed doeth represent, and then adding them altogether, I finde the totall to be 1500, which also will amount by the multiplication of the grosse masse of 300 by the common price 5, wherefore it appeareth well wrought.

Master. Nowe will I make the Alligation to proue your cunning somewhat better: but because you shall not thinke your selfe pressed so much, I will also note the differences, as in this example you may see where I haue alli-



gate 1 with 6 and 8, and therefore haue I set against 1 both their differences, that is 1 & 3. Likewise because 2 is combined with 8 and 10, I set before him their differences, 3 and 5. Against 3 I haue set onely 5, which is the difference of 10, with whom 3 is combined onely. Likewise 6 is onely alligate to 1, & therefore is the difference of 1 from 5: which is 4 onely set against it: 8 is linked with 1 and 2, and therefore hath he against him both their differences, 4 and 3, and 10 is ioyned with 2 and 3, therefore hath he their differences 3 & 2. And because of ease for you, in an other columne I haue set the differences reduced into one number, for euerie seuerall sort, and haue also added them together, whereby appeareth that they make 33, and so consequently you see the workes of the Golden Rule set forth for the fyre seuerall drugges, I haue added letters a, b, c, &c. as before. But I would not wish you to cleaue still to these elementarie aides, but accustome memorie to trust her selfe, so shall occasion of negligence be best auoyded. And as for the proofe, trye it at more leysure, because the time nowe is shorte, and you sufficiently instructed in that proofe. And there resteth diuerse thinges behinde yet, of which I woulde gladlie giue you some taste before our departure.

Scholer. But if it may please you to let me see

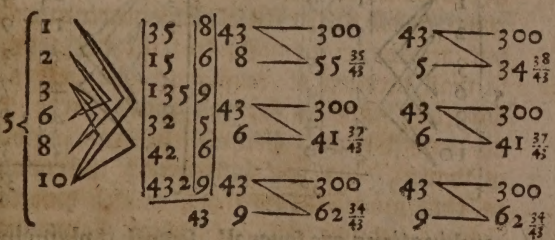
ſée all the variations of this queſtion, before you goe from it, for me thinketh I could varie it two or three wayes more yet.

Maſter. I am content to ſée you make two or three variations, but I would be loth to ſtay to ſée all the variations, for it may be varied aboue 300 wayes, although many of them would not well ſerue to this purpoſe.

Scholer. I thought it impoſſible to make ſo many variations.

Maſter. Maruel not thereat, for ſome queſtions of this rule may be varied aboue 1000 wayes, but I would haue you forget ſuch fantaſies, til a time of more leyſure. And now goe forward with ſome variation of this queſtion.

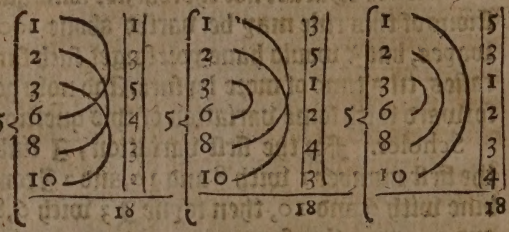
Scholer. For the firſt variation, I linke the firſt number 1 with 8 and 10, and 2 I combine with 6 and 10, then ioyne I 3 with 6, 8, and 10, as in this forme.

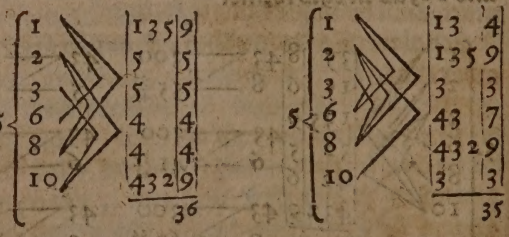


And ſo doeth there appeare the portion of weight for euerie kinde of drugge in this mixture. Nowe for the triall.

Master. May stay there, you shall not neede to make triall in one example so often, or if you list to doe it by your selfe, I am content. But nowe set forth (for declaration that you conceiue the rule) two or three examples of seuerall combinations, and then will we passe to some other example, and so ende this Rule.

Scholer. As it pleaseth you so will I doe. And these be the varieties in which as the





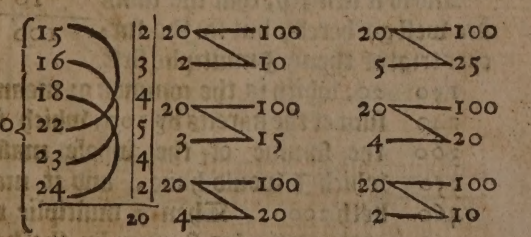
combinations are seuerall, so doth it plainelie appeare, that the differences by which the proportion of each seuerall kind is taken, are also seuerall. And yet I see in the three first of these

fiue

fiue varieties, and in one other before, the totall summe of the differences to be one, that is to say 18, whereby I perceiue that the variety of their mixture doth depende of the varietie of their differences seueral, and not of the varietie of their totall summe.

Master. So is it. And seeing you conceiue it so well, I will make an ende of this rule, onely exhibiting to you one question or two of the mixture of mettals, that by it you may deuise other like, and exercise your selfe therein also, because the vse of it serueth often in busines of charge, not so much for Goldsmithes, as for coynage in Mintes. First I demaund you this question. If a Mintmaster haue gold of 22 karectes, some of 23 karectes, some of 24. Againe, some of 15, some of 16, some of 18 karectes, and would mixe them so, that he might haue 100 ounces of 20 karectes, howe much shall he take of euerie sorte? A question of mixing of gold.

Scholer. To knowe that, I aunswere in order thus.



Master. You haue wrought the question well, but howe chaunced you made no doubt of that newe name Karecte.

Scholer. Because I thought it out of time to demaund such questions nowe, seeing you make so much hast to ende: and againe in this case the proportion of the numbers is sufficient for my purpose in this worke, trusting, that an other time you will instruct me as well of this, as of sundrie other things which I haue heard you talke of, so I haue a great desire to knowe them.

Master. Your answere is reasonable, and your request and trust with Gods helpe I intend to satisfie. And to goe forward with this matter, let me see your examination of this last worke.

Scholer. First for the one part I adde together all the particular summes as they appeare in the worke, and they make 100, as here by their Addition doth appeare.

	10
	15
	20
	25
	20
	10
	100

And so it seemeth, that the sums are well gathered, but for the further trial of them, I multiplie first

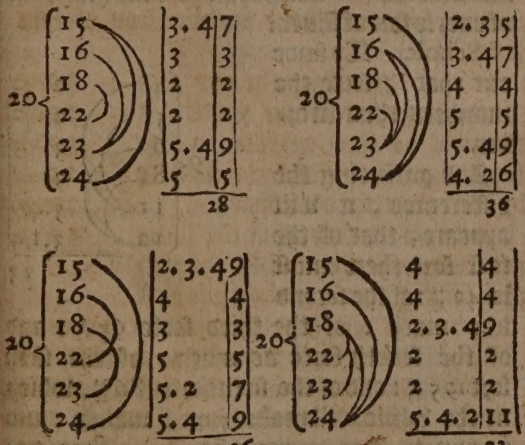
150	20, which is the common or meane
240	sum of the karects by 100, which is
360	the summe of the whole masse
550	which I would haue, and it ma-
460	keth 2000. Then I multiplie e-
240	uerie particular summe by the ka-
2909	rects that it doeth containe, as 10

by 15, and that maketh 150.

Likewise I multiplie 15 by 16, and it yéldeth 240: so 20 by 18 maketh 360. And 25 by 22 yéldeth 550: likewise 20 by 23 bringeth forth 460, and last of al 10 multiplied by 24, yéldeth 240: which summes all ioyned together make 2000, that doth agrée with the like summe before, wherefore I may wel say, that the worke is good. And nowe if it please you I would set forth some varieties of this question, to prowe my wit.

Master. Goe to, let me sée.

Scholer. Here be foure varieties.



And

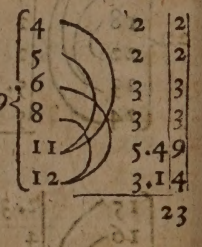
And more yet I could make, but not like to the number that you spake of in the variation of the other question.

Master. That will I teach you at more leysure, seeing it is a thing rather of pleasure than of any necessitie.

A question of mixing of siluer.

But nowe for your exercise in this Rule, one other question I will propose. A minte master hath 6 ingottes of siluer of sundrie fineneste, some of 4 ounces fine, and some of 5 ounces, some of 6, and other of 8, some of 11, and other of 12, and his desire is to mixe 500 pound waight, so that in the whole masse euerie pound waight should beare 9 ounces of fine siluer, howe much shall he take (say you) of euerie sort of siluer?

Scholer. To finde out that I sette the numbers thus in order.



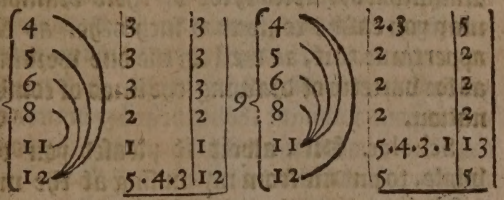
And gathering the differences, it will appeare, that of the first sort there must be 43 $\frac{11}{23}$: of the second like much: of the third sorte 65 $\frac{5}{23}$: and of the fourth sorte as much: of the fifth sort 195 $\frac{15}{23}$: and of the sixth sorte 86 $\frac{21}{23}$. which in the whole will make 500 ℔ waight: and in ounces after 9 ounces fine 4500, that is of the first sort 173 $\frac{21}{23}$: and of the second sorte

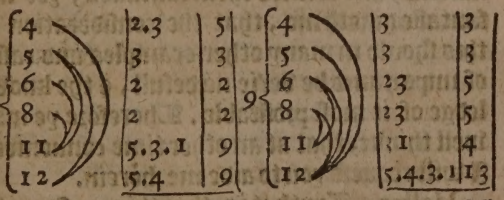
217

217 2/23: of the third sorte 391 7/23: of the fourth sorte 521 17/23: of the fifth sorte 2152 4/23, and of the sixth sorte 1043 11/23, which all together doe make 4500 ounces, agreeable to the multiplication of 9 by 500.

Master. This is well done of you, therefore nowe make three or foure varieties, and so an ende of this rule.

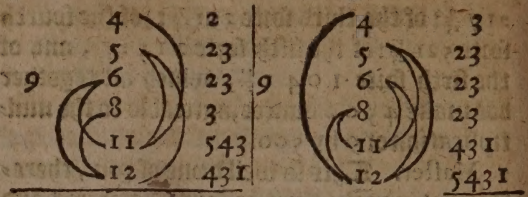
Sc. These 4 varieties I set for example.





Master. And by these it appeareth, that you can finde out more, with which I will not nowe meddle, saue onely for to shewe you an easie helpe in drawing the lines of Combination, I will set forth two varieties here.

And



And this shall suffice nowe for the rule of Alligation or mixture, for by these examples may you easilie conjecture such other as doe appertaine to it, as well for the due working, as for varietie of drawing the lines of combination.

Scholer. Sir, albeit it pleased you ere while, to put me from my musing at the manifold varieties, that may fal in these combinations, and termed them fantasies, yet my fantasie giueth me, that the consideration of this should in many other examples and cases of importance be verie needefull, & the knowledge of it most profitable. Therefore ye may well thinke, that at an other time conuenient I will request you to aide me herein.

Master. Truth it is, that this consideration may fall in practise as well Politike as Philosophicall, and sundrie wayes in them be applyed, therefore when time shall fall fit for the discussing of this consideration, you shall not want my helping hande.

The

The rule of Falshood.

NOwe will I briefely also teach you somewhat of the rule of Falshood, which beareth his name, not for that it teacheth any fraude or Falshode, but for that by false numbers taken at all aduentures, it teacheth howe to finde those true numbers that you seeke for. The occasion of the name.

Scholer. So might any other rule be called the rule of Falshode, for they worke by wrong numbers, and by them finde out the right numbers, so doth the rule of Alligation, the rule of Felowship, & the golden rule partly.

Master. In the Golden rule, the rule of Felowship, and the rule of Alligation, although the numbers that you worke by, bee not the true numbers that you seeke for, yet are they numbers in iust proportion, and are found by orderly worke: whereas in this rule the numbers are not taken in any proportion, nor found by orderly worke, but taken at all aduentures.

And therefore I sometimes being merie with my friends, and talking of such questions, haue caused them that proponed such questions to call vnto them such children or Ideots as happened to be in the place, and to take

take their answere, declaring that I would make them solue those questions, that seemed so doubtfull.

And indeede I did answere to the question and worke the triall thereof also by those answeres which they happened at all aduentures to make, which numbers seeing they be taken as m[illegible] falsse, therefore is this rule called the rule of falsse positions, and for briefenesse, the rule of Falshood, which rule for readinesse of remembrance, I haue compriſed in these fewe verses following, in forme of an obscure Riddle.

Geſſe at this worke as hap doth leade,
By chaunce to truth you may proceede.
And first worke by the queſtion,
Although no truth therein be done.
Such falſehoode is ſo good a ground,
That truth by it will ſoone be found.
From many bate to many moe,
From too fewe take too fewe alſo.
With too much ioyne too fewe againe.
To too fewe adde too many plaine.
In croſſe waies multiplie contrarie kinde,
All truth by falſhoode for to finde.

The ſence of theſe verſes, and the ſumme of this rule, is this.

When any queſtion is propoſed appertaining to this rule firſt imagine any number that you liſt, which you ſhall name the firſt

first position, & put it in stead of the true number, and then worke with it as the question importeth: and if you haue misled, then is the last number of that worke eyther too great or too litle: that shall you note as hereafter shall be taught you, and you shall call it the first errour.

Then begin againe, and take an other number which shall be called the second position, and worke by the question: if you haue misled againe, note the excesse or default as it is, and call that the second errour. Then multiplie crosse waies the first position by the second error, and againe the second position by the first errour, and note their totals seuerally by the names of totals. Then marke whether the two errors were both like, that is to say, both too much, or both too litle: or whether they be vnlike, that is, the one too much, and the other too litle, for if they be like, then shall you subtract the one totall from the other (I meane the lesser from the greater) and the Remainer shall be your diuidende, so must you abate the lesser errour out of the greater, and the residue shall be the diuisor. Now diuide the diuident by that diuisor, and the quotient will shewe you the true number that you seeke for. But and if the errors be vnlike, then must you adde both those totals (which you noted) together, and take that whole number for the diuident, so shall

shall you adde both the errors together, and that whole number shall be the diuisor, and the quotient of that Diuision shall giue you the true number that the question seeketh for, and this is the whole rule.

Scholer. This rule seemeth so vnlike any other, that without some example I shall not easilie vnderstande it.

A question of Masondrie.

Master. Therefore take this example: A Mason was bound to builde a wall in 40 dayes, and it was couenanted so with him, that euerie day that he wrought, he shoulde haue for his wages 2 s 1 d, and euerie day that he wrought not, he should be amerced 2 s 6 d, so that when the wall was made, and the reckening taken of the dayes that he wrought, and of the other that he wrought not, the Mason had clearely but 5 s, 5 d, for his worke. Nowe doe I demaund, howe many dayes did he worke of those 40, and howe many did he not worke?

Scholer. I pray you expresse the order of the worke, that I may partly by imitation, and partly by comparing it with the rule, be able againe to doe the like.

Master. This order shall you keepe in the worke of this rule: first take some number (as you list) at aduenture, as for example: I say he played 12 dayes & wrought 28 dayes. Nowe cast you the wages of euerie day, and see whether it wil agree with the summe of

of 5 s, 5 d.

Scholer. The 28 daies that he wrought after 25 pence the day, yeldeth 700 pence. Then the 12 daies that he wrought not, at 30 pence each day, doth amount to 360 pence, which if I abate out of 700 pence, there resteth 340: but you say he had not so much.

Master. He had but 65 pence, and by this supposition he should haue had 340: therefore is this summe too much by 275, which sum I must set downe after this sort, as you see here, where first I haue made a crosse (commonly called Saint Andrewes crosse) at the ouer corner on the left hand

12 ✕
275+

I haue set the first position 12, and at the other corner vnder it, I haue set 275 which is the first error, with this figure +, which betokeneth too much, as this line, —— plain without a crosse line, betokeneth too litle. On the right hand of the crosse I haue left two like rowmes for the second position and his error. Therefore to prosecute the worke, I suppose he played 16 daies, and wrought 24.

Scholer. I was a while in doubt why you named the daies of his working, seeing they be not set in the figure: and I doubted howe you knewe them, or else whether that you did suppose them at all aduentures, as you did the dayes that he played: but nowe I ga-

ther, that seeing 40 daies is the whole time limited, then the daies that he played being supposed the rest of 40, must nedes be the daies that he wrought, and therefore 28 followed 12 of necessitie, and 24 followeth 16 also of necessitie: but yet I scarce perceiue why you set not in the figures as well 28 as 12.

☞ Master. It forceth not which of them I take, so that in the second position I take the numbers of the same nature that is here both of working daies, or both of idle: but now examine you this second position.

Scholer. If he played 16 dayes, then abating 16 times 30 d, the summe will be 480 d. And for 24 daies that he wrought, euerie day yelding 25 d, the totall is 600 pence: so that abating 480 out of 600, there resteth 120: and as you say it should be but 65, therefore it is too much by 55, that must be set on the right hand of the figure at the neather part, and ouer it on the same side 16, which is the second position, thus.

12 16

And as I gather by your words it were al one if I did set 28 in 275 + 55 + stead of 12, & 24 in stead of 16.

☞ Master. So were it. But this shall you marke, that of what nature soeuer the two positions be, of the same nature is the quotient. Therefore when the positions in this

Note.

question

question are 12 and 16, which both being numbers of the playing dayes, the quotient shall declare the true numbers of playing dayes, where as if the positions had beene 28 and 24, which are supposed to be the working dayes, then would the quotient declare the true number of the working dayes, and not of playing dayes as it will doe nowe. And therefore to continue the worke of this question, and to finde the true number of playing dayes, I must multiplie crosse wayes the first position 12 by 55, that is the second error, and the totall will be 660, then I multiplie 275 by 16, and it yeeldeth 4400. Nowe because the errors are like, that is to say, both too much, I must subtract 660, out of 4400, and so remaineth 3740, which is the dividend. Againe I must subtract the lesser error 55 out of 275 that is the greater error, and there will remaine 220, which shall be the divisor, then dividing 3740 by 220, the quotient will be 17. Wherefore I say nowe constantly, that 17 is the true number of dayes that the Mason playde: and then it followeth: that he wrought 23 daies, and so is the question answered.

Nowe for the order of triall of this worke there needeth none other proofe but only this, to worke with this number according to the question, and if it agree, then appeareth the number to be it that you would haue. As

The proofe of this rule.

here nowe seeing he wrought 23 daies, and must haue for euerie day 25 pence, the whole summe commeth to 575. Then againe seeing he played 17 dayes, and must abate 30 pence for euerie day, the whole summe of the abatement will be 510: therefore I subtract 510 out of 575, and there will remain 65, which maketh 5 s, 5 d, the cleare wages of the Mason for his worke, according to the question.

Scholer. Nowe I trust I vnderstande the worke and the rule so well (and the better by this proofe) that I can be able to doe the like. And for a proofe I take the same question all saue the last number, where I will suppose that he had 10 s for his wages cleare. And now to gesse at the number of ỹ dayes that he wrought, I suppose first that he wrought 20 dayes, then say I, if he wrought 20 dayes, his wages must be 500 d, then did he play other 20 daies, for which must be abated 600 d, and then he loseth 100 d. And so am I at a stay, for it is not like vnto your former worke.

Master. You should haue required of me some question, and not haue taken a question of your owne fantasying, vntill you were more expert in this arte: for so might you as well happen on an impossible question as on a possible: but now to goe forward, consider that this number is too litle by 220, seeing he should gaine by your supposition 120 d, and in this position he loseth 100, those both make 220

which

which you shall set downe for the first error with this signe —, betokening too litle, as here in this forme following doth appeare.

And nowe for the rest goe forward your selfe once againe. 20

Scholer. As my error hath vttered my follie, so it hath procured me better vnderstanding. 220—

Nowe therefore considering this position not to solue the question, I take an other, supposing that he wrought 30 days, then for his wages he must be allowed 750 ð, and for the 10 daies which he wrought not, he must abate 300 ð, and so remaineth cleare 450 ð: but it should be onely 120 ð, therefore is it too much by 330, which I set downe in the figure with the former position, and his error and the figure appeareth thus. 20 30

Now must I multiplie in crosse wayes 220, by 30, and it wil be 6600. 220— 330+

Then againe I multiplie 330 by 20, & it will be also 6600. Wherefore if I shall subtract the one out of the other, there will remaine nothing to be the Diuident.

Master. In this you forget your selfe againe: for in as much as the signes in the errors be vnlike, therefore must you worke by Addition, adding together those two totals to

make the diuident, and also adding the two errors to make the diuisor. And because you shall no more forget this part of the rule, take this briefe remembrance:

Vnlike require Addition,
And like desire Subtraction.

Scholer. You meane, that if the errors haue like signes, then must the diuident and the Diuisor be made by Subtraction, as is taught before: And if those signes be vnlike (as in this last example they be) then must I by Addition gather the diuident and the diuisor. Therefore must I adde 6600 to 6609, and it will be 13200, which shall be the diuident. Then againe I adde 220 to 330, and it will be 550, which must be the diuisor: wherfore diuiding 13200 by 550, the quotient will be 24, whereby I knowe that the Mason wrought 24 dayes, and then it followeth that he played 16 daies.

Master. Examine your worke whether it be agreeable to the question or no.

Scholer. For 24 daies worke, the wages must be 600 d, and for 16 dayes which the Mason wrought not, there must be abated 480, and then remaineth cleare to the Mason 120 pence, as the question importeth, wherefore it is euident, that 24 is the true number of daies that he wrought.

Master.

Master. Although you seeme nowe to vnderstand this worke, yet to acquaint your minde the better with the newe trade of this rule, I thinke it good to propone to you fiue or sixe examples more, before I make an ende of it.

Scholer. Sir I thanke you that you doe so consider my commoditie and profite in knowledge, for vndoubtedly it is practise and exercise that maketh men prompt and expert in euerie kinde of knowledge.

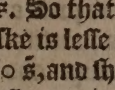
Master. You say well so that they followe some certaine precepts to gouerne and rule their practise by, else may practise procure custome of error, and a repugnance to exactnesse of knowledge, namely as long as the error is not plainely knowne to the vulgare sort. But to returne to our worke. There is a seruant that hath bought of veluet and damaske for his maister 40 yards, the veluet at 20 s a yard, and the Damaske at 12 s, and when he commeth home, his maister demaundeth of him howe much he hath bought of each sort: I cannot tell (saith he) exactly, but this I knowe, that I paide for damaske 48 s more than I paide for veluet, nowe must you gesse howe many yards there is of each sort. A question of wares.

Scholer. Although the gesse seemeth difficult, yet I will proue what I can doe: for I remember your saying, that it forceth not howe fonde or false the gesse be, so it be some-

what to the question, and not an answere of a contrarie matter.

Therefore first I imagine that he bought 20 yards of Damaske, for which he should pay after the former price 240 shillings: then must he néedes haue of veluet other 20 yards (to make vp the 40 yards) and that would cost 400 s. So that the totall of the price of the damaske is lesse than the summe paide for veluet 160 s, and should be more by 48. Therefore the first error is 208 too little. Then begin I againe, and suppose he bought of Damaske 30 yards that cost 360 s, then had he but 10 yards of veluet, which cost 200 s: and nowe the price of Damaske is greater than the price of the Veluet by 160 shillings, and should be but 48, therefore is the second error 112 too much, which I set in forme of a figure as here doeth appeare.

Then doe I multiplie in crosse wayes 280 by 30, and the summe will be 6240. Also I multiplie 112 by 20, and there wil amount 2240. And in as much as the signes of the errors be vnlike, I knowe I must worke by Addition, therfore adde I those two totals together, and they make 8480, which is the diuident: then adde I also the two errors together, 208 and 112, and they make 320, which is the diuisor. Where-



fore

fore diuiding 8480, by 320, the quotient wil be 26½, which is the true summe of yardes of Damaske that he bought: and in Veluet 13 yards½, and that appeared by examination thus: 26½ yardes of Damaske at 12 s the yard maketh 318 s, then in veluet he had but 13 yards and½ that cost 270 s, at 20 s the yard. Nowe subtract 270 out of 318, and there will remaine 48, which is the number of shillings that the damaske did cost more than the veluet.

Master. Nowe shal you haue a question of an other kinde. A questiō of debt.

There are three men that doe owe money to me, and I haue forgotten what the totall summe is, and what the particulars be.

Scholer. Why? then it is impossible to knowe the debt.

Master. Peace, ye are too hastie: there is more helpe in it than you yet see: I haue three seuerall notes, whereby it appeareth that I did conferre their debts together, and founde the debt of the first and the second to amount to 47 l, the debt of the first man and the third man did make 71 l, and the second man his debt with the third, did rise to 88 l. Now can you tell what euerie man did owe, and what was the whole totall?

Scholer. Nay in good faith: but as I perceiue that it must be found by coniecture, so will I gesse at it, supposing that the first man did

did owe 20 l, & the second man 30, & the third.

Master Nay stay there, for you are too far gone alreadie, you may not suppose a severall summe for everie man, for it is ynough to suppose one summe for the first man, and let the other rise as the question importeth. Therefore seeing you set the first man his debt to be 20 l, the second man cannot owe 30 l, for the declaration is that their debtes added together, did make 47 l, so must the second man his debt be but 27 l. Nowe this second debt with the third must make 88, therefore subtract 27 out of 88, and there will remaine 61, as the third man his debt. Then saith the declaration, that the first and thirde mens debts doe make 71: but by this supposition they make 81 that is 10 too much: which I must set for the first error. Now worke you the second position.

Scholer. I suppose the first mans debt to be 24 l, then must the second mans debt (by your declaration) be but 23 l, seeing both they make but 47 l. And the second man his debt with the third, doe make 88 l, and the second man oweth but 23, therefore the thirde man must owe 65 l. Nowe the third mans debt with the first, should make by the declaration 71 l, and they doe make 89 l: that is 18 l too much: and that is the seconde error, which I set downe with the first, and their position in this forme, and then doe I multiplie

plie in crosse wayes 20 by 18, and it is 360. Also 10 by 24 maketh 240. And because ỹ signes of the erors be like, I must worke by Subtraction: therefore I subtract 240 out of 360, and there resteth 120, which is the diuident: then doe I subtract 10 out of 18 by the same reason, and so is the diuisor 8, which is found 15 times in 120, therfore I say that the first man did owe 15 l, & then the second man must owe 32 l, for those 2 doe make 47 l, and the third man his debt is 56, for so much remaineth if I bate 15 out of 71, or if I take 32 out of 88.

20 24
10+ 18+

Master. For the third example take this easie question for the varietie in worke. Two men hauing seuerall summes which I know not, doe thus talke together: the first saith to the second, If you giue me 2 s̄ of yonr money, then shall I haue 3 times so much money as you: the second aunswereth, It were more reason, that our summes were made equall, and so will it be, if you giue me 3 s̄ of your money. Now gesse what each of them had. The third question.

Scholer. I imagine that the first had 9 s̄. Note.

Master. Consider euermore in your imaginations that you take a likely summe, as in this question take such a summe that hauing 2 added vnto it, may be diuided into 3 partes euen.

Scholer.

Scholer. Why? I remember you saide before, it forced not howe fondly soeuer I gessed.

Master. As for the possibilitie of the solution it is trueth, but for easinesse in worke, the aptest numbers are most conuenient.

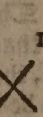
Scholer. I thought no lesse, and therefore I toke 9 as an apt number to be parted into 3: but I perceiue I should haue considered the aptnesse of that partition after the addition of 2 vnto it, and then 7 had bene more meeter.

Master. That is trueth, and then shoulde the second man his summe be 5: for although he haue nowe but the third part of 9, that is 3, yet you must remember that he lent the first man 2, and so had he 5.

Scholer. Then to goe forward: if the second man had 3 of the first man, then should he haue 8, and the first man but 4, so hath he double to the first man: yet he saide in the question they should haue equall: wherefore it appeareth that he hath 4 to much. Therefore I note that error with his supposition, and gesse againe that he hath 10 s: whereunto I adde 2 shillings borrowed of the second man, and then hath he 12 shillings, so the second man hath remaining but foure, whereunto if I adde the 2 that he lent to the first man, so had he but 6 s at the beginning. Then take 3 shillings from the first man, and

giue

gine to the second, then hath the first man but 7, and the second hath 9, which are not equall, but there are 2 too many, wherefore I set downe both the positions with their errors, as here you see, and multiplie a crosse, so commeth there 40 and 14: and because the signes be like, I take 14 out of 40, and so resteth 26 to be the diuident, then likewise I take 2 out of 4, and there resteth 2, by which I diuide 26, and the quotient will be 13, which is the summe that the first man had. And so appeareth that two being added thereto, the summe will be 15, so hath the second man nowe but 5, and before he had 7: then take three from the first, and put to his seuen, so haue each of them 10, and that is equall, as the question would.



Master. For the fourth example take this question. One man saide to another: I thinke you had this yeare two thousand lambes: so had I saide the other: but what with paying the tithe of them, and then three seuerall losses, they are much abated: for at one time I lost halfe as many as I haue nowe left: and at an other time the third part of so many: and the third time $\frac{1}{4}$ so many. Now gesse you howe many are left. The fourth example.

Scholer. Because here is mention made of certaine parts, I must take a number that may

may haue all those parts: that is to say, $\frac{1}{2}$ $\frac{1}{3}$ & $\frac{1}{4}$, which wil be 24, howbeit 12 hath the same parts. Therefore first I take 12 to be the number that doth remaine, so hath he lost 6, 4, and 3, that is 13, and in the whole 25, but it should be 2000.

Master. We are deceiued yet still: you haue forgotten the 10 part, which must be defalked, that is 200, so there remaineth but 1800 and nowe goe on againe.

Scholer. Then to finde the error, I take 25 out of 1800, and there remaineth 1775 too fewe, which I set for the first error. Then for the second position I take 24, whose halfe is 12, the third part 8, and the quarter 6, whereby riseth 50, which is too litle by 1750, therefore I set downe both the positiõs with their errors thus.

12	24
1775	1750

And multiplie in crosse wayes 1775 by 24, whereof commeth 42600. Also I multiplie 1750 by 12, and there ariseth 21000. And because the signes are like, I doe subtract the one from the other, and so remaineth the diuident 21600: then doe I subtract 1750 out of 1775, and there resteth 25: by which I diuide 21600, and the quotient is 864, whereof the halfe is 432, and the third part is 288, the quarter is 216, which all being added

added together will make 1800. 864
And if you adde thereto the tenth 432
which was abated before, then wil 288
the whole summe be 2000. And 216
nowe doeth there come a question 1800
to my memorie which was demanded of me, but I was not able to aunswere to it, and nowe me thinketh I could solue it.

Master. Propone your question.

Scholer. There is supposed a Lawe made, A question of sheepe & tillage. that (for further tillage) euerie man that doeth keepe sheepe, shall for euerie 10 sheepe eare and sowe one acre of ground: and for his allowance in sheepe pasture, there is appointed for euerie foure sheepe one acre of pasture. Nowe is there a rich sheepemaster which hath 7000 acres of ground, and would gladly keepe as many sheepe as he might by that statute, I demaund howe many sheepe shall he keepe.

Master. Aunswere to the question your selfe.

Scholer. First I suppose he may keepe 500 sheepe, and for them he shall haue in pasture after the rate of 4 sheepe to an acre, 125 acres, and in arable ground 50 acres, that is 175 in all: but this error is too litle by 6825. Therefore I gesse againe, that he may keepe 1000 sheepe, that is in pasture 250 acres: and in tillage 100 acres, which maketh 350: that is too litle by 6650.

These

500 1000

6825— 6650—

These both errours with their positions I set down as you see, and multiply in crosse 6825 by 1000, and it maketh 6825000. Then I multiplie 6650 by 500, and it doeth amount to 3325000, which summe I doe subtract out of the first, and there remaineth 3500000 as the diuident. Also I doe subtract the lesser error out of the greater, and so remaineth 175, by which I diuide the saide diuident, and the quotient will be 20000, so that I see that by this rate he that hath 7000 acres of ground, may keepe 20000 sheepe: and thereby I coniecture, that many men may keepe so many sheepe, for many men (as the common talke is) haue so many acres of ground.

Master. That talke is not likely, for so much ground is in compasse aboue 48¾ miles: Leaue this talke and returne to your questions, least your pointing be scarce wel taken.

Scholer. Indeede I doe remember, that the Egyptians did grudge so much against sheepeheards, till at length they smarted for it, and yet they were but small sheepemasters to some men that be nowe, and the sheepe are waxen so fierce now and so mightie, that none can withstand them but the Lion.

Master. I perceiue you talke as you heare some

some other: but to the other worke of your question: both this last question, and the next before might be wrought without the second position, by the rule of proportion, as this. When in this question ye founde in the first error, that for 500 sheepe, there must bee 175 acres, then might you reduce it to the Golden rule, thus. An other way of working.

175 — 500
7000 — 20000

If 175 acres wil admit in allowance 500 sheepe, than 7000 wil haue 20000. And so by one position with the helpe of the Golden rule may you aunswere that question. Likewise for the question of Lambes, when you had found that 12 came of 25, you might haue set the figure thus as ye see I haue saide:

25 — 12
1800 — 864

If 25 doe leaue but 12, what shall 1800 leaue? and it would appeare to be 864.

Scholer. Sir, I thanke you for this aide, for it doeth much shorten the worke of this rule.

Master. Yet againe I will shewe you an other way, to aunswere to this last question without this rule of False position, and that by the rule of Felowship, for it appeareth in the proponing of the question, that 10 sheepe must haue in pasture 2 acres and $\frac{1}{2}$, and for them must there be eared but one acre: so it An other way yet.

loweth, that for two acres eared, there must be 5 set to pasture. And if you put them both into one summe, they will make 7. Therfore loke what proportion 7 being this totall, doth beare to 5 and to 2, such proportions shall any totall in this question beare to the pasture ground, and the eared ground.

Scholer. This serueth wonderous aptly. Therefore to prowe it, I demaund this by the former suppositiõ: If a man haue 300 Acres, how much shall he leaue in pasture, and howe much shall he turne to tillage? You say that as 7 is to 5, so shall 300 be to the acres of pasture: and as 7 is to 2, so is 300 to the acres of tillage, whereof for both I haue set examples here following, whereby appeareth that of Pasture there shall be $214\frac{2}{7}$ Acres, and of Tillage $85\frac{5}{7}$, which both summes added together, do make 300.

7 — 5 — 300 — $214\frac{2}{7}$

7 — 2 — 300 — $85\frac{5}{7}$

An other question.

Master. Nowe take an other example: A man hath three siluer cuppes with one couer, the couer wayeth 18 vnces, the second cuppe wayeth euen halfe the weight of the first and the third. Nowe if the couer be put to the first cuppe, they wey iust as much as all the three cuppes doe wey: and if the couer be ioyned with the second cuppe, they wey as much as the second twice, and the third: and if the couer

couer be put to the third cup, they will make twice as much as the first and the second cup. Nowe trie you what was the iust weight of euerie cup.

Scholer. I doe set the weight of the first cup to be 9 ounces: then in as much as these two (that is to say, the couer and the first cup) doe wey the weight of the three cuppes, I see that the three cups must wey 27 ounces, for so much is 18 and 9. Also because the first and the third doe wey double so much as the second, therefore is it the third part of that weight, that is 9, and then would it followe, that the third cup also should wey 9 ounces, but then the question saith, that the couer being ioyned to the second cuppe, they wey as much as the second twice, and the third once, that should be 27, and so it doth: then being ioyned with the third cuppe, they should wey twice as much as the first & the second, that should be 36, and they wey but 27, so is that error 9 to litle. Then begin I againe, and say, that the first cuppe doeth wey 12 vnces, which I ioyne with the couer, and they make 30 ounces: then seeing the second is $\frac{1}{3}$ of that weight, it must needes wey 10 ounces, and the third must wey 8 ounces, seeing the first and the third must wey 20 ounces. Nowe put I the couer to the second cup, and they wey 28 ounces, which should be euen so: then ioyne I the couer with the third cup, and so should

it wey twice the first, & the second, that is 44 ounces, and they doe wey but 26, that is 18 two litle: those errours with their positions I set downe, and multiplie in crosse waies 9 by 12, wherof commeth 108. Also 9 by 18, and that yéeldeth 162: and in as much as the signes be like, I abate the lesser out of the greater, and there doeth remaine 54. Then doe I also abate the lesser error from the greater, and so remaineth 9, by which I diuide 54, and the quotient is 6: which I take for the true weight of the first cup: which being ioyned with the couer must wey as much as the three cups, so do they wey but 24 ounces. Then séeing the second cup is the third part of that weight, for the other two cuppes (you say) must wey double his weight, the weight of the second cuppe is 8 ounces, and so the weight of the third must be 10 ounces. Nowe put the couer to the second cup, and it will make 26 ounces: that must be the weight of the second twice, and the third once, that is twice 8, and once 10, and so is it. Againe, put the couer to the third cup of 10 ounces, and they must wey twice as much as the first and the second, that is 28: and so is all agréeable.

9 12
9— 18—

Master. Then answere to this question.

A question of water. There is a Cesterne with foure cockes, con-

containing 72 barrels of water: and if the greatest cocke be opened, the water wil auoid cleane in 6 houres: at the second cocke it will aske eight houres: at the third cocke it will auoide in no lesse than nine houres: and at the smallest it will require twelue houres: Nowe I demaund, in what space, will it auoide, all the cockes being set open?

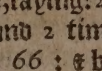
Scholer. First I imagine that it wil auoid in two houres.

Master. Then must there auoide by the first cocke $\frac{2}{3}$ of the water, that is 24 barrels, and by the second cocke $\frac{1}{4}$ that is 18, and by the third cocke $\frac{2}{5}$ that is 16 barrels, and by the smallest cocke $\frac{1}{8}$, that is 12 barrels, all which summes put together doe make 70, as by their addition it doth appeare, but it should be 72, therefore the error is 2 too fewe.

Scholer. Then I begin againe by your fauour, because I thinke I vnderstand the worke, and put three houres for the due time: so shall there run out at the greatest cocke $\frac{1}{2}$, that is 36 barrels, & at the second hole $\frac{3}{8}$, that is 27, and at the third cocke $\frac{1}{3}$, that is 24, and at the smallest hole $\frac{1}{4}$, that is 18 barrels, which altogether doe make 105, and should be but 72, so is it too much by 33, therefore doe I set the errors in order of the figure with their positions, and worke by multiplication,

	24
	18
	16
	12
	70

in croſſe, ſaying: 2 times 3 is 6, and 2 times 33 maketh 66: & becauſe the ſignes are vnlike, I muſt adde thoſe two totals together, which make 72: alſo I adde the two errors, and they make 35, by which I diuide 72, and the quotient riſeth $2\frac{2}{35}$, whereby I ſee that all the cockes being ſet open, the water will auoide in 2 houres and $\frac{2}{35}$ of an houre.



Maſter. This exerciſe maketh you to growe expert in the rule. Therefore I wil inure you ſomewhat more with a queſtion or two.

A queſtiō of partēners.

There were two men that had bēene partēners, and had in account betwēene them 300 duckets: whereof the one ſhould haue for his part 180, and the other 120: but in the parting of them they fell at variance, ſo that ech of them catched as many as he could: yet afterward being reconciled, they agréed that he which had gotten moſt part of them, ſhould lay downe $\frac{1}{4}$ of them againe, and he that had gotten leaſt, ſhould lay downe $\frac{1}{3}$ of thoſe which he had taken, and then parting them vnto two equall parts, each man to haue halfe thereof, and ſo had they their iuſt portions, as they ought: nowe I demaund of you what each of them had gotten by the ſkambling.

Scholer. I ſuppoſe he that had leaſt, got

108 duckets, then the other had 192: wherefore in laying downe againe of the 192, there was put downe $\frac{3}{4}$ that is 144, and so had he left but 48. Also of the 108: there was laide downe 36, that is $\frac{1}{3}$, and so he had left 72. Then I put together 144, and 36, and it maketh 180 which I part into two partes euen, and so commeth 90 to be giuen to ech of them: which sum put to 72, maketh 162, and ioyned to 48, it maketh 138: and nowe I doubt howe I shall goe forward.

Master. You neede not to take but one of them which you list, the greater or the smaller, for all commeth to one purpose: and so may you compare it that you take to any of the other sums, remembring that you make comparison to the same in the second worke: as for example of the first part. If you compare 138 with the lesser summe due, that is 120, so is it 18 to much: and if you compare it with the greater summe, then is it 42 to litle. Againe, if you compare 162 to the greater summe, the error will be 18 as it was in the other: but it will haue a contrarie signe: and if you compare it with the lesser summe, it will be 42 to much: so that the error both waies is eyther 18 or 42: and as for the signes it litle forceth, for in them is nothing considered here, but likenesse and vnlikenesse, which in this case, doth neither further nor hinder. But nowe goe on with the worke. Note.

Scholer. If it be so, then am I out of my greatest doubt. Then I ioyne that 90 (which I found as the halfe of the latter partition) vnto 48, which is left with the one man, and so hath he 138, which (I may say) is 18 too many, for the least should be but 120: that error doe I note, and then make a newe position, supposing the one man to haue 204, and the other to haue 96, wherefore of the 204 there must be laide downe 153, and so remaineth with him 51. Also of the 96 there must be laide downe $\frac{1}{3}$, that is 32, and so resteth with that man 64. Nowe of the 153 and 32 I make one summe as 185, which I must diuide into two equall parts, and so each man shall haue $92\frac{1}{2}$, whereunto if I adde their former portions reserued, then the one shall haue $156\frac{1}{2}$, and the other hath $143\frac{1}{2}$. Wherfore I take the lesser sum now againe, as I did before, that is $143\frac{1}{2}$, and finde that he hath too many by $23\frac{1}{2}$, for he should haue but 120, so haue I for my two positiõs two errors, which I set downe, as here may be sæne, each error vnder his position, and then by the rule I doe multiply in crosse ways 108 by $23\frac{1}{2}$ and there riseth 2538 which I note, then again I multiplie 96 by 18, & thereof amounteth 1728.

108	96
18+	$23\frac{1}{2}$+

Nowe

Nowe because the signes are both like, that is both to many, I must worke by Subtraction, and so abating 1728 out of 2538, there will rest for the diuident 810: then for the diuisor I subtract 18 out of $23\frac{1}{2}$, and there remaineth $5\frac{1}{2}$, by which I diuide 810, and the quotient will be $147\frac{3}{11}$, which is the iust portion of him that had the least summe. And if I doe subtract it out of 300 being the totall summe, then will there remaine $152\frac{8}{11}$ as the portion that the other did get.

Master. For the proofe of this worke, you may chose whether you will examine those numbers according to the forme of the question, or else worke by other two positions for to finde the second number: and if those positions bring the same numbers that did amoũt by the first two positions, then doth ech worke confirme other.

Scholer. By your patience, I will proue both wayes, not onely to see their agreement, but also to accustom my mind to those workes: for I perceiue it is exercise that must be the chiefe engrauer of these rules in my memorie.

Master. You consider it well: then goe to.

Scholer. First I will by two other positions trie to finde the portion of him which had most.

Master. Although you may doe it with any positions, yet to see the agreement of your worke

worke the better, take the same positions that you did before, comparing them nowe to the greater, as you did before vnto the lesser.

Scholer. Then I suppose, that he that had most, had 192, so had the other 108. Nowe if I take $\frac{3}{4}$ out of 192, that will be 144, & there will rest to that man but 48. And from the second which had 108, if I take $\frac{1}{3}$, that is 36, there will remaine to him 72: then ioyning 144 with 36, it will make 180, the halfe whereof being 90. If I adde to each of those two mens portions remaining with them, the one shall haue 138, and the other 162, of which two I take the greater (that is 162) and set it to be 18 to fewe, for it should be 180, that error I note vnder his position. Then for the second position I take (as I did before) 204 for the one, and so resteth 96 for the other: then take I $\frac{3}{4}$ of 204, and it wil be 153, and there resteth to him 51. Also of the 96 I take $\frac{1}{3}$ that is 32, and there remaineth to him 64. Nowe put I that 32 to 153, and it yeldeth 185: which being parted in equall values, maketh 92 $\frac{1}{2}$ to be added to each mans remainder, and so the one hath 143 $\frac{1}{2}$, and the other 156 $\frac{1}{2}$: wherefore I take the greatest summe, and it is 23 $\frac{1}{2}$ too litle, that do I note also, and set both these errors vnder their positions, as in this example following doeth appeare.

And then multiplying 192 by 23 $\frac{1}{2}$, there doeth

doth arise 4512. 192 204

Againe, I multiplie 204 by 18, and it maketh 3672, which I doe subtract out of 4512, 18— $23\frac{1}{2}$

because the signes be like, and there resteth 840 for the diuident: then subtracting 18 out of $23\frac{1}{2}$, there will remaine $5\frac{1}{2}$, which I must take for the diuisor. And so diuiding 840 by $5\frac{1}{2}$, the quotient will be $152\frac{8}{11}$, whereby I haue found an agreeable sum to that which I found by the former positions, for him that had most, which if I doe subtract out of 300, that is the totall, there will rest $147\frac{3}{11}$, which was the portion of him ỹ had the least part.

Master. So by diuerse positions you see, that one doeth confirme the worke of the other. Nowe examine those two numbers by the forme of the question, and so shall you prowe your worke good also.

Scholer. If that he which gate most, had $152\frac{8}{11}$, then must he lay downe $\frac{3}{4}$ of his sum, that is $114\frac{6}{11}$, and so shall remaine with him but onely $38\frac{2}{11}$. The other which had least, that is $147\frac{3}{11}$ must put downe of his sum $\frac{1}{3}$, that is $49\frac{1}{11}$, and so doeth there remaine with him yet $98\frac{2}{11}$. Then do I adde together $114\frac{6}{11}$ and $49\frac{1}{11}$, and it will make $163\frac{7}{11}$, which I must part into two equall parts, and that wil be $81\frac{9}{11}$ to be giuen to each of them: so putting $81\frac{9}{11}$ vnto $38\frac{2}{11}$, there doth amount 120 iust,

which

which is the true portion of him that shoulde haue the lesser summe: and adding 81 2/11 to 98 2/11, the totall will be 180, the true portion of the other. And so is the worke by this proofe also tried to be good. And this I marke by the way, that in their scambling, he gate most (as it chanceth often) that ought to haue had least by iust partition.

Master. Let your studie be to learne truth and iust arte of Proportion, and to distribute and parte according thereunto, as often as occasion shall be ministred. And here would I make an end of this rule, saue that I remember one pleasant question which I can not ouerpasse, which I wil declare somwhat largely, because you shall as well vnderstand some reason in the pleasant inuention, as apt proceeding in the wittie working thereof.

An example of mixture of golde and siluer.

Hiero king of the Syracusans in Sicilia, had caused to be made a crowne of Gold of a wonderfull waight, to be offered for his good successe in wars: in making whereof, the Goldesmith fraudulently toke out a certain portion of golde, and put in siluer for it, so that there was nothing abated of the full weight, although there was much of the value diminished. Which thing at length being vttered, (as no euill can alwayes lie hid) the king was sore moued, and being desirous to trie the trueth without breaking of the Crowne, proponed the doubt to Archimedes, vnto whose

wit

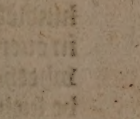
wit nothing seemed vnpossible, which although presently he could not aunswere vnto, yet hee had good hope to diuise some policie for that inuention. And so musing thereon, as he chaunced to enter into a baine full of water to wash him, he obserued that as his bodie entred into the baine, the water did run ouer the Tub: whereby his readie wit of such small effects coniecturing greater workes, conceiued by and by a reason of solution to the Kings question, and therefore reioycing exceedingly more than if he had gotten the Crowne it selfe, forgate that he was naked, and so ran home, crying as he ran, εὕρηκα, εὕρηκα, I haue found, I haue found. And thereupon caused two massie peeces, one of golde, and an other of siluer to be prepared of the same weight that the saide crowne was of: and considering that golde is heauier of nature than siluer, and therefore golde of like weight with siluer, must needes occupie lesse rowme, by reason it is more compact and sound in substance, he was assured, that putting the masse of golde into a vessell brimme ful of water, there would not so much water run ouer, as when he should put in the siluer masse of the like weight. Wherefore he tried both, and noted not onely the quantities of the water at each time, but also the difference or excesse of the one aboue the other, whereby he learned what proportion in quantitie is betweene golde and siluer of equall weight.

weight. And then putting the crowne it selfe into the vessel of water brimme ful (as before) marked howe much water did run out then, and comparing it with the water that ranne out when the golde was put in, noted howe much it did exceede that: and likewise comparing it to the water that ran out of the siluer, marked how much it was lesse than that: and by those proportions found out the iust quantitie of golde that was taken out of the crowne, and howe much siluer was put in stead of it. But seeing Vitruuius which writeth this historie, doth not declare the particular worke of this triall, it shall be no inconuenience to suppose an example for declarations sake, wherein although the true and iust proportions be not expressed, yet the forme of trial shal be truely set forth. And for an example, I suppose the weight of the crowne to be 8 pound, & so of ech the other two masses. And when the masse of golde was put into the water, I imagine that there ran out 2 pound of water: and when the masse of siluer was put in, I suppose there ran out 3 pound $\frac{1}{2}$. Againe when the crowne was put in, there ran out 2 pound $\frac{1}{2}$. Now to knowe what quantitie of siluer was in the crowne, worke by the rule of false position, and imagine that there was 2 pound of siluer: then must there be 6 pound of golde. then say thus by the rule of proportion: If 8 pound of gold doe expel 2 pound of water, what

shal

shall 6 pound expell? and it will be 1 pound $\frac{1}{10}$. Againe for the siluer: If 8 pound of siluer expell 3 pound $\frac{1}{2}$ of water, what shall 2 pound of siluer put out? it will be $\frac{7}{8}$. Now adde those two weights of water together, and they wil make 2 pound $\frac{3}{8}$, and it should be by the suppositiõn 2 pound $\frac{1}{2}$, so is it too much by $\frac{1}{8}$.

Scholer. Now doe I vnderstand the worke as I thinke, therefore I pray you let me worke the rest of the question. And because this first supposition did erre, I note that position, and his errour, & take a newe position, estéming the siluer to be but one pound, so must there be in golde 7 pound. Then say I: If 8 pound of golde doe yéld 2 pound of water, what shall 7 pound yéld? and it will be 1 pound $\frac{3}{4}$. Again if 8 pound of siluer expell 3 pound $\frac{1}{2}$ of water, what shall 1 pound expell? and it will be $\frac{7}{16}$. Nowe must I adde those two summes together, and they make 2 pound $\frac{3}{16}$, & they should make 2 pound $\frac{1}{2}$, so is it too litle by $\frac{1}{16}$. Therefore I set the positions with their errors in order, as here followeth. And then I multiplie in crossewaies 2 by $\frac{1}{16}$, and it maketh $\frac{1}{8}$: likewise 1 multiplied by $\frac{1}{8}$: maketh $\frac{1}{8}$. And because the signes be vnlike, I must adde those two summes which make $\frac{1}{4}$, and that is the diuident. Againe I must adde $\frac{1}{8}$ to $\frac{1}{16}$, and it will be



be $\frac{3}{16}$, that is the diuisor. Nowe I shal diuide $\frac{1}{4}$ by $\frac{3}{16}$, and the quotient will be $\frac{16}{12}$, that is 1 $\frac{1}{3}$, whereby I knowe that there was put 1 ℔ and $\frac{1}{3}$ of siluer into the crowne, and so much golde taken out for it.

Master. Proue it now by examination according to the question.

Scholer. If there were 1 pound $\frac{1}{3}$ of siluer, then was there of golde 6 pound $\frac{2}{3}$. Nowe say I by the rule of proportion:

If eight pound of golde expell two pound of water, what shall 6 pound $\frac{2}{3}$ expell?

8 — 2
$6\frac{2}{3}$ — $1\frac{1}{3}$

It wil be one pound $\frac{1}{3}$. Again,

8 — $3\frac{1}{2}$
$1\frac{1}{3}$ — $\frac{7}{12}$

If 8 pound of siluer expell 3 pound $\frac{1}{2}$ of water, what shall 1 $\frac{1}{3}$ expell? It will be $\frac{7}{12}$.

Nowe must I adde together 1 pound $\frac{1}{3}$, and $\frac{7}{12}$, and they will make 2 pound $\frac{9}{36}$, that is 2 pound $\frac{1}{4}$, according to the supposition of the question, whereby I perceiue the worke to be wel donne. And as I cannot but much reioyce of this excellent inuention, so my desire is kindled vehemently to be perfectly instructed in euerie part thereof, & namely in this point, whether the portion betweene water and gold be such, that for 8 pound of golde into a vessell full of water, there shall run out 2 pound of water: and for as much siluer, whether 3 pound $\frac{1}{2}$ of water would auoide?

Master. I perceiue your meaning, and con-

coniecture your imagination to be thus: that if you knewe the exact proportion betweene golde & siluer and water both in their weight and quantities, then could you eaſilie finde out the mixtures of them, which thing I haue reserued for an other worke that intreateth of such matters especially. And at this time you muſt consider, that you learne Arithmetike, which intreateth of ỹ maner to solue doubtful questiõs touching number, wout regard what matter is signified by that number, else were it necessarie in Arithmetike to teach all Arts, seeing in it may be moued questions of all artes. But seeing you are so desirous to know this thing, I will tell it you in such a sort, that you shall practise your arte in finding it, and propoũd it in forme of a question. Gold beareth greater proportion to water than siluer doth, and their two proportions be in proportion together as $\frac{48}{25}$. But to helpe you somewhat in this riddle, you shall note that the proportion of quicke siluer vnto water, is the iuſt midle number proportionall in Progreſſion Geometricall, betweene the proportions of golde and siluer vnto water. And his proportion is $\frac{290}{21}$. Nowe if you will knowe the iuſt numbers of these two proportions, then muſt you finde out 3 numbers in Progreſſion Geometricall, whereof the midlemoſt muſt be $\frac{290}{21}$, and the firſt muſt be vnto the laſt, as 25 to 48. And thus I will leaue you to finde those numbers

Note.

A queſtiõ of the proportion of golde, siluer and quicke siluer vnto water.

when you be at leysure.

Scholer. Yet sir I thanke you heartilie for this much, for nowe I see the possibilitie to finde them out. Howbeit, because this question seemeth strange, if it might please you to instruct me somewhat in the order of working for it, I should the more easily finde the true working.

Master. You desire too much ease if you will studie for nothing: therefore to occasion you to studie the better, I will leaue this doubt wholly to your owne search. But as touching the generalitie of the rule, Archimedes needed not to take two masses of golde and siluer equall in weight with the crowne, for the proportion might as well be found in any other weight, yea although the masse of golde were of one weight and the masse of siluer of another. As for example. If the crowne were of 8 pound weight, as I did suppose, and I haue not so much other fine golde, but onely one pound, and trying that by water, and finding that it doeth expel but $\frac{3}{4}$ of an ounce of water, yet then by it may I inferre, that 8 pound of gold would expell 6 ounces of water. And likewise of the siluer: whereof if I had but 2 pound, and finde that it doeth expell three ounces of water, then might I affirme that 8 pound would expell 12 ounces, that is 1 pound weight: and so is it, as good as if the 3 masses were all of one weight. And thus for

for this time I will make an end of this other part of Arithmetike.

Scholer. Although I cannot sufficiently thanke you for this, yet your promise made me to loke for the art of extraction of rootes, whereof hitherto I haue learned nothing.

Master. I will not breake my promise, but intend (God willing) to performe it within these three or foure moneths, if I perceiue this my paines to be well taken in the meane season. And you shall not repent the tarrying for it: for it shall be increased by the tarrying. And in the meane time, you shall take this Addition, not for the second part of Arithmetike which I promised, but for an augmentation of the first part, vnto which I would haue annexed the extraction of rootes, square and cubike, namely for examples of the statute of Assise of wood, but that in the second part I must write of diuers other rootes, and thought it best to reserue those rules also with their examples vnto the same second part.

Scholer. Sir, although I cannot recompence your goodnesse, yet I shall alwayes doe mine endeuour to occasion you not to repent your benefite on me thus employed.

Master. That recompence is sufficient for your part.

FINIS.

The third part

or Addition to this Booke entreateth of briefe Rules, called Rules of Practise, of rare, pleasant and commodious effect, abridged *into a briefer Method than hitherto* hath beene published.

*With diuerse other necessa-*rie Rules, Tables, and Questions not onely profitable for Merchants, but also for Gentlemen, and *all other Occupyers whatsoeuer*, as by the Contents of this Booke may appeare.

Set forth by *John Mellis* Schoolemaster.

The first chapter of this Addition entreateth of briefe Rules, called Rules of PRACTISE, with diuerse necessarie Questions profitable, not *onely for Merchants, but also for all* other Occupyers whatsoeuer.

1. Rule. THE working of Multiplication in Practise, is no other thing than a certaine maner of multiplying of one kinde by another: whereupon is brought forth the product of the proponed number, which is accomplished by the meanes of Diuision in taking the halfe, the third, the fourth, the fift, or such other parts of the summe which is to be multiplied.

And for the better vnderstanding of such conuersions: you shall vnderstand that in the maner and vse of these rules of Practise, you ought first to knowe the euen or aliquot parts of a shilling, which in this Table following doth appeare.

Item d { 6 4 3 2 1 } is the { ½ ⅓ ¼ ⅙ 1/12 } of a s.

Where-

Wherein as you see according to the order of these rules of Practise at 6 d the yard of any thing, you must take the ½ of your number which is to be multiplied, and the product that commeth therof shall be shillings, if any vnite doe remaine it is 6 d.

For 4 d take the ⅓ of the number that is to be multiplied, and the product also produceth shillings if any vnites doe remaine, each one shall be worth in value 4 pence. The like is to be vnderstode of the other 3, &c.

Example I.

At 6 d the yarde: what 379 yards?

189 s—6 d

II.

At 4 d the yarde: what 104 yards?

34 s—8 d

III.

At 3 d the yarde: what 5014 yards

1253 s—6 d

IIII.

At 2 d the yarde: what 532 yards?

88 s—8 d

V.

At 1 penie the yarde: what 409 yards?

34 s—1 d

Here you may see in the first example that 379 yards at 6 d the yard, are worth 189 s 6 d, in taking the ½ of 379. And in the second example the 104 yards at 4 d the yard, are

worth 34 s 8 d in taking ȳ⅓ of 104. Likewise in the third example 5014 yards at 3 d the yard bringeth forth 1253 s 6 d in taking the ¼ of 5014. Also in the fourth example at 2 d the yard maketh 88 s 8 d.

And lastly in the fifth example: 409 yards at 1 d the yard, amounteth to 34 s, and 1 d, in taking ȳ 1/12 of 409: And so is to be done of al other questions ȳ like, when the number of ȳ pence is any of the eue or aliquot parts of 12 d

Item to bring the products of these shillings & all other the like into pounds is verie easie in dividing of it into your minde by 20, for it is to be understoode that as often as 20 is found in that product, so many pounds doth it containe, which with facilitie to performe, alwaies strike of the figure toward your right hand, with a right downe dash of your pen for the 0 that appertaineth to 20. And then begin at the left hand, in taking the ½ of the rest. And if at the last any vnite doe remaine, the same shall be ioyned with the figure that is cut off, which shall represent the odde shillings contained in that worke.

As for example in your third question at 3 d the yard which amounteth to 1253 s 6 d, the product whereof maketh 62 l 13 s 6 d: as here you may se is easily performed by this example.

1
125|3
62—13—6

Item also for the working of 1 penie the yard

yard, it is something harsh and hard to take the $\frac{1}{11}$ of some products: Therfore to ease that hard worke, you shall first bring your deliuered summe into groates, by taking the $\frac{1}{4}$ part of the product. And if any vnites remaine of that $\frac{1}{4}$ part, as sometimes there may, they are pence: and must be signified with a line from the groats with their title of pence: And because that 60 groates maketh a pound or twentie shillings, strike of the first figure toward your right hand for the o that appertaineth to 60 (as you did euen nowe for the o that belongeth to 20:) then in taking the $\frac{1}{5}$ of that product, if there doe remaine any vnites, the same shall you ioyne with the figure that you cut off, esteeming the as groats: which keepe in your minde. And by taking the $\frac{1}{5}$ part of them, you shall turne into shillings: and so haue you done: as for example by a question or two hereafter proponed shal more plainely by the worke appeare.

At 1 d the yard what 5 4 3 6 8 yards

1 3 5 9 | 2 groates

$\frac{1}{8}$ £—226—10 s—8 d

Here in taking the $\frac{1}{8}$ part of 1 3 5 5: in comming to the last worke the $\frac{1}{8}$ part of 39 being taken, the remainer is 3, which ioyned with the 2 that was cut off, maketh 32 groats: which conuerted into shillings by taking the $\frac{1}{3}$ part: maketh as appeareth 10 s. 8 d. Many other wayes there are, but none more

more apt for a young learner to vnderstand than this: wherefore this one way well impressed in memorie is better than 20 wayes doubtfully vnderstode.

At 1 d the yard, what 4532 yards?

113	3 groates—1 d
℔ 18——17——9 d	

At 1 d the yard, what 64768 yards?

1619	2 groats
℔—269—17—4 d	

2. Rule. NOwe followeth also to be vnderstood that if the number of pence be not an aliquot part of 12, you must reduce them into some aliquot part of 12. And after the aforesaide maner, you shall make of them 2 or 3 products as nede shal require, and adde them together into one summe. And here for thy furtherance appeareth a note of the order of their parts, as they are to be taken.

For pence
5. take. 3 ℔ 2
7. take. 4 ℔ 3
8. take. 4 ℔ 4
9. take. 6 ℔ 3
10. take. 6 ℔ 4
11. take. 6. 4 ℔ 1
} or
4 and 1
6 and 1
6 and 2
4. 4 ℔ 1
4. 4 ℔ 2
4. 4 ℔ 3

Here in the first note of this Table at 5 d, you shall first take for 3 d the ¼ of the number that

that is to be multiplied: And likewise for 2 d the $\frac{1}{8}$ of the same number, adding together both the products. But if you will worke by 4 and 1, you must for 4 d first take the $\frac{1}{3}$ of the number that is to be multiplied. And for 1 d take the $\frac{1}{12}$ of the whole sum, or rather which is more better for 1 d, you may take the $\frac{1}{4}$ of the product which did come of the 4 d: Because that 1 d is the $\frac{1}{4}$ of 4 d: The total sums of these two numbers shall be the solution to the question. And in like maner is to be done of all others, as by these examples following shall appeare.

I.

At 5 pence the yard what—758 yards?

3 d	189—6 d
2 d	126—4 d
	315 s—10 d

Otherwise.

At 5 pence the yard what—758 yards?

4 d	252—8 d
1 d	63—2 d
	315 s—10 d

II.

At 7 d the elle, what 563 elles?

4 d	187—8 d
3 d	140—9 d
	328 s—5 d

III.

At 8 pence ŷ pound what 112 pounds?

4 d	37—4 d
4 d	37—4 d
	74 s—8 d

Other-

Otherwise.

At 8 pence ỹ pound what 112 pounds?

6 d	56—0
2 d	18—8
	74 s̄—8 d

IIII.

At 9 pence the elle what 356 elles?

6 d	178—0
3 d	89—0
	267 s̄—0 d

V.

At 10 pence ỹ péce what 795 péces?

6 d	297—6
4 d	265—0
	662 s̄—6 d

VI.

At 11 d the pound what 7579 pound?

6 d	3789—6
4 d	2526—4
1 d	631—7
	694 | 7 s̄—5 d
	£ 347—7 s̄—5 d

Here in this first example where it is demaunded (at 5 d the yard) what will 758 cost: First for 3 d I take the $\frac{1}{2}$ of 758: ẽ thereof commeth 189 s̄—6 d: Then for 2 d I take the $\frac{1}{3}$ of the same product which amounteth to 126 s̄ 4 d, these two summes added together, doe make 315 s̄, 10 d: And so much are the 758 yards worth at 5 d the yard.

Item also for the same againe: First for 4 d I take the $\frac{1}{5}$ of 758: and thereof commeth 252 s̄ 8 d: then for 1 penie I take the $\frac{1}{4}$ of the same product, that is to say of 252 s̄—8 d, and it yéldeth me 63 s̄ 2 d: which both added together

gether maketh 315 ß—10 ð, as before.

Item for 7 ð there is taken the $\frac{1}{2}$ and the $\frac{1}{4}$ of the whole summe: which is to be multiplyed, and adde them together, that is to say, first, for 4 ð there is taken the $\frac{1}{3}$ of 563: which comes to 187 ß—8 ð, as appeareth by the worke: and for 3 ð there is taken the $\frac{1}{4}$ of the whole summe which amounteth to 140 ß 9 ð, Both which products added together maketh 328 ß—5 ð: And so much comes 563 elles to at 7 ð the elle.

Item, for the first 8 ð there is taken for 4 ð the $\frac{1}{2}$ of the whole summe: and an other $\frac{1}{3}$ for the other 4 ð, which added together as in the example doth evidently appeare, amounteth to 74 ß—8 ð.

Againe, for the second worke of 112 ℔, there is taken first the $\frac{1}{2}$ of the whole summe for 6 ð, which comes to 56 ß: then for the 2 ð you haue to take $\frac{1}{3}$ of the whole summe, or if you will the $\frac{1}{3}$ of the product that came of 6 ð, eyther which maketh 18 ß 8 ð. These two summes being added together doe make 74 ß 8 ð, as in the third example appeareth.

Item, for 9 ð there is taken for 6 ð the $\frac{1}{2}$ of the whole summe: and the $\frac{1}{4}$ of the whole sum for 3 ð, or otherwise for the 3 ð you may take the $\frac{1}{2}$ of the product that came of 6 ð, because 3 ð is the $\frac{1}{2}$ of 6 ð: which added together as plainely appeareth in the fourth example, amounteth to 267 ß—0 ð.

Item

Item, for 10 d, first there is taken for 6 d the ½ of the whole summe, which amounteth to 397 s 6 d. Then for 4 d there is found 265 s: both which added together maketh 662 s 6 d, as appeareth in the first example. It may also be wrought, as appeareth by the second note in the table by 4 d twice taken, and the ½ of the product of 4 d: or else by the ⅕ of the whole summe, &c.

Item, for 11 d, there is first taken the ½ for 6 d: then the ⅓ of the whole summe for 4 d: lastly, the ¼ of ŷ last product for 1 d: All which 3 summes added together, maketh in s 6947. 5 d, and in pounds 347—7 s—5 d.

3. Rule. ITem likewise by the same reason, when you will multiplie (by shillings) any number that is under 20 s, you shall haue in the product pounds, if you knowe the euen or aliquot parts of 20, which are here in this litle Table set downe to sight.

Item s { 10, 5, 4, 2, 1 } is the { ½, ¼, ⅕, 1/10, 1/20 } of one l.

So that for 10 s which is the ½ of a pound, you may take the ½ of the number which is to be multiplied: and you shal haue in your product

duct pounds: if a vnite doe remaine, it shal be worth 10 s.

Likewise for 5 s you must take the $\frac{1}{4}$ of the number which is to be multiplied: And if there doe remaine any vnites, they shall be fourth parts of a pound, euerie vnite being in value 5 s.

For 4 s take the $\frac{1}{5}$ of the number which is to be multiplied: And if there doe remaine any vnites, they shall be fift parts of a pound, ech vnite being worth 4 s.

For 2 s you must take the $\frac{1}{10}$ of the number to be multiplied: wherefore to take the $\frac{1}{10}$ of any number, you must cut off the last figure of the same number (which is nearest your right hand) from all the other figures with a small right down line or dash with a pen, and so haue you done: for all the other figures which do remaine toward your left hand from the same figure that you doe separate shall be the saide $\frac{1}{10}$ of a pound: And that figure so separated towards your right hand, shall be so many péeces of 2 s the péece: the which figure you must double to make thereof the true number, of s, as by the example shal appeare.

Finally, for 1 s néedeth small worke, for it is so many shillings as be proponed in the summe, which to bring into poundes hath béene alreadie taught in the first rule.

Example.

At

At 10 s the péce
What are—6543—péces worth?

½ £—3273—10 s

At 5 s the Elle
What comes—4373—Elles to?

¼ £—1093—5 s

At 4 s the yard
What—7839 yards?

⅕ £—1567—16 s

At 2 s the pound weight
What are—752|7—pounds weight worth?

1/10 £—752—14 s

At 1 s the péce
What comes—757|3 péces to?

1/20 £—378—13 s

4. Rule. NExtly, nowe followeth in order to be vnderstoode, that if the number of shillings be not some euen or aliquot part of 20, you must then conuert the same number of shillinges into the aliquot partes of 20, and thereof make two or three products, as néede shall require: which donne, adde them together, and bring them into poundes. And here for thy furtherance I haue set downe a note of the order of their parts, as they are to be taken.

If or

s̄				s̄	
3		2 ₤ 1		13	10.2 ₤ 1
6		4 ₤ 2	5. ₤ 1.	14	10. ₤ 4
7		5 ₤ 2		15	10. ₤ 5
8	of	4 ₤ 4	or 5.2.1.	16	of 10. 5. 1
9		5 ₤ 4	4.4.1.	17	10. 5. 2
11		10 ₤ 1		18	10. 4. 4
12		10 ₤ 2		19	10. 5. 4

For 3 s̄ according to the tenor that you see is expressed in the Table, you must first take for 2 s̄ the $\frac{1}{10}$ of the number that is to be multiplied: Then for 1 s̄ you must take the $\frac{1}{2}$ of the product which did come of the same $\frac{1}{10}$ part, which two summes added together, produceth the effect desired.

Item, for 6 s̄ according to the note set forth in the table, first for 4 s̄ I take the $\frac{1}{5}$ of the number that is to be multiplied: Then for 2 s̄ the $\frac{1}{5}$ of the product that came of 4 s̄, and adde them together.

Or else, as appeareth also in the table, for 5 s̄ you may take the $\frac{1}{2}$ ₤ the $\frac{1}{5}$ part of the product that came of 5 s̄, and adde them together.

Item, for 7 s̄, first take for 5 s̄ the $\frac{1}{4}$ of the product, that is to be multiplied, then for 2 s̄, take the $\frac{1}{10}$ of the number that is to be multiplied, and adde them together, &c.

Item, for 8 s̄, according to reason, and the intent of the Table, for the first 4 s̄ take the

$\frac{1}{5}$ of the product, and the same number againe for the other 4 s, and adde them together.

Item, for 9 s: first for 5 s take the $\frac{1}{2}$: then for 4 s take the $\frac{1}{5}$: and adde them together.

Otherwise as you see by the intent of the table, worke twice for 4 s, as was taught euen nowe for 8: and then take the $\frac{1}{2}$ of the last product for the 1 s: But 5 and 4 is the shorter.

Item, for 11 s, first dispatch 10 s: for which you must take the $\frac{1}{2}$ of the product, then lastly for 1 s take the $\frac{1}{10}$ part of the summe produced of the $\frac{1}{2}$ of the product, and adde them together.

Item, for 12 s where I will end with the first part of my Table: First take the $\frac{1}{2}$ for 10 s. And then for 2 s take the $\frac{1}{5}$ of the summe that came of 10 s, and adde them together: or else, if you please for 2 s you may take the $\frac{1}{10}$ of the whole giuen number.

To write more of the maner of taking the true parts, I omit. The desirous practitioner will (no doubt) conceiue it. Also the Table is some aide to helpe the vnperfect, whereupon by and by I will set downe three or foure of these notes in examples, and the rest I will leaue to thine owne industrie and practise to labour vpon.

This is the order most commonly vsed in Practise when the number of the s is not an aliquot part of a pound. But (louing Reader) after I haue touched the euen or aliquot parts of

of a ℔ that falleth out in ℔ and s̄, I will deliuer two newe rules that shall drowne this common order quite and cleane: wherein shall be comprehended in one line or working both euen and odde part of s̄ vnder 20: without regard whether it be an aliquot or not an aliquot part: which two rules, when they come in place, I commit to thy friendly iudgement in working.

Nowe followe the examples vpon the notes before saide.

At 6 s̄ the yard

What	—3215—	yardes
4 s̄	643—	
2 s̄	321—10	
	℔—964—10 s̄	

Otherwise by Multiplication of 6.

	3215—	
6 s̄	19290	
	℔—964—10 s̄	

At 7 s̄ the Elle

What	—4563	Elles
5 s̄	1140..	15
2 s̄	456..	6
	℔—1597..	1 s̄

Otherwise by multiplication of 7.

4563

7s 3194|1

1597.. 1s

At 8 s the pèce what 7563 pèces?

4s 1512.. 12

4s 1512.. 12

£—3025.. 4s

Otherwise by Multiplication.

7563

8s 6050|4

£—3025—4s

At 13 s the pèce what 401 pèces?

10s 200.. 10

2s 40.. 2

1s 20.. 1

£—260.. 13s

Otherwise by Multiplication.

401

13s 1203

401

5213

£.. 260—13s

These

These & such like Questions of compound numbers which I haue here in this fourth rule for orders sake set downe, for that it hath bin heretofore a common course of worke, I account but as superfluous. For in the 8 and 9 Chapters of this my simple Addition shal appeare that the giuen price of any euen or od number of shillings eyther vnder or aboue 20, shal be wrought at one or two workings at the most, howe difficult soeuer the Question be.

ITem, there resteth yet a kinde of Practise, howe to bring pence into pounds at the first working: whereupon you must vnderstande, that 240 pence maketh one pound, or 20 s. In consideration whereof I cut off the last figure or 0: and there remaineth but 24 (of which 24) 8 d is the $\frac{1}{3}$ part thereof: 6 d is the $\frac{1}{4}$ part, 4 d the $\frac{1}{6}$ part: and 2 d is the $\frac{1}{12}$ part thereof. 5. Rule.

Whereupon if it were demaunded what 1486 yards or poūds of any thing commeth to: at 8 d the yard, in pricking or cutting off the first figure towards your right hande: for the 0 that appertaineth to 240: There is remaining of the saide summe 148, whereout I taking the $\frac{1}{3}$ part, and it commeth to 49 l, and there resteth 1: which 1 I put to the 6, that I pricke or cut off, and it maketh 16 peces of 8 pence, which I double to make into groates,

and they make 32, whereof the $\frac{1}{3}$ part maketh 10 s, and there remaineth $\frac{2}{3}$ s, which is 8 pence, whereby it followeth, that the 1486 yards at 8 pence the yard, maketh 49 l, 10 s 8 d, as by the example shall appeare.

Item for 6 pence, take the $\frac{1}{4}$ part of the number from the prickt figure: And if any vnites doe remaine, they are so many fire pences, whereof taking the $\frac{1}{2}$ they are shillings, if there doe remaine yet one, it is in value 6 pence.

Item for 4 pence, take the $\frac{1}{6}$ part of the number from the prickt figure: If any vnites remaine, they are so many groats, which to conuert into shillings take the $\frac{1}{3}$ part: And if any thing yet remaine, they are thirds of shillings, each one in value being worth 4 pence.

Item for 3 pence, take the $\frac{1}{8}$ part from the prickt figure, if any vnites remaine, they are so many peces of 3 pence whereof in taking the $\frac{1}{4}$ part, maketh shillings: If any thing yet remaine, they are fourth partes of shillings, eche one being in value 3 pence.

Item, for 2 pence as appeareth also by the table, take the $\frac{1}{12}$ part of the number from the prickt figure: If any thing remaine, they are so many peces of 2 pence, which by taking the $\frac{1}{2}$ part, you shall turne into shillings: and if any vnites remaine, they are so many firt part of shillings, or peces of 2 d, whether you wil.

If

If one cost 8 d
What are 1486 worth?

maketh £— 49 · —12—0 d

If one cost 6 d
What are 7865 worth?

maketh £—196 · —12—6 d

At 4 d the yard
What are 8736 yardes worth?

maketh £—145 · —12—0 d

If one cost 3 d
What are—9874 worth?

maketh £—123 · —8—6 d

At 2 d the Elle
What comes 7894 Elles to?

maketh £— 65 · 15—8 d

BVt if your number of pence be not an aliquot or euen part of 24: then must you bring them into the aliquot parts of 24, and make thereof diuers products, which must be added together, as by the questions hereafter following shall appeare. 6. Rule.

Item for 5 d, first take for 3 d, then for 2 d: and adde them together according to the instruction of the second rule: Or else first take

for 4 d, then for 1 d.

Item for 7 d, first take for 4 d: then for 3 d and adde them together.

Item for 9, first take for 6 d: then for 3 d, and adde them together.

Item for 10 d, first take for 6 d: then for 4 d, and adde them together.

Item for 11 d first take for 8 d, then for 3 d, and adde them together: as by these examples following doth appeare.

Examples.

If one yard cost 5 d
what are 7596 worth?

4 d	126	—12
1 d	31	—13
maketh pouds	158—	5 s

Otherwise.

1—5—7596

3 d	94	—19
2 d	63	—6
maketh pounds	158	—5 s

If one cost 7 d
what are 987 worth?

4 d	16	—9—
3 d	12	—6—9
maketh pounds	28—	15—9 d

Other-

Otherwise.

1—7—987

6 d	24	—13—	0
1 d	4	— 2—	3
maketh pounds	—28—	15—	9

If one cost 9 d
what are 987 worth?

6 d	24	—13—	6
4 d	12	—6—	9
maketh pounds	37	—0—	3

Otherwise.

1—9—987

3 d	12	—6—	9
maketh pounds	—37	—0—	3

If one cost 10 d
what are 987 worth?

6 d	24	—13—	6
4 d	16	— 9—	0
maketh pounds	—41	— 2—	6

If one cost 11 d
what are 987 worth?

8 d	32	—18—	
3 d	12	6—	9
maketh pounds	45	4—	9

But

But if you haue any shillings and pence to be multiplied together: Then are you to take for the shillings according to the instruction of the third rule: And for the pence according to the first rule before mentioned: vnlesse you can spie the aduantage thereof, and thereby helpe your selfe: as appeareth in this second example, where first I worke for 6 d: which is to be rebated out of the giuen number, and I haue 719 l 11 s my desire.

At 19 s 6 d the yard
what 738 yards

	738	
10 s	369 · —	
5 s	184 · — 10	
4 s	147 · — 12	
6 d	18 · — 9	
l	719 · — 11 s	

Otherwise by Rebating.

738	
6 d	18 · — 9
l	719 — 11 s

The like againe is done by rebating as by these 2 examples appeareth:

At 18 s the Elle what 418 Elles		
2 s	41 · — 16	
l	376	— 4 s

At 16 s the Elle what 517 Elles		
4 s	103 · — 8	
l	413	— 12 s

And

And nowe I will touch a litle the euen parts of a pound that falleth out in pence and shillings, whereof for those parts you shall take such like part of the giuen number that is to be multiplied, as the price of that giuen number beareth in proportion to a pound, which also for thy better aide is here set downe. 7. Rule.

1 s. 8 d.
2. 6
3. 4
6. 8

} is the { 1/12 1/8 1/6 1/3 } part of a l.

Item first for 1 shilling 8 pence take the 1/12 part of the giuen number, and if any thing do remaine, they are twelue parts of a pound, each one being in value 1 shilling 8 pence.

Item for 2 shillings 6 pence, take the 1/8 part of the number that is to be multiplied. And if any thing doe remaine, they are eight parts of a pound each one being in value 2 shillings 6 pence.

Item for 3 shillings 4 pence as appeareth by the table, you must take the 1/6 part of the giuen number. And if any thing doe remaine, they are 6 parts of a pound, each one being in value 3 shillings 4 pence.

Item for 6 shillings 8 pence take the 1/3 part of the number that is to be multiplied: And if any vnites doe remaine, they are thirds of a pound,

a pound euerie one being worth 6 shillings 8 pence.

Other infinite numbers there are, that may be reduced by abbreuiation into the proportionate parts of a pound: as 16 shillings 8 pence maketh $\frac{1}{3}$: which 16 shillings 8 pence is easily reduced into groates by multiplying 16 by 3, and therto adde 2, which maketh 50 groats: Then set 60 the groates of a pound vnder 50, cutting of the 2 ciphers as is here performed in the margent. And then haue you brought 16 shillings 8 pence into the knowne parts of a pound which maketh $\frac{1}{6}$.

16—8
3
5 | 0
6 | 0

But yet gentle Reader, for thy further instruction, I haue hereunto annexed in a Table, howe pence and shillings beareth proportion to a pound, which I commit to thy friendly benevolence, it will be some aide vnto the vngrounded practitioner: but I count him the best workeman that can presently reduce his giuen price vnto the knowne & proportionate parts of a pound.

s	d	p	s	d	p
0	2	1/120	8	4	5/12
0	3	1/80	8	9	7/10
0	4	1/60	9		9/20
0	6	1/40	10		1/2
0	8	1/30	11		11/20
1	0	1/20	11	3	19/10
1	3	1/16	11	8	7/12
1	8	1/12	12		3/5
2	0	1/10	13		13/20
2	6	1/8	13	4	2/3
3	0	3/20	13	9	11/10
3	4	1/6	14		7/10
3	9	3/10	15		3/4
4	0	1/5	16		4/5
5		1/4	16	8	5/6
6		3/10	17		17/20
6	3	5/16	17	6	7/8
6	8	1/3	18		9/10
7		7/20	18	4	11/12
7	6	3/8	18	9	15/16
8		2/5	19		19/20

Here followe 4. Examples vpon the 4. notes deliuered.

At

At 1 s 8 d the yard
what are 3 8 8 4 yards worth?

maketh pounds —323—13—4 d

At 2 s 6 d the yard
what are 4 5 6 3 yards worth?

maketh pounds —570—7—6 d

At 6 s 8 d the Elle
what comes 7 5 6 2 Elles to?

maketh pounds—2 5 2 0—13—4 d

Nowe by custome you are able to worke by all sorts of summes being deliuered in shillings and pence, as 1 shilling 1 penie, 2 shillings 2 pence, 3 shillings 3 pence, and so of all other: wishing you to haue some considerations of your questions, when they are set downe, for there are many subtill abbreuiations, and great aduantages to be gotten, and easilie to be perceiued.

As 3 s 8 d of 2 s and 1 s 8 d.

4 s 2 d, of 3 s 4 d, and 10 d, which 10 is $\frac{1}{4}$ of 3 s 4 d.

5 s 8 d of 4 s, and 1 s 8 d.

5 s 10 d, of 5 s and 10 d, which 10 d is $\frac{1}{6}$ of 5 s.

And by this meane when you haue taken one product, you may oftentimes vpon the same

same take an other more briefely than vpon the summe which is to be multiplied, &c.

NOwe gentle Reader that you haue séene the vertue of the euen or aliquot parts of a pound in shillings alone, and also in the aliquot parts of shillings and pence: according to my promise hereafter followeth a briefer and easier method for any euen number of shillings either vnder or aboue 20, than euer yet hath béene published: Notwithstanding Master Humfrey Baker, whose trauel is worthie commendation, & whom for knowledge sake I reuerence, hath in some part touched this first part, though not in this method. The worke of the rule is both pleasant, readie and brief, as by the varietie of the examples deliuered therupon shal appeare. And first I wil set forth a question, thereby the better to expresse or teach you the order thereof: which is this. 8. Rule.

If one yard cost 6s what 8574?
1 6 8574

maketh pounds ——2572· —4

To the vnderstanding of this example, after you haue set downe your giuen number in forme of the rule of 3, with a line drawen vnder it, you shall presently set a pricke vnder your first figure 4, towards your right hande, draw-

drawing from the pricke as heretofore hath beene practised, a litle short line, thereto set downe the shillings anone, which done, multiplie the first figure 4 by 6 the value of your price, (which here you see standeth in sight aboue the line) it maketh 24: which is 1 pound 4 shillings. The 1 pound keepe to carie to the next place, and the 4 shillings set downe at the ende of the prescribed line towardes your right hand. Thus haue you done nowe with 6 aboue the line, and also with 4 in the first place (for the pricke vnder 4 doeth represent that 4 hath done his office.) Then secondarily for a generall rule take but the ½ of the giuen price which here is 3, which 3 is the number that shall nowe continue the rest of the multiplication and ende the worke, whereupon I multiplie 3 into 7 standing in the second place it maketh 21, and with the 1 I I kept in minde 22, set downe 2 and keepe 2 in minde working according to the rule of Multiplication, deliuering the tennes in minde in their due place, which done, the product from the pricke to your left hand representeth the pounds and the other at the end of the line the shillings as appeareth by the examples.

Note. A generall rule.

If one yard cost 2 s̄ what 7536?

1 2 7536

maketh pounds— 753 '—12 s̄

If

If one yard cost 4 s what 8792?
1 4 8792

maketh pounds— 1758 l —8 s

If one peece cost 6 s what 9537?
1 6 9537

maketh pounds— 2861 l —2 s

If one cost 8 s what 7509?
1 8 7509

maketh pounds— 3003 l —12 s

If one cost 12 what 5794?
1 12 5794

maketh pounds— 3476 l —8 s

If one cost 14 s what 3705?
1 14 3705

maketh pounds— 2593 l —10 s

If one cost 18 s what 5703?
1 18 5703

maketh pounds— 5132 l —14 s

If one cost 22 s what 953
1 22 953

maketh pounds— 1048 l 6 s

Let these suffice gentle reader for an entrance into euen numbers: And nowe I will shewe the like rule for any odde or vneuen parts of a pound.

9. Rule. TO helpe you to the vnderstanding of these other questions that hereafter followe: where in my first example the giuen number is 6487. At 3 shillings the yard: I multiplie 3 aboue the line into 7, it maketh 21. The 1 shilling I set downe, and the 1 I keepe. Nowe am I to take the $\frac{1}{2}$ of 3, which because it is an odde number I cannot. Therefore I shall keepe and continue my multiplication by 3 still, and worke by the $\frac{1}{2}$ of the rest of the giuen figures or numbers, to wit 648. And first the $\frac{1}{2}$ of 8 which is 4 multiplied into 3 maketh 12, thereto ioyne the 1 I in minde, it maketh 13: set downe 3 keepe one. Then againe multiplie by 2 the $\frac{1}{2}$ of 4, it maketh 6, and with 1 in minde it maketh 7. Then lastly take the $\frac{1}{2}$ of 6 which is 3, saying 3 times 3 is 9, which 9 set downe, and so is the question aunswered as appeareth by the practise, & the examples following.

At 3 s̄ the yard what 6487?

1 3 6487.

maketh pounds —973.—1 s̄

If one yard cost 5 s̄ what 4269?

1	5	4269

maketh pounds— 1067 l.—5 s̄

At 7 s̄ the Elle what 6489?

1	7	6489

maketh pounds— 2271 l. —3 s̄

If one Elle cost 9 s̄ what 2807?

1	9	2807

maketh pounds — 1263 l.—3 s̄

At 11 s̄ the pistolet what 8263?

1	11	8263

maketh pounds — 4544 l.—13 s̄

If one peece cost 13 s̄ what 4629?

1	13	4629

maketh pounds— 3008 l.—17 s̄

But nowe note gentle Reader, when the giuen price falleth vpon any odde number. As 3.5.7.9.11.13. &c. Then it is to be presupposed, that the giuen summe to be multiplyed must be a summe made of euen numbers, as 2. 4. 6. 8. 10. &c. else cannot that question be wrought at one line or working.

Prouiding alwayes that it may beare an odde figure in the first place towardes your right hand: as appeareth in these 6 examples which last were wrought, and such like, &c.

which may beare an odde number for the price, and be done at one line or working verie well.

But if the giuen price be an odde number, and the summe to be multiplied odde numbers also. Then can it not be done at one working, but requireth the aide of 2 workings, for odde with odde will not agree, which notwithstanding to bring to passe, take this for a generall rule: First worke for the euen number, contained in that question, or giuen price, according as you haue learned, and then afterwards for the one odde shilling, take the ½ of the summe giuen to be multiplied, omitting the first prickt place, as was taught for the working of one shilling in my first rule of Practise, and adde those two together, & you shall haue your desire.

A generall rule.

Example.

At 3 s the yard
what are 7539 yards worth?

2 s	753 ·	—18
1 s	376 ·	—19
maketh pounds—	1130	—17 s

At 7 s the Elle what 7539?

1	7	7539	
6 s		2261 ·	—14
1 s		376 ·	—19
maketh pounds—		2638	—13

At

At 13 s the yard what 7534?

1	13	7534		
12 s		4520	—	8
1 s		376	—14	
maketh pounds		4897	—2	

And thus haue I abbridged into these two rules howe to bring any number of shillings: whatsoeuer they be into poũds, with a briefer method than euer yet hath bẽne published, which I commende vnto thy friendely censure and iudgement in the vse of practise thereof.

If one coll 6 s 5 d what 1231?

1	6	5	1231		
6 s			309	—	6
4 d			20	—10	—4
1 d			5	—2	—7
maketh pounds			394	—18	—11

At 14 s 2 d what 2825?

1	14	2	2825		
14 s			1977	—10—	
2 d			23	—10—	10
maketh pounds			2001	—00—	10 d

At 16 s 4 d what 2531?

1	16	4	2531		
16 s			2024	—16	
4 d			42	—3—	8
maketh pounds			2066	—19—	8

At 3 s the Pistolet what 8325?

1	3	8325
maketh pounds		1248 · —15 s

At 7 s the crowne what 6529?

1	7	6529
maketh pounds		2285 · —3 s

At 9 s the peece what 6567?

1	9	6567
maketh pounds		2955 · —3 s

These three last questions may seeme something hard, yet are they easie ynough if you marke them well, if I should explane them, then are they too easie: therfore I leaue them to whet the mindes of the desirous.

10. Rule. ITem, when any one of the sums which is to be multiplied, is composed of many denominations: and the giuen number but of one figure alone. Then shall you multiplie all the denominations of the other summe by the same one figure, beginning first with that summe which is least in value towards your right hand, and bring the product of those d into shillings, and the product of the shillings into pounds, as by this example doth appeare.

At 3 l 7 s 4 d what are 9 worth?

Maketh	l 30	6	0 d

But

BVt if in any of the summes that are to be multiplied there be a broken number: First worke for the whole according to the instructions that you haue learned: and then take such part of the giuen price, as that broken number beareth in proportion to the price as in the example. After you haue wrought for 3 s and for 6 d, then are you to take the ½ of 3 s 6 d for the ½ yard, and adde that to the summe: So adding all 3 products together, which maketh 43 l 2 s 9 d the iust price of 246 ½ Elles: and thus must you doe of all other. 11. Rule.

At 3 s 6 d the Elle what 246 ½?

1	3	6	246 ½
3 s			36 —18
6 d			6 —3
½			1—9
maketh			43 —2—9

At 16 s 4 d the peece what 14 ¾?

1	16	4	14 ¾
16 s			11 —4—
4 d			0 —4—8
¾			0—12—3
maketh ponnds			12—00—11 d

If one peece coſt 4 l 3 s 6 ½ d what are 12 peeces worth?

4 l	48
3 s	1—16
6 d	0 —6
½	0 —0—6
maketh pounds	50 —2—6 d

The proofe.

If 12 peeces cost 50 l. 2 s. 6 d what one peece?

	50	2	6
maketh pounds	4	—3	—6½

12. Rule. ITem touching the maner howe to vnderstand the order of this question, and others the like: first seeke how many times 12 is contained in 50, which is 4 times, and so resteth 2 l, which 2 l conuerted into shillings, and ioyned with the other 2 shillings, maketh 42 shillings, wherin is found 12 three times: resteth 6 shillings, which turned into pence, putting thereto the 6 d in the first place, it maketh 78: wherein 12 is found 6 times, resteth 6 d, which containeth 12 but ½ a time, put that ½ to the 6 d: And then the solution is 4 l 3 s 6½, as appeareth by the practise thereof.

13. Rule. ITem, the like is to be done of any thing that is bought or solde after 5 score to the hundreth, or the Quintall: As for example.

If

If 100 l. cost 27 l. 13 s. 4 d.
What one pound?

27—13—4	
20	
s.	5 | 53 12
	1 | 10 5 | 3
d.	6 | 4 | 0 2 10 | 03
maketh	5 s.—6 3/5 d.

I haue wrought this at length for the aide of the young learner, because he should vnderstand howe all the multiplication is set downe.

27—13—4	
20	
s.	5 | 53 12
d.	6 | 40 2 100 5
makes	5 s 6 2/5 d.

But to worke it more neately, it is by a litle vnderstanding ended thus.

Item to the vnderstanding of this and such like questions, the right downe line is al the guide, which is pulled downe close by 20, as you see in the example, where 27 pound 13 shillings is reduced all into shillings, and maketh 553 shillings.

The 5 towards your left hand being separated

rated with the hanging or right downe line, is the iust number of shillings that aunswereth to the question: Secondly, 5 shillings is multiplied by 12 to reduce them to pence, putting to the 4 d, it yeeldeth for the multiplication of the first figure two 110: the one beyond the line towards the left hand, is 1 penie towards the rest of the price: then 53 also multiplied by 1 yeeldeth 53: but the 5 behinde the line towards the left hand, is also 5 pence more, towards the price, which 1 and 5 I adde together vnder the line: it maketh 6 d. So is there found nowe as appeareth by the Titles of shillings 8 pence 5 shillings 6 pence.

Finally, I come nowe on this side the line towards the right hand, and vnder 12 I finde first 10, and then 3, which added together maketh 40, vnder which 40 you must put the 100, and it maketh $\frac{40}{100}$, which abbreuiated commeth to $\frac{2}{5}$. So the iust price of one pound after 5 score to the hundreth, maketh 5 s 6 $\frac{2}{5}$ d

One example more, and so wil I leaue this rule.

If 100 cost $10\frac{3}{4}$ ℔

What 9874

100 •• $10\frac{3}{4}$ •• 9874

6 ℔	246 •	17	0
4 ℔	164 •	11	4
$\frac{2}{4}$ ℔	20 •	11	5
$\frac{1}{4}$	10 •	5	$8\frac{1}{2}$

maketh

ℓ	4	42 ••	5 • $5\frac{1}{2}$
ś	8	20	
		45	
		12	
℔	5	$45\frac{1}{2}$ / 100	$\frac{91}{200}$ parts of a ℔.

Also the like may be done of our vsuall weights here in England (which is 112 ℓ for euerie hundreth weight) in case you knowe the aliquot parts of a hundreth weight, which are these, 56 ℓ, 28 ℓ, 14 ℓ, and 7 ℓ: For 56 ℓ is the $\frac{1}{2}$ of 112 pound, 28 ℓ is the $\frac{1}{4}$ of 112 ℓ, 14 ℓ is the $\frac{1}{8}$, and 7 ℓ is the $\frac{1}{16}$ part.

Therefore for 56 ℓ take the $\frac{1}{2}$ of the summe of money that 112 ℓ weight is worth.

For 28 ℓ take the $\frac{1}{4}$ of the summe of money that 112 ℓ weight is worth.

For 14 ℓ, take the $\frac{1}{8}$ of the summe that 112 ℓ is worth.

And for 7 ℓ, the $\frac{1}{16}$ of the summe of money that C is worth.

As for example: At 17 ℓ 19 ś the hundreth pounds

pounds weight, that is to say the 112 ℔, what shall 3 quarterns and 7 ℔ coll?

1 · ·	17 · 19 · ·	3—7
2 quarterns	8 · 19 · 6 ·	
1 quarterne	4 · 9 · 9	
7 pounds	1 · 2 · 5 $\frac{1}{4}$	
maketh pounds	14 · 11 · · 8 $\frac{1}{4}$	

The second Chapter entreateth of the Reduction of diuers measures to others value by Rules of Practise.

15. Rule. NOwe will I shewe a fewe examples of Practise in reducing of measures: as Els, Yardes, Braces, pawnes of Jeanes, &c. Much more I would haue touched but that I feare the booke will rise to too great a volume.

In 864 Elles of Antwerpe, howe many yeardes of London?

864	864
432	216
216	648
maketh 648———	yeardes of London.

Item, in these and such like questions of Flemmish measure to be brought into yardes English: first take the $\frac{5}{7}$ of the giuen number, as

as appeareth in the first example towardes your left hande. Then take the $\frac{1}{2}$ of that product, or the $\frac{1}{4}$ of the giuen number, and adde those 2 products together, they shalbe yardes English, as by the example you may perceiue.

The second example towards your right hand is yet briefer than the first, whose worke is this: take the $\frac{1}{3}$ of the deliuered number, and that product, subtract out of the giuen number, and the rest sheweth your desire. Of these two wayes vse which you thinke best.

The proofe.

In 648 yardes of London,
howe many Elles of Antwerpe?

648

216

maketh 864 Elles of Antwerpe.

Item for the vnderstanding of this worke: first take the $\frac{1}{3}$ part of the yardes of London, which found, adde that $\frac{1}{3}$ part, and the yardes together, as appeareth by the practise: and the product sheweth the Elles of Antwerpe.

In

In 320 yardes of London, howe many Elles of Antwerpe maketh $426\frac{2}{3}$ Elles.

320 yardes	Proofe. $426\frac{2}{3}$ elles
$\frac{1}{2}$ $106\frac{2}{3}$	$\frac{1}{2}$ $106\frac{2}{3}$
$426\frac{2}{3}$ elles	320 yardes

Other Reductions.

Item you shall vnderstande, that for as much as 6 Braces of Millane make 5 Elles of Antwerpe, whereupon, according to the rules of Practise, you may reduce the one into the other by the like reasons aforesaide, in taking the $\frac{1}{5}$ part, and then subtract the same to make Elles of Antwerpe: And againe by the contrarie in taking the $\frac{1}{5}$ part, with adding the giuen number, to turne the Elles to Braces, as for example.

In 876 Braces how many elles of Antwerp?

876 Braces	The contrarie.
146	730 Elles Flem.
Elles 730 Antwerp	146
	876 Braces.

Elles 730 Antwerp.

182 $\frac{1}{2}$

Yardes 547 $\frac{1}{2}$ English.

Thus

Thus appeareth, that 876 Braces by practise, make 730 Elles Flemmish: which Elles Flemmish reduced into English yardes by the rules aforesaide, make 547 ½ yardes.

So againe vpon the same first question of Braces: I would knowe howe many yardes English they make. 17. Rule.

After the rate that 100 Braces are worth 62 ½ yardes.

876 Braces

438
109 ½

I answere 547 ½ Yardes

Item, to the vnderstanding of this worke and such like, first take the ½ of the giuen braces, and after take the ¼ of that halfe, or the ⅛ of the giuen number, & adde them together, and the product are also yardes English.

Item 3 Elles of Rochell make 5 Elles at Lisbone: So likewise 3 Elles at Lions make 5 Elles at Antwerpe. 18. Rule.

To worke these and such like, double the Elles of Lions, and the Elles of Rochel: and from their products subtract the ⅕: And the rest shall be Elles of Antwerpe, or Elles of Lisbone.

Example.

Example.

In 63 elles of Lyons how many elles of Antwerpe?	In 100 Elles of Rochell howe many Elles of Lisbone.
63 63	100
126	200
$\frac{1}{6}$) 21	$\frac{1}{6}$) $33\frac{2}{3}$
Ans. 105 elles Ant.	Ans. $166\frac{1}{3}$ els of Lisb.

Touching the proofe or returne of these & such like questions for a generall rule, you shall first take the $\frac{1}{5}$ of the giuen number: And adde that $\frac{1}{5}$ & the giuen number together, and the $\frac{1}{2}$ of that product shall be your desire.

Example.

In 105 Elles of Antwerp, howe many Elles of Lions?	In $166\frac{2}{3}$ elles of Lisbone, howe many elles of Rochell?
$\frac{1}{5}$) 105 21	$\frac{1}{5}$) $166\frac{2}{3}$ $33\frac{1}{3}$
$\frac{1}{2}$) 126	$\frac{1}{2}$) 200
Ans. 63 els of Lyõs.	Ans. 100 els of Roch.

The

The third Chapter treateth of the order and worke of the Rule of 3 in broken Numbers, after the trade of Merchants, *digressing something from* Master *Records*, which is comprehended in 3 Rules.

NOwe that I haue somewhat intreated of the Rules of Practise, I will giue a fewe instructions after my simple order, for the working of the Rule of 3 in broken numbers, wherein I shall nede to say the lesse, because I hope the studious Learner, that hath trauelled any thing in the Ground of Artes, is not vnfurnished of knowledge capable to vnderstand me. But before I deliuer any instructions for broken numbers, I will propone a question, which shall be wrought 3 sundrie wayes, thereby to shewe as it were 3 degrées of comparison: howe farre the Rule of 3, in broken, for more speede of worke, differeth from whole, which I rather set downe for a viewe, that the studious herein may be more desirous to attaine broken: leauing any more to discourse in Dialogue forme, but onely to giue instructions where nede is: and in the rest to put forth the questions with their aunsweres.

My first question is this.

If one yarde cost 6 s — 8 d
What are ——789 worth at that rate?

1 —— 6 —— 8 —— 789
12 80
80 63120 d.

Here the product of the sum are pence according to the nature of the midle number.

x x
x 3 7 | x
6 3 x 2 0 | 5 2 6 0 (263
x 2 2 2 2 | 2 2 2 0
x x x

I aunswere 263 l

1 —— 6 2/3 —— 789
3 20 20
15780

Here the product of the summe is s, according to the nature of the midle number.

x
x 5 7 5 0 | 5 2 6 0 | 263 pounds.
3 3 3 3 | 2 2 2 0

1 —— 1/3 —— 789
3 1 1
789.

Here the product is pounds according to the title of the second number.

x
789 (263
333
I answere 263 £

Now that you haue séene the 3 former vertues of the Rule of 3, whose products haue first brought forth ð, next s̃, and lastly pounds: I will deliuer 3 notes in order following, and with them a dosen questions, that shall shewe the worke of the Rule of 3 in broken numbers or Fractions.

1 The first foure shall be sundrie questions of a Fraction comming in the seconde place.

2 The second foure shall be of 2 Fractions comming in the second or third place.

3 The third foure of Fractions in al three places.

My first question is this.

If one yarde cost me 3 s̃ —4 ð what are 756 worth at that price? 1. Rule.

In setting downe the question to performe the work I turne 4 ð into the part of a s̃, which is $\frac{1}{3}$, and then the question standeth thus.

1 ——— $3\frac{1}{3}$ ——— 756

TO the readie working of this question, and all such other like, my first note is this: which take for a generall rule, that when any one Fraction shall come either in the second or third place, that the denominator of that Fraction or Fractions, must alwayes be brought vnto the number or Numerator of the first place: and thereby multiplied the one into the other.

And this benefite is alwayes gotten by the vertue of bringing the denominator of the second numbers fraction vnto the first place. For the fraction in the midle number is nowe released: and the product that commeth of the multiplication, is of the nature and like denomination of the whole number in the second place which here are shillings.

Whereupon nowe to worke the question I bring 3 the denominator of the Fraction in the second place, vnto my first number 1 with a line set vnder 1 thus: and the 3 vnder it $\frac{1}{3}$ thus: saying, once 3 is 3 my Diuisor: That done, reduce $3\frac{1}{3}$ saying, 3 times 3 is 9, and the 1 ouer 3 make 10: my second number in the rule of 3: by which 10 I doe multiplie my last number 756 as appeareth by the worke thereof, and it yeldeth 7560 s̄ my diuident.

Then diuiding 7560 by 3 my diuisor, it yeldeth in quotient 2520 s̄, which maketh 126 pounds, as appeareth here most plainely, both

both by the example and the worke.

At 3 s̄ 4 d̄ the yarde what 756 yardes?

1/3	—— 3	1/3	··	756
				10
		10		7560 s̄

x	x		
7560	2520	126	I aunkwere 126 l.
3333	2220		

Yet otherwise vpon the same question, altering the price nowe into the proportion it beareth to a pound, for the 3 s̄ 4 d̄ is 1/6 part of a pound: which example first standeth thus as appeareth on the left hand, and afterwards wrought as appeareth on the right hand.

1	—— 1/6 ——	756	1/6 —— 1/6		756
			1		1
					756 pounds.

As soone as I haue caried 6 the denominator of my midle number vnto my first place, as afore hath beene taught: I pul downe one, the numerator of 6, with a line vnder 6 thus: 6/1: And that one of custome I pull downe in sight being the figure that I shall multiplie my third or last number by, according to the tenor of the rule of three: And because one can

neither multiplie nor yet diuide (though here it is set downe in forme of Multiplication the rather for your vnderstanding) the product of the multiplication according to the declaration of this my first rule or note is conuerted into the title of my second number, which here are pounds: Nowe followeth the diuision performed by my diuisor 6, to make an ende of the question.

```
x 3
756 (126: which maketh 126 l as before.
666
```

And thus much for the varietie in working that question.

And nowe followeth an other.

If one yarde of cotten cost $8\frac{1}{4}$, what 859?

```
1 ——— 8 1/4 ——————— 859 ——— 1
4          33          33
                      2577
                    2577
                   ------
                   28347

2(3 |          | x1 | l   s—d
28347 | 708 (6 | 590 | 29—10—6—1/4 d.
4444  | 42     | 220 |
```

This

This question was also wrought like the first, and bringeth forth 29 ℔—10 ß—6 ¾ ð, the price of 859 yardes.

If 7 ℔ of any thing cost 3 ℔—10 ß, what comes 987 pound to?

7——3 ½——987

2 7 7

14 6909

(7
6909 | 493 7/14 | ½ I answere 493 ℔ 10 ß.
14

Notes vpon my second Rule for two Fractions comming in the second and third place.

My first question is this.

2. Rule. IF one Elle cost 13 ß 4 ð: what halfe a quarter or ⅛ of an Elle?

Answere. First bring 13 ß 4 ð into the parts of a ℔, which is ⅔ and then will the question stand thus.

1——⅔——⅛

Item for the performance of this worke doe as before was taught in the first Rule, first

bring 3 the denominator of the second fraction vnto your first number 1: setting a line vnder 1 thus: Saying once 3 is 3: that done, bring 8 the denominator of the third fraction setting it vnder 3, and multiplie them together, saying 3 times 8 makes 24: which 24 is your diuisor: (Nowe haue you done with the denominator 3, and also with the denominator 8.) Therefore you shall put a line vnder 3 thus. And the like line also vnder 8, setting or pulling downe vnder them their owne numerators, that is 2 vnder 3, and also 1 vnder 8, as appeareth in the example, which numerators for a generall rule are euermore to be pulled down, of custome in sight to multiplie the one by the other, according to the tenor of the Rule of 3. Then I multiplie the one by the other, saying: once 2 is 2, which signifieth 2 ℔, being of the nature and like denomination of the midle number, which 2 ℔ is to be reduced into shillinges, otherwise it cannot be diuided by my first number 24. Then diuiding 40 by 24, the quotient bringeth forth 1 $\frac{2}{3}$ s̃, So much is $\frac{2}{3}$ of an elle worth after that rate. Otherwise although 2 pound could not be diuided by 24: yet it might haue beene abbreuiated to $\frac{1}{12}$ of a pound: which is worth 1 s̃ 8 pence, as before.

1 — 2/3 — 1/8

3 2 1 16

8 20 40 (1 2/3 5/8

24 24

24 40

Second queſtion.

If one pound of any weight coſt 13 ſhillings 4 pence, what are 7/8 of the pound worth after that rate: Anſwere. Reduce the 13 ſhillings 4 pence into the partes of a pound: which is 2/3: and then will the queſtion ſtande thus.

£

1 — 2/3 — 7/8

Item for the vnderſtanding of this, if you marke well the laſt example, this and the reſt lyeth open, and nedeth ſmall inſtruction. For as you did laſt, ſo nowe againe bring the denominator of the ſecond and third fraction vnto the firſt figure 1, multiplying the one into the other, which maketh alſo 24 your diuiſor.

Then making a line vnder 3 thus, and a line vnder 8 thus, and pulling downe their Numerators vnder each figure, that is, 2 vnder 3, and 7 vnder 8, which as I ſaide before for a generall rule I pull downe of cuſtome in ſight: to be the two numbers that of duetie ought

ought to be multiplied together, which done I bring 2 being the lesser figure vnder 7: multiplying them together, it maketh 14: which are of the nature of the midle number. That is to wit pounds: which 14 cannot aptly bee diuided among 24, therefore are reduced into s, as is plainely to be seene in the example. Then 280 s parted among 24, yeeldeth for his quotient 11 s 8 d your desire, and the iust price of $\frac{7}{8}$ of an elle. Otherwise, 14, though it could not be diuided by 24, might by mediation or diuision in broken numbers haue beene diuided or abbreuiated to $\frac{7}{12}$, which in effect being reduced to his knowen parts, maketh 11 s 8 d as before. But my good will & meaning is to aide young beginners. Therefore haue I reduced the 14 l into s, which is the easier way.

Nowe followeth the Example.

				(1
1	2	7	4	
3	3	8	46	
8	2	7	280	11 $\frac{7}{8}$
14		2	244	
		14	2	
		20		
		280 s		

The third Example.

If

If one yarde cost me 2 s̄ 6 d̄, what 345 ¼ yardes?

Answere. First put 6 d̄ into the partes of a s̄: and then the question standeth thus.

1——2 ½——345 ¼:

Item, to the readie vnderstanding of this and all such like, according as before hath bin declared. Bring the denominators of the second and third fractions vnto the first place, multiplying them the one into the other, all which make 8 your Diuisor common. Then next reduce your seconde number, saying 2 times 2 is 4: and 1, is 5: as was taught in the example aforesaide. Lastly reduce your third number 345 ¼ al into fourths, and they make 1 3 8 1: which 1 3 8 1 is to be multiplied by 5, according to the tenor of the rule of 3: which done, maketh 6 9 0 5 s̄: And diuided by 8 your diuisor, yeldeth in quotient 863 ⅛ s̄, which maketh in pounds 43 l̄—3 s̄—1 d̄ ½: And so much are the 345 and ¼ yardes worth at that price.

The same question wrought againe but 2 s̄ 6 d̄, is nowe conuerted into the partes of a l̄, and standeth thus:

1———⅛———345 ¼

Item, after I haue brought here my second and third fraction vnto my first place, and found 32 to be my diuisor: hauing thus furnished

shed my first place with all thinges vnto him belonging (which is meant of bringing and multiplying the Denominators of the second and third Fractions into him) I then goe in hand to see what is to doe in my second place, where presently of [illegible]me I pull downe my Numerator 1 vnder 8. beeing the figure in sight that shall [illegible]ltiplie my third number.

Then lastly I reduce 345 $\frac{1}{4}$ al into fourths, as afore was practised, which maketh 1381, the which 1381 I am to multiplie by one my second number: they are nothing increased, but by the Metamorph of my worke, they are nowe 1381 pound: being of the nature of the midle number, as I haue often shewed you, which diuided by 32 my diuisor, yeldeth 43 pound and $\frac{5}{32}$: which $\frac{5}{32}$ of a £ reduced into knowen numbers, make 3 shillings 1 d $\frac{1}{2}$ as before.

Example.

x

x0(5

1——$\frac{1}{8}$——345 $\frac{1}{4}$ x38.x 5

32 1 1381 3xx (43——

3 32

3. Rule. Nowe followe 4 other questions which are in all three places, broken numbers or whole and broken together.

Item

Item first for the finding out of your diuisor: you shal take this for a most certaine and generall rule. That you must multiplie the numerator of the first number in the question by the denominator of the second: And also all that againe by the denominator of the third: and the totall thereof shall be your diuisor.

Secondarily for a generall rule to finde out your diuident, multiplie the denominator of the first number by the numerator of the second, and the whole therof by the denominator of the third. And the totall thereof shall euermore be your diuident.

Nowe for an example I propone this question, thereby to make my meaning the more plaine, and to shewe you as I haue done in the rest the maner and order of the worke.

If $\frac{2}{3}$ of any weight or measure cost $\frac{5}{6}$ of a £ or 20 s, what are $\frac{3}{8}$ of the like weight or measure worth after that rate?

Example.

$\frac{2}{3}$ —— $\frac{5}{6}$ —— $\frac{3}{8}$

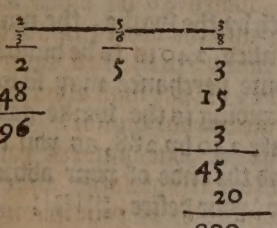
Item for the more plainer vnderstanding hereof, and all other the like, in broken numbers: First you shall pull downe 2 the Numerator of the first number or fraction, with a line vnder $\frac{2}{3}$ thus: that done according as you haue

haue learned before, bring 6 the denominator of the second fraction, and set it vnder 2 multiplying the one into the other which maketh 12. Then lastly bring 8 the denominator of the third fraction, and set it vnder 12, multiplying that 12 by 8, which amounteth to 96: (or else for more briefe multiplie 6 by 8: saying 6 times 8 makes 48, which 48 set vnder 2, and multiplie the one into the other, maketh 96 as before.) And this 96 is the first number in the rule of three. That shall alwayes for a most generall rule be your diuisor.

Secondly to worke for your diuident you shall as hath bene sufficiently declared afore, pull downe 5 the numerator of your second fraction. And set it vnder 6 with a line vnder 6 thus. That done as you knowe, you are to pul downe 3 the numerator of the third fraction, and set it vnder 8 with a line vnder 8 thus, multiplying the one into the other, according to the tenor of the rule of 3, which maketh 15. Then according to my note, forget not to bring the denominator of the first fraction which is 3 vnder 15, and multiplie them together, which maketh 45, which 45 is your diuident. Which 45 are of the nature or denomination of the midle number, as I haue oft taught you before. And therefore are 45 l, which aptly cannot be diuided by 96. Therefore you shall reduce that 45 l into s, as you see is performed in the example, which amoun-

amounteth to 900 s̄, which diuided by 96 your diuisor, it yeldeth 9 s̄, and $\frac{15}{32}$ of a s̄, which in lesser termes is $\frac{3}{8}$, which $\frac{3}{8}$ in money maketh 4 $\frac{1}{2}$ d̄: And so much will the aforesaide $\frac{2}{3}$ cost, as by the worke following shall appeare.

Example.



Otherwise though 45 could not be diuided by 96: yet by Diuision in broken numbers it might haue bene abbreuiated to $\frac{15}{32}$ of a l̄, which reduced into knowen parts wil make 9 s̄ 4 $\frac{1}{2}$ d̄ as before.

Nowe my second example shall be the proofe of this question.

If $\frac{3}{8}$ yardes cost $\frac{15}{32}$ of a l̄ or 20 s̄, what shal $\frac{2}{3}$ cost?

Answere. Worke as was taught you before, and you shall haue your desire.

Here

$\frac{3}{8}$	$\frac{15}{32}$	$\frac{2}{3}$
3	15	2
32	2	
96	30	
3	8	
288	240	

Here as appeareth by the worke, the multiplications being ended, 240 is to be diuided by 288, which to some perchance may seeme hard, yet notwithstanding is the worke good: Therefore abbreuiate 240 by 288, as you see here is practised: and the ende of your abbreuiation shal come to $\frac{5}{6}$ your desire. $\frac{240}{288}\left|\frac{30}{36}\right|\frac{5}{6}$

Otherwise.

240	120	60	30	5
288	144	72	36	6

Otherwise.

240	40	5
288	48	6

The third question.

If $\frac{3}{4}$ elles cost 13 s 4 d, what $156\frac{1}{2}$ elles?

Answere. To worke this question the shortest way: reduce 13 s 4 d into the parts of a £ which is $\frac{2}{3}$.

Then as you did afore, after you haue set downe the question, the numerator of the first fraction 3, is pulled downe vnder 4, and the De-

Denominators of the other 2 fractions multiplied into him which maketh 18 your diuisor.

Then the numerator of the second fraction 2 is pulled downe vnder 3 of custome nowe in sight readie to multiplie my third number by: which is performed as sone as the last number 156 ½ is reduced into halfes.

Then lastly I multiplie that product by 4, the denominator of my first fraction: it yeldeth 2504: which I diuide by 18, and my quotient is 139 l. ⅑ of a remaining, which is worth 2 s. 2 ⅔ d. And so much wil 156 ½ elles cost, as by the worke following doth appeare.

¼	⅔	156 ½	x
3	2	313	47
6		2	76 (2
18		626	2504 | 139 ⅑
		4	1888
		2504	xx

The fourth Example.

If 2 ½ Elles cost 1 ⅓ l. what commeth 29 ¾ elles to?

Item to the workemanship of this question: first reduce your first number to one direct Numerator: in saying 2 times 2 is 4, and 1 is 5: Then bring the Multiplication of the Denominators of the second and third Fracti-

ons, which maketh 12: and multiplie that 12 by 5 your first numerator, it maketh 60, which is your diuisor.

Then the reduction of the second number, which is 5 multiplied by 117 the product of ỹ last numbers reduction, make 585, which 585 yet resteth to be multiplied by 2, the denominator of the fraction in the first place yeeldeth 1170: which diuided by your diuisor 60: yeeldeth 19 ℔—10 ſ as appeareth by the worke thereof.

Thus hauing nowe touched the 12 questions whereof I first pretended, which with diligence and oft practise I trust are sufficient to aide the desirous, vnto the working of any broken numbers, I will nowe intreate of diuers necessarie rules incident vnto traffike as hereafter followeth.

The fourth Chapter treateth of Losse and Gaine, in the trade of Merchandise.

IF one yard cost 6 ſ—8 ð: and the same is solde againe for 8 ſ—6 ſ: the question is, what is gained in 100 ℔, laying out on such commoditie.

Answere. The rule of thrée direct, applyeth two maner of wayes to doe the same, the one is to say: If $6\frac{1}{2}$ giue $8\frac{1}{2}$, what giueth 100:

Mul-

Multiplie and diuide, and loke what your quotient bringeth forth aboue your laying out is the neate gaines, and the solution to your question: If you followe the worke, your quotient will bring forth 127 l—10 s̄, which is 27 l—10 s̄ more than your principall, and so much is gained in the 100 l laying out.

Item, to worke it the other way, which I take the nearest, seeke the difference betwixt the iust price, and the ouer price, which is 1 s̄ 10 d̄: Then say by the rule of three: If 6 2/3 s̄ gaine 1 5/6 s̄, what shall 100 l gaine? Multiplie and diuide, and you shal find 27 l—10 s̄, and so much is gained in 100 l laying out.

Use which of these two wayes you thinke good.

The Proofe.

If a yard of cloth be deliuered for 8 s̄—6 d̄, whereupon was gained after the rate of 27 l 10 s̄ in 100 l laying out. The question is what the yarde cost at the first hand?

Answere. Put your gaines to 100 l, all maketh 127 l—10 s̄: then say, if 127 l—10 s̄ giue but 100 l, what giueth 8 1/2 s̄. Worke, and you shall finde 6 s̄—8 d̄, the true solution to your question.

Yet an other Branch or proofe vpon the same first Question.

If one yarde cost 6 s—8 d, the question is what price the same is to be solde againe for, to gaine 27 l—10 s in 100 l laying out.

Answere. Say by the rule of three, if 100 l giue 127 l—10 s, what giueth 6 $\frac{2}{3}$ s? Multiplie and diuide, and you shall finde 8 s 6 d, your true solution.

If one elle cost 7 s—8 d, and solde againe for 8 s—6 d Question: What is gained in 20 l laying out, in such commodities.

Answere. Seeke the difference betwixt the iust price and the ouer price, which is 10 d, and then applie the rule of three, as before is taught, saying, If 7 $\frac{2}{3}$ s giue $\frac{1}{2}$ s, what giueth 20 l? Multiplie and diuide, and you shal finde 2 l—3 $\frac{11}{23}$ s, and so much is gained in 20 pound laying out.

The Proofe also by an example of losse.

A Merchant hath bought Holland cloth at 8 s—6 d the elle, which proueth not to his expectation, whereupon he is content to lose 2 l—3 s—$\frac{1}{2}\frac{1}{3}$ s in 20 l laying out. The question is what price ought to be made of the elle abating this losse.

Answere. Doe as before in gaines hath beene taught, putting 2 l—3 $\frac{11}{23}$ s to your 20 l, all together maketh 22—3 $\frac{11}{23}$ s. Then say by the rule of three: If 22—3 $\frac{11}{23}$ s giue but 20 l, what

what shall come of 8 ½ s? worke and you shall find 7 s—8 d the iust price that the elle ought to be solde for after the rate of this losse.

Thus it appeareth euidently as in companie the rule is appliable as well to gaines and losse.

If 20 ¼ yardes cost 36 l—10 s howe shall I sell the same againe to gaine ⅓ of the principall, or to make of 3,4: which is all one.

Answere. By the rule of 3, if 3 doe giue 4, what will 36 ½ giue? Multiplie and diuide, and you shall finde 48 l—⅔. Then say again, if 20 ¼ yardes doe giue 48 ⅔ poundes, as well principall as gaine, what will one yarde be worth at that price? Multiplie and diuide, and you shall finde 2 l—8 16/243 s.

If one Elle of cloth cost me 8 s—8 d, and afterwards I sell 10 ½ Elles thereof for 5 l 13 s—4 d, I would knowe whether I winne or lose: and how much vpon the 100 l of money.

Answere. See first, at 8 s—8 d the Elle, what 10 ½ elles comes to, and you shall find 4 l 11 s, and I sold the same for 5 l—13 s—4 d, so that I did gaine vpon the 10 ½ yardes 1—2—4 d. Then if you would know how much is gained in the 100 l, say by the rule of three, If 4 l—11 s did gain 22 ½ s, what wil 100 l

gaine? Multiplie and diuide, and you shall finde —24 l.—10 s.—10 $\frac{10}{21}$ d. And so much is gained in the 100 l. of money.

If 12 $\frac{1}{2}$ yardes cost me 11 l.—5 s. and I sel the yarde again for 16 s. the question is, whether I doe winne or lose, and howe much in or vpon the pound of money.

Answere. Loke what the 12 $\frac{1}{2}$ yards come to at 16 s the yarde, and you shall finde 10 l. But they cost 11 l.—5 s. So there is lost vpon the whole 1 l.—5 s. Then to knowe howe much is lost in the pound, say by the rule of 3, if 11 $\frac{1}{4}$ l doe lose 1 $\frac{1}{4}$ l, what will 1 l lose. Multiplie and diuide, and you shall finde 2 s 2 $\frac{2}{3}$ d, and so much is lost in the l of money.

If I sell the C weight of any commoditie for 4 l, whereupon I do lose after 10 l in the 100 l, I demaund how much I shall lose or gaine in the 100 l, if in case I had solde the same for 4 l—10 s.

Answere. Say if 90 l yéld 100, how much will 4 giue? Multiplie and diuide, and you shall finde 4 $\frac{4}{9}$. Then say againe, if 4 $\frac{4}{9}$ giue me 4 $\frac{1}{2}$, what will 100 come to? Multiplie and diuide, and you shall finde 101 l $\frac{1}{2}$, which is more than 100 l by 1 l—5 s. And so much is gained in the 100 l.

A Merchant hath solde Currans for the summe of 430 l, and he hath gained therein after 10 l in the 100 l. The question is to knowe

knowe how much he gained in all.

Answere. Say by the rule of three, If 100 ℔ doe gaine 10 ℔, what wil 430 ℔ gaine? Multiplie and diuide, and you shall finde 39 ℔—1 s̄—9 $\frac{9}{11}$ d̄, and so much hath he gained in all.

If one yarde be worth 28 $\frac{1}{2}$ s̄, for how much shal 10 yardes be sold to gaine after 8 ℔—6 s̄ 8 d̄ in the 100 ℔.

Answere. First adde 8 ℔—6 s̄ 8—d̄ to 100: Then say, if 100 ℔ doe giue 108 $\frac{1}{3}$ for principal and gaine, what will 28 $\frac{1}{2}$ s̄ principall yelde? Multiplie and diuide, and you shall finde 30 $\frac{7}{8}$ s̄. Then say againe by the rule of three, If 1 yarde doe giue 30 $\frac{7}{8}$ s̄ (which is as well the principall as the gaine, what shall 10 yardes giue? Multiplie and diuide, and you shal finde 15 ℔—8 s̄—9 d̄. And for the same price shal the 10 yards be solde, for to gaine after the rate of 8 ℔—6 s̄—8 d̄ vpon the 100 ℔.

A Branch or Proofe out of this Question.

A Merchant hath solde Clothes for 15 ℔—8 s̄—9 d̄, and he hath gained in the whole, the summe of 1 ℔—3 s̄—9 d̄. The question is to know how much he hath gained in the 100 ℔.

Answere. To knowe this, first rebate the gaines from the price, and there will remaine

14 ℔—5 ß—0 ð. Then say by the rule of 3 direct, if 14 ℔ ¼ giue me 1 ℔—3 ß—¾ ð, what will 100 ℔ giue? Multiplie and diuide, & you shall finde 8 ℔—6 ß—8 ð the effect desired: the proofe is apparant in the question before.

Yet an other Branch or Proofe of the first Question.

If 10 yardes be deliuered for 15 ℔ 8—9, whereupon was gained after the rate of 8 ℔ 6 ß—8 ð vpon the 100 ℔, the question is, what the yarde did cost at the first hand.

Answere. First say by the rule of three, if 10 with principall and gaine yéeld 15 ℔—8 ¼, what shall 1 yéeld? Multiplie and diuide, and you shall finde 30 ¾ ß. Then say againe by the rule of three, if 108 ⅓ principall and gaine giue but 100, what shall 30 ¾ ß of principall and gaine yéeld? worke, and you shall find 28 ⅓ ß. And so much did the yarde cost at the first penie.

If one yard cost 36 ß, howe much shall 12 yardes be solde for to gaine after the rate of 10 ℔ in the 100 ℔.

Answere. First say, if 100 giue 110 ℔ principall and gaine, what will 36 ß giue? Multiplie and diuide, and you shall finde 39 ⅗ ß. Then say againe by the rule of three: If 1 yarde of principall and gaine yéelde 39 ⅗ ß, what

what shall 12 yardes gaine? Multiplie and diuide, and you shall finde 23 l—15 $\frac{1}{5}$ s, which $\frac{1}{5}$ s in knowen number is 2 $\frac{2}{5}$ d. And for the same price shall the 12 yards be solde, to gaine after the rate of 10 in the 100.

The Proofe.

If 12 yards be solde for 23 l—15 s—2 $\frac{2}{5}$ d, whereupon is gained after 10 l in the 100 l. The question is, what the yarde cost at the first penie.

Answere. First say, if 12 giue 23 l—15 $\frac{1}{5}$ s, what 1 yarde? Multiplie and diuide, and you shall finde 39 $\frac{3}{5}$ s. Then say againe by the rule of 3, if 110 l giue but 100, what shall 39 $\frac{3}{5}$ giue? Worke and you shall finde 36 s, the iust price of the yarde at the first hand.

Item, when one Merchant selleth wares to another, & he giueth to the buyer 1 l—6 s 8 d vpon the score, or 20 l. The question is, howe much shall the buyer gaine vpon the 100 l after that rate.

Answere. First adde 1 l—6 s—8 d vnto 20 l, and they are 21 $\frac{1}{3}$. Then say, if 20 l giue 21 $\frac{1}{3}$, what shal 100 giue? Multiplie and diuide, and you shal finde 106 $\frac{2}{3}$. So the buyer get-

getteth after the rate of $6\frac{2}{3}$ ℔ vpon the 100 ℔.

Gentle Reader, other necessarie questions appertaining to Losse and Gaine, you shall haue in the eight Chapter of this Treatise.

The fifth Chapter entreateth of Losse and Gaine vpon Time, wrought by the double Rule of three: or by the Rule of three composed, which is contained in foure speciall selected branches or questions of diuers formes, each one of them springing from the first question, and each one of them also beeing a proofe to other, &c.

IF one yarde cost me 2 s 8 d readie money, and after I sell the same againe for 2 s 10 d, to be paide for it at the ende of 3 moneths. The question is, what I gaine vpon the 100 ℔ in 12 moneths.

Answere. First say, if $2\frac{2}{3}$ gaine $\frac{1}{8}$, what shall 100 ℔ gaine? Multiplie and diuide, and you shall finde $6\frac{1}{4}$ ℔. Then say againe by the rule of three, if three moneths gaine $6\frac{1}{4}$ ℔, what shall 12 moneths gaine? Worke, and you shall finde 25 ℔, and so much shall I gaine in 12 moneths after that rate.

Item you may also worke it at one working by the first part of the rule of 3 composed, say

saying, if $2\frac{2}{3}$ s in three moneths doe gaine $\frac{1}{2}$ of a s, (which is 2 d) that will 100 l gaine in 12 moneths: which for thy further encouragement, the worke of this one example, I haue here put downe, to verifie that I affirmed in the first part of this Ground of Artes, that this rule, and so all others more reioyceth in Broken than in Whole.

s	moneths	s	l	moneths
$2\frac{2}{3}$	3	$\frac{1}{2}$	100	12
8				100
3		2		1200
24		7		3
6		14		3600
144		3600	25	
		144		
		14		

Where the Multiplication and the Diuision being ended maketh 25 l your desire.

If a yarde be deliuered for 2 s—10 d, to be paide at 3 moneths, whereupon was gained after the rate of 25 l in the 100 for 12 moneths. The question is nowe, what the yarde cost at the first hand.

Answere. First say, if 12 moneths gaine 25 l, what shall 3 moneths gaine? Worke, and you shall finde $6\frac{1}{4}$ l. Then say againe the

second

second time, if 106 ¼ £ giue but 100, what shal 2 ⅝ s̄ giue? Worke, and you shall finde 2 s̄ 8 d̄, which is the iust price that the yarde cost at the first hand.

If one yarde of Cloth cost me 2 s̄—8 d̄ readie money, for what terme shall I sell the same againe for 2 s̄—10 d̄, so that I might gaine after the rate of 25 £ vpon the 100 £ in 12 moneths.

Answere. First say, if 2 ⅝ gaine ⅝, what shall 100 £ gaine? Multiplie and diuide, and you shall finde 6 ¼ £. Then say againe for the second worke, if 25 £ become of 12 moneths, what shall come of 6 ¼? Worke, and you shall finde 3 moneths the iust terme of time that the cloth ought to be deliuered at 2 s̄ 10 d̄ to gaine 25 £ vpon the 100 £ in 12 moneths.

If one yarde cost me 2 s̄ 8 d̄ readie money, for what price shall I sell the same againe to be paide at the ende of three moneths, so that I may gaine after the rate of 25 £ in the 100 £ for 12 moneths.

Answere. First say, if 12 gaine 25 £, what shall 3 moneths gaine? Multiplie and diuide, and you shall finde 6 ¼ £. Then say for the second worke, if 100 £ giue 106 ¼, what giueth 2 ⅝ s̄? Worke and you shall finde 2 s̄—10 d̄, and for that price must the yarde be solde to gaine after 25 £ in the 100 £ for 12 moneths.

Ma-

Many other of these questions I might here haue deliuered, but for feare the booke would rise to too thicke a volume, and so to make the price so much the dearer, wherby it might not be so partable to my Countriemen as I wish it. But these foure, I haue of purpose framed in this order, hauing relation one to an other, assuring you, that what questions soeuer may be proponed within the compasse of this rule, you shall finde by one of these foure to make a solutiō. And moreouer, diuers other are yet to be deliuered: where the Creditor giueth diuers dayes of payment, which can neuer be well wrought, nor yet vnderstoode, vnlesse you can first finde by arte, the iust time that all those payments, howe different soeuer they be, ought to be paid at once: whereupon first I thinke good here to giue some instructions into such a Rule, for it is the onely aide for the finishing of such questions as hereafter shall followe.

The

The sixth Chapter entreateth of Rules of Payment, which is a right necessarie Rule, and one of the chiefest handmaids that attendeth vpon buying and selling, &c.

Example.

A Merchant doth owe a summe of money, whereof the $\frac{1}{3}$ is to be paide at 6 moneths, the $\frac{1}{4}$ at 8 moneths, and the rest at a yeare. If he would pay at one payment, the question is, what time ought to be giuen him.

Answere. I haue omitted the quantitie of the summe, for you shall vnderstand, the rule is appliable, and yeldeth a true solution to what summe soeuer shall be proponed. But nowe for orders sake in teaching, I doe imagine the summe to be 60 l., whereupon the maner of this worke is to multiplie the proportionate part of the money by the time, as in companie. Then 20 being the first paiment and the $\frac{1}{3}$ of 60, which $\frac{1}{3}$ multiplied in broken numbers by 6, his time of payment maketh $\frac{6}{1}$, which in whole numbers, as appeareth by the example in the margent, maketh 2 moneths: next 30, which is the $\frac{1}{2}$ multiplied by his terme 8, yeldeth 4 months, then the rest which

$\frac{1}{3}$ by $\frac{6}{1}$	
$\frac{6}{3}$	2 months
$\frac{1}{2}$ by $\frac{8}{1}$	
$\frac{8}{2}$	4 months
$\frac{1}{6}$ by $\frac{12}{1}$	
$\frac{12}{6}$	2 months
	8 months

is 10 l. must nedes be abbreuiated into ỹ proportion it beareth to 60, which is $\frac{1}{6}$, which $\frac{1}{6}$ multiplied by his time 12 moneths, produceth $\frac{12}{6}$, maketh 2 moneths. All which added together, as appeareth in the margent, maketh 8 moneths, which is the iuſt time that all thoſe payments ought to be paide at once.

A Merchant hath 800 l. to pay, the $\frac{1}{3}$ thereof readie money the $\frac{1}{4}$ at two moneths, the $\frac{1}{5}$ at foure moneths, and the reſt at a yere. The queſtion is, if he would pay all at one payment, what time ought to be giuen him.

Anſwere. The readie money is neuer multiplied: then $\frac{1}{3}$ multiplyed by 2 moneths as you did before, maketh $\frac{1}{3}$, then $\frac{1}{3}$ by 4 produceth 2 moneths, as appeareth here in the margent. But nowe for the reſt of the money you can not multiplie it vntill you haue ſought what proportion it beareth to 800 l. Therefore you muſt ſubtract the readie monie, the $\frac{1}{4}$ & the $\frac{1}{5}$ out of the principall. The reſt will be $66\frac{2}{3}$ l., which you muſt loke what part it beareth to the principall, which you ſhall finde to be $\frac{1}{12}$, the ſame you muſt alſo multiplie by his time 12 moneths, and it yeldeth 1 moneth, ſo all make $3\frac{1}{3}$ moneths as appeareth in the margent.

$\frac{1}{4}$	$\frac{2}{3}$
$\frac{2}{4}$	$\frac{1}{3}$
$\frac{1}{2}$	$\frac{4}{3}$
$\frac{4}{2}$	2
$\frac{1}{12}$	$\frac{12}{3}$
$\frac{12}{12}$	1
	$3\frac{1}{3}$

A Merchant is to pay 1200 l, in 3 termes, That is to wit, 400 l at 2 wéekes, and 600 l at 4 moneths, lastly 200 l at 5 moneths. The question is in what time, they ought to be paide at once.

Answere. Proportionate the partes, and you shall finde that 400 is $\frac{1}{3}$ part, and for 600 you shall finde $\frac{1}{2}$, & likewise 200 is the $\frac{1}{5}$ part, which multiplie by their times as before, & you shall haue $\frac{2}{3}$ wéekes, more 8 wéekes, and lastly $3\frac{1}{2}$ wéekes, which together maketh 12 wéekes, or 3 moneths your desire.

A merchant is to pay 600 l in 3 termes, whereof 100 l is payde present, more 300 l at 20 dayes, and the rest at 5 moneths, accounting 30 dayes to a moneth. The question is, what time ought these paymentes to bée payde at once.

Answere. Worke and you shall finde 2 moneths.

The

The seuenth Chapter entreateth of buying and selling in the trade of Marchandise, wherin is taken part readie money, and diuerse dayes of payment giuen for the rest, and what is woon or lost in the 100 ℔ forbearance for 12 moneths more or lesse according to the quantitie of money, or proportion of time, &c.

A Merchant hath bought Sattens which cost 8 s̄ the yarde readie money. And he selleth the same againe to another man for 10 s̄ the yarde, but he giueth two dayes for the payment, that is to say three moneths for the one halfe, and fiue moneths for the other halfe. The question is to knowe howe much the seller doth gaine vpon 100 ℔ in 12 months after that rate.

Answere. Seeke first by the rules of payment, at what time those two paymentes ought to be paide at once, and you shall finde 4 moneths, at which time the second Merchant ought to haue paide the whole entire payment. And therefore say by the first part of the rule of three composed If 8 s̄ in 4 moneths do gaine 2 s̄, what will 100 ℔ gaine in 12 moneths

8—4—2—100—12

4 — 12

32 — 1200 — 2400 | 75

2 — 32

2400

Multiplie and diuide, and you shall finde 75 ℔ as appeareth in the example, and so much doth the first merchant gaine vpon 100 ℔ in 12 moneths.

A merchant hath solde 50 clothes at 9 ½ ℔ the peece, to be paide the one ⅓ at 4 moneths, the ⅓ at 5 moneths, and the ⅓ at 7 moneths, and the sellers minde is to take no more but after 8 ℔ in the 100 for 12 moneths. The question is nowe what the first Merchant gaineth in the sale of these clothes after that rate.

Answere. First loke what the 50 clothes come to at that price, and you shall finde 475 ℔. Then secondly, according to your direction in the Rules of payment, seeke at what time all the payments are to be performed at once. And you shal find 4 ⅓ moneths. Then thirdly say by the first part of the rule of 3 composed, If 100 ℔ in 12 moneths gaine 8 ℔, what will 475 ℔ gaine in 4 ⅓ moneths? Worke, and you shal finde 18 ℔, and ⅓ of a ℔, which is the neat gaines that the first merchant hath after the rate aforesaide.

A merchant hath bought Holland at 7 s 3 d the Elle readie money, and he selleth the same againe for 8 s 4 d the Elle, to be paide ⅓ part in readie money, more ⅓ part at 3 moneths, and the rest at 4 moneths. The question

question is now to knowe howe much the first merchant doth gaine vpon the 100 ℔ in 12 moneths after the same rate.

Answere. According to the direction deliuered you in the rule of payment, the readie money is not to be multiplied. Then working for the other 2 payments, to finde out the true proportion at what time they ought to be paid at once, you shall finde for $\frac{1}{3}$ at 2 moneths $\frac{2}{3}$ of a moneth. And the rest of the money which is $\frac{5}{12}$ multiplied by his terme 4 moneths, yeeldeth $1\frac{2}{3}$ moneths, both which added together make 2 and $\frac{1}{3}$ moneths, the iust time that both the payments ought to be performed at once. And therefore say by the first part of the rule of 3 composed, if $7\frac{1}{4}$ in $2\frac{2}{3}$ moneths doe gaine $\frac{13}{240}$ of a ℔, what shall 100 ℔ gaine in 12 moneths after that rate, worke and you shall finde $76\frac{172}{203}$ pounds. And so much doth he gaine vpon 10 pound in 12 moneths.

A merchant hath bought 30 clothes at 6 ℔ the peece for readie money. Afterward he selleth 10 of them for 7 ℔ the peece, for three moneths terme. And the other 20 he selleth for 8 ℔ the peece for foure moneths terme. The question is nowe, what he gaineth vpon 100 ℔ in 12 moneths.

Answere. First finde the value of the 30 Clothes, which amount to 180 ℔. Secondarily, seeke what the tenne peeces come to at 7 ℔, and what the 20 peeces come to at 8 ℔

the one comes to 70, and the other to 160: both which together make 230, which is 50 l. more than they cost. Thirdly, as I haue taught you in the rule of payment, proportionate the first and second prices, vnto the proportion they beare vnto 230 the product of their two prices, and you shall finde $\frac{7}{23}$ for the first, and $\frac{16}{23}$ for the latter. Then fourthly, multiplie those parts by their times, and you shall haue $\frac{21}{23}$ and $\frac{64}{23}$: both which together maketh 3 whole moneths, and $\frac{16}{23}$ of a moneth, which is the iust time that both those payments are to be paide at once.

Then say by the first part of the Rule of 3 composed: If 180 l in $3\frac{16}{23}$ moneths doe gaine 50 l, what shall 100 gaine in 12 moneths? Multiplie & diuide, and you shal finde $90\frac{10}{11}$ l. And so much doth he gaine vpon 100 l in 12 moneths.

A merchant hath bought Synamon which cost him 9 s the l readie money. The question is nowe at what price he ought to sell the C weight. To wit 112 l, to be paide the $\frac{1}{4}$ at 2 moneths, and the residue at the ende of 3 moneths, so that he may gaine after the rate of 10 l vpon a 100 l for 12 moneths.

Answere. Seeke first by the Rules of payment what terme both the payments ought to be paide at once, where the $\frac{1}{4}$ multiplied by his terme 2 moneths, maketh $\frac{1}{2}$ moneths.

Like-

Likewise the next payment which is ¾ multiplied by his terme 3 moneths, maketh 2 ¼ moneths, both which added together, maketh 2 ¾ moneths, which is the time, that both the payments ought to be paide at once. Then say by the Rule of 3, if 12 moneths doe giue me 10 l, what will 2 ¾ moneths giue? Multiplie and diuide, and you shal finde 2 7/24 l. Then say againe by the rule of 3: If one l. cost me 9 s, what will 112 l cost? Multiplie and diuide, and you shall finde 50 l—8 s. Then say once againe, If 100 l doe giue 102 7/24, what will 50 5/12 l giue? Multiplie and diuide, and you shall finde 51 l—11 s—1 ½ d. And for that price ought I to sell 112 l of Synamon to be paide at two seuerall payments aforesaide, to gaine thereby after the rate of 10 l. vpon the 100 l in 12 moneths.

Briefe Rules for our hundreth weight here at London, which is after 11 l for the C.

Item who that multiplieth the pence that one pound weight is worth by 7, and diuideth the product by 15, shall find how many pounds in money the 112 pound waight is worth.

And contrariwise, he that multiplieth the pounds that 112 l waight is worth by 15. and diuideth the product by 7, shall finde how

many pence in money the 112 pound weight is worth.

Example.

At 10 d the pound weight, what is 112 l weight worth?

Answere. Multiplie 10 by 7, and thereof commeth 70, the which diuide by 15, and you shall finde 4 2/3 l. And thus the 112 l is worth 4 l—13—4 d after the rate of 10 d the l aforesaide.

At 6 l the 112 l weight what is one pound worth?

Answere. Multiplie 6 l by 15, and thereof commeth 90: the which diuide by 7, and you shal finde 12 d 6/7. So much is one pound worth when the 112 l did cost 6 pounds.

The eight Chapter entreateth of Tares and allowances of merchandise solde by weight, and of losses and gaines therein, &c.

AT 16 l the 100 suttle, what shall 895 l suttle be worth in giuing 4 l weight vpon euerie 100 for treate.

Answere. Adde 4 vnto 100, and you shall haue 104. Then say by the rule of three, If

104

104 be worth 16 l, what are 895 l worth? Multiplie and diuide, and you shal finde 237 l 13—10 $\frac{2}{13}$ d. And so much shall the 895 pound weight be worth.

Item at 3 s 4 d the pound weight, what shall 754 $\frac{1}{2}$ be worth in giuing 4 l weight vpon euerie 100 for treate.

Answere. See first by the rule of three, what the 100 pound is worth, saying: If 1 cost 3 $\frac{1}{3}$ s, what 100? Multiplie and diuide, and you shall finde 16 l $\frac{2}{3}$. Then adde 4 vnto 100, and they are 104. Then say againe by the rule of three, if 104 be solde for 16 $\frac{2}{3}$, for howe much shall 754 $\frac{1}{2}$ be solde for? Multiplie and diuide, and you shall finde 120 l—18 s 3 $\frac{3}{13}$ d. And for so much shall the 754 $\frac{1}{2}$ be solde for, at 3 s—4 d the pound in giuing 4 vpon the 100.

Other necessarie briefe rules there are for the finding of treats, or casting vp of chestes of sugar, &c. which for that it is a mysterie, I omit. If any lacke instruction, that way they shall finde me readie to pleasure them.

Item if 100 l be worth 36 s 8 d, what shal 800 l be worth in rebating 4 pound vpon euerie 100 for tare and cloffe.

Answere. Multiplie 860 by 4, and thereof commeth 3440, the which diuide by 100, and you shall haue 34 $\frac{2}{5}$ l, abate 34 $\frac{2}{5}$ from 860, &

there will remaine 825 3/4. Then say by the rule of three, If 100 l cost 36 2/3 s, what will 825 3/4 cost after that rate? Multiplie and diuide, and you shall finde 15—2—8 16/33. And so much shall the 860 cost in rebating 4 pound vpon euerie 100: for tare and cloffe.

Item whether doth he lose more that giueth 4 l vpon the 100: or he that rebateth 4 l vpon the 100.

Answere. First note that he that giueth 4 l vpon 100, giueth 104 for 100. And he which rebateth 4 l vpon the 100, giueth the 100 for 96. Therefore say by the rule of three, if 104 be deliuered for 100, for how much shall the 100 be deliuered? Multiplie and diuide, and you shall finde 96 8/13, and he which rebateth 4 in the 100, maketh but 96 of 100, so that he loseth 4 in the 100, and the other which giueth 4 vpon the 100, loseth but 3 11/13 vpō the 100. Thus you may see that he which rebateth 4 in the 100, loseth more by 11/13 in the 100 l, than the other which gaue 4 vpon the 100, for tare and cloffe.

If 100 l of any thing cost me 23 s 4 d, the question is howe I shall sell the l to gaine after the rate of 10 l vpon the 100 l.

Answere. Say by the rule of three, if 100 l giue 110 l, what shall 23 1/3 s giue? Multiply and diuide, and you shall finde 1 l 17/20. Then say

againe, if 100 ℔ be worth 1 $\frac{17}{20}$ ℔, what is one pound worth? Multiplie and diuide, and you shall finde 3 ð $\frac{3}{17}$. And so much as the ℔ worth in gaining 10 ℔ vpon the 100 ℔.

Item a Grocer hath bought C weight of a commoditie for 6 ℔ 10 s̃. The question is now to knowe howe many pounds thereof he shall sell for 32 s̃ 4 ð to gaine 20 s̃ in the C. weight.

Answere. Adde 20 s̃ vnto 6 ℔ 10 s̃, & they make 7 ℔—10 s̃. Then say if 7 $\frac{1}{2}$ yelde me 112 ℔, what shall 1 $\frac{1}{2}$ ℔ yeld? Multiplie and diuide, and you shall finde 24 ℔ $\frac{8}{3}$. And so many pound ought he to sell to gaine 20 shillings in his C. waight.

If one pound weight cost 3 s̃—4 ð, and I sell the same againe for 4 s̃, what is gained in 100 pound of money laide out in that commoditie.

Answere. You may say if 3 $\frac{1}{3}$ s̃ giue 4 s̃, what will 100 ℔ giue? But then when you haue found, you must subtract 100 ℔ out of the product, the rest is your neate gaine, or else to produce the neate gaine in your worke at the first. Then subtract the iust price out of the ouerprice, as I taught before in the first beginning of losse and gaine, and your conclusion shall be all one. Multiplie and diuide by which of the two wayes you thinke good, and you shall finde that he gaineth 22 ℔—4 s̃ 5 $\frac{1}{3}$ ð

in

in the 100 pound.

Item if the pound weight which cost 4 s̄, be solde againe for 3 s̄—4 d̄, I demaund what is lost in the 100 ℔ of money?

Answere. Say if 4 s̄ lose ⅓ s̄, what shall 100 ℔ lose? Multiplie and diuide, and you shall finde 16 ℔—13 s̄—4 d̄, and so much is lost vpon the 100 ℔ of money.

Item if C weight of any commoditie cost 45 ℔, and the buyer repenting would lose 5 ℔ in the 100 ℔ of money, I demaund howe the pound may be solde: his losse to be neither more nor lesse than after the rate aforesaid of 5 by the hundreth?

Answere. By the rule of 3, if 100 ℔ lose 5 ℔, what shall 45 ℔ lose? Worke and you shall finde 2 ¼ ℔, which rebated from the principall 45, resteth 42 ℔—15 s̄. Lastly say if 112 ℔ yeeldeth but 42—15 s̄, what 1 pound? Multiplie and diuide, and you shall finde 7 s̄ 7 d̄ 17/28. And so much is the pound worth after that losse.

A Grocer hath bought 3 peeces of raisins, weying 175 ½ ℔, 182 ¼ ℔, 191 ℔: tare for each fraile 2 ¼ ℔, at 25 ½ s̄ the C weight. The question is what they amount to in money.

I answere 6 ℔ 3 s̄—4 d̄ 23/28

A

A Grocer hath bought 3 sacks of Almonds waying 267 ½ ℔ tare 2 ℔: 257 ½ ℔ tare 2 ½ ℔, 252 ℔ tare 3 ℔ at 2 s 10 ½ the pound what amount they to in money?

I answere 110 ℔—12—3 ¾ d.

The ninth Chapter treateth of Lengthes and Breadthes of Arras, and other clothes with other questions incident vnto Length and Breadth.

IF a pece of Arras be 7 elles and ¾ long, and 5 elles, and ⅔ broad: how many elles square doth the same pece containe?

Answere. Multiplie the length by the breadth, that is to say 7 ¾ by 5 ⅔. And thereof will come 43 11/12 elles: so many elles square doth the same pece containe.

Item more a pece of Arras doth containe 22 elles square, and the same being in length 3 ¼ elles, I demaunde howe many elles in breadth the same pece doth containe?

Answere. Diuide 22 elles by 3 ¼, & thereof commeth 6 10/13. So many elles doth the same containe in breadth.

Item more a Merchant hath 3 ¼ elles of Ar-

Arras, at 1 $\frac{3}{4}$ elles broad, which he wil change with an other man for a peece of Arras, that is $\frac{7}{8}$ elles square. The question is howe many elles of that squarenesse ought the first Merchant to haue.

Answere. Multiplie the first merchants peece his length by his breadth, and you shall finde it containeth 5 $\frac{5}{12}$ elles, which 5 $\frac{5}{12}$ Elles you shall diuide by $\frac{7}{8}$, and you shal find 6 $\frac{4}{21}$ els, and so many elles of that squarenesse, ought the latter merchant to giue the first.

Item a student hath bought 3 $\frac{1}{2}$ yardes of broade cloth, at 7 quarters broade, to make a Gowne, and should line the same throughout with Lambe, at a foote square each skin: the question is now how many skinnes he ought to haue?

Answere. Seeke first the number of yards square that his cloth containeth, which to do, multiplie 3 $\frac{1}{2}$ his length, by 1 $\frac{3}{4}$ his breadth, and you shall finde 6 $\frac{1}{8}$ yardes square, then say by the rule of three, if 1 yarde square giue 9 foote, what shall 6 $\frac{1}{8}$ worke, and you shal finde 55 $\frac{1}{8}$ skinnes.

Item more, a Lawier hath a rich peece of seeling come home, which is 24 foote and three Inches long, and 7 foote and 2 $\frac{1}{2}$ Inches high: the Joiner is to be paide by the yard square: the question is, how many yardes this peece con-

containeth?

Anſ. Multiplie his length by his breadth, that is to wit 24 ¼ foote by 7 5/12 foote, and you ſhall finde 174 77/96 foote ſquare, which 174 you ſhall diuide by 9 (for ſo many fot make a yard ſquare, and you ſhall finde 19 yards, 3 foote, and 77/96 of a foote, and ſo many yardes doth this peece holde.

Item, bought a peece of Holland cloth conteyning 36 Elles ½ Flemmiſh. The queſtion is, howe many Elles Engliſh it makes?

Anſwere. You muſt note, that 5 Elles Flemmiſh doe make but 3 Elles Engliſh. Therefore ſay by the rule of three, if 5 Elles Flemmiſh make but 3 Elles Engliſh, howe many Elles Engliſh will 36 ½ Elles Flemmiſh make? Multiplie and diuide, and you ſhall finde 21 ⅗, and ſo many Elles Engliſh doth 36 ½ Elles Flemmiſh containe. The like is to be done of all others.

Item more, I haue bought 342 Elles Flemmiſh, of Arras worke at two Elles broade, Flemmiſh, and I would line the ſame with Elle broade Canuas of Engliſh meaſure. The queſtion is, howe many Elles Engliſh will ſerue my turne?

Anſwere. For as much as 3 Elles Engliſh are worth 5 Elles Flemmiſh, therefore put 3 elles Engliſh into his ſquare, in multi-

plying

plying 3 by himselfe, which maketh 9. Likewise multiplie the English elle which is 5 quarters euerie way into himselfe squarely, and you shall finde 25. Then multiplie 342 which is the length of the peece, by 2 which is the breadth, and thereof commeth 684, then say by the rule of 3, as before: if 25 Elles square of Flemmish measure be worth 9 elles square of English measure, what are 684 of Flemmish measure? Multiplie and diuide, and you shall finde 246 $\frac{6}{25}$ Elles English.

The same is also wrought by the backer Rule of 3, in seeking the squares contained in the Flemmish Elle of 2 Elles broade (which are 18): and also in seeking the squares conteined in the English elle (which are 25) then say by the rule of 3 backeward, if 18 quarters require 34 elles, what shal 25 quarters giue? Multiplie and diuide by the rule of 3. Reuerse, and you shall finde as before 246 $\frac{6}{25}$ elles English.

Item more, at 2 s 4 d the Flemmish Elle, what is the English elle worth after the rate?

Answere. Say if 3 quarters giue 3 $\frac{1}{3}$ s, what giueth 5 quarters? Multiplie and diuide, and you shall finde 5 s — 6 $\frac{4}{9}$ d.

Item more, at 8 s 4 d the Flemmish Elle square, what is the English Elle worth after

ter that rate.

Answere. According to the reason of the last question, consider that a Flemmish Elle square is equall to 9 quarters of a yarde English, and an English elle square is equall to 25 quarters of a yarde. Therefore say by the rule of 3, if 9 quarters giue $8\frac{1}{3}$ s, what 25 quarters? Worke and finde 23 s—1 $\frac{7}{9}$ d. And so much is the English Elle worth.

Item more at 6 s—8 d the Elle square, what shall a peece of cloth cost that is $7\frac{1}{2}$ elles long, and $3\frac{1}{4}$ elles broade?

Answere. Multiplie the breadth by the length, and you shall finde $24\frac{3}{8}$ elles square. Then say by the rule of three, if 1 elle square cost $6\frac{2}{3}$ s, what $24\frac{3}{8}$? Multiplie and diuide, and you shall finde 8 l—2 s—6 d, & so much the same peece of cloth shall cost.

Item more, a Mercer solde 3 peeces of silke. To wit $24\frac{1}{4}$, $23\frac{1}{2}$ 25, and 25 yardes at $9\frac{1}{4}$ s the yarde, and was glad to receiue in part of paiment againe, a Cloth containing $34\frac{1}{2}$ yardes at $7\frac{2}{3}$ s the yarde. The question is nowe, what the Debitor is in the Creditors debt. Worke, and you shall finde he oweth the mercer 22 l—3 s—$2\frac{1}{4}$ d.

The

The tenth Chapter entreateth of the reducing of the Pawnes of Geanes into Englisth yardes.

NOte, that 100 Pawnes doe make 26 yardes, whereupon 3 Pawnes $\frac{13}{15}$ do make 1 yarde, and 1 Pawne after that rate and proportion is $\frac{13}{50}$ of a yarde.

In 4563 Pawnes of Geanes, howe many yardes English?

Answere. Say by the rule of three, if 100 pawnes doe make 26 yardes, what wil 4563 pawnes make? Multiplie and diuide, and you shall find 1186 yardes $\frac{19}{50}$. So many yards doe 4563 pawnes make.

Otherwise, take some other number at your pleasure, as 10 pawnes, which is the $\frac{1}{10}$ part of 100, then to finde his proportion, take the $\frac{1}{10}$ part of 26, which is $2\frac{3}{5}$, and then say also by ỹ rule of 3, if 10 pawnes giue $2\frac{3}{5}$ yards, what will 4563 pawnes giue? Worke, and you shall finde $1186\frac{19}{50}$ yardes, as before.

More, at 2 s̄—6 d̄ the pawne of Geanes, what will the English yarde be worth after that rate?

Answere. Say by the rule of three, if $\frac{13}{50}$ of a yarde cost $2\frac{1}{2}$ s̄, what one yarde? Multiply, and

and diuide, and you ſhal finde 9 s—7 $\frac{5}{13}$ d.

More, if 346 $\frac{1}{2}$ Pawnes coſt 30 l—13 s 4 d ſterling, what is that the Engliſh yarde after the rate?

Anſwere. Say by the rule of thrée, if 346 $\frac{1}{2}$ Pawnes coſt 30 $\frac{2}{3}$ l, what are 3 $\frac{11}{13}$ Pawnes worth (for ſo many Pawnes make a yarde:) Multiplie and diuide, and you ſhall finde $\frac{9200}{27027}$ parts of a pound, which in knowen numbers is worth 6 s—9 d $\frac{6271}{9009}$.

The eleuenth Chapter intreateth of Rules of Loane and Intereſt, with certaine neceſſarie Queſtions and Proofes incident thereunto, &c.

ITem lent my friend 326 l for 5 $\frac{1}{2}$ moneths ſimplie without any Intereſt, vpon condition, to haue the like courteſie againe when I néede. But when I came to borowe, he could ſpare me but 149 l—8 s—4 d, the queſtion is now howe long time I ought to haue the vſe thereof, to counteruaile my friendſhip before time ſhewed him.

Anſwere. Say by the backer rule of thrée, if 326 giue 5 $\frac{1}{2}$ moneths, what time wil 149 l $\frac{5}{12}$ giue? Multiplie and diuide, and you ſhall finde 12 moneths, and ſo long time ought I

to vse his money.

The Proofe.

Item, lent my friend 149 ℔—8 ʃ—4 ð, for 12 moneths. The question is nowe, howe much money he ought to lende me againe for 5 ½ moneths to recompence my friendeship shewed him.

Answere. Say by the backer or reuerse rule of 3: if 12 moneths giue 149 5/12, what shal 5 ½ moneths giue? Worke, and you shal finde 326 ℔, and so much ought he to lende me to requite my gentlenesse or good turne.

Two other Braunches yet more for Proofe out of the same questions.

Item lent my friend 149 ℔—8 ʃ—4 ð for 12 moneths, to haue the like friendship again when I neede. And comming to borowe of him, he very courteously toke me 326 ℔ (for that he could well then spare the same.) The question is nowe, howe long I ought to occupie it, not vsurping friendship, but in his due time to restore it againe.

Answere. Say by the Rule of three Reuerse, if 149 5/12 giue 12 moneths, what shall 326 ℔ giue? Multiplie and diuide, & you shall finde, that at 5 ½ moneths terme, I ought to restore it againe.

Proofe

Proofe.

Item lent my friend 326 ℔ for 5 ½ moneths, The queſtion is nowe, howe many pounds he ought to lende me for 12 moneths to recompence the pleaſure againe.

Anſwere. Worke by the rule of three Reuerſe, as you haue done before, and you ſhall finde 149 ℔—8 ſ—4 d.

Againe, foure other ſelected queſtions of Loane and Intereſt, all out of one braunch, and each one alſo a neceſſarie Queſtion, and a particular proofe to other.

ITem, lent my friend 430 ℔ at Intereſt for 3 moneths to receiue after the rate of 8 ℔ in the 100 for 12 moneths. The queſtion is, what the Intereſt commeth to?

You may if you pleaſe worke it at 2 workings by the rule of 3 direct, in ſaying: If 12 moneths giue 8 pound, what giueth 3 moneths? Multiplie and diuide, and it giueth 2 pound.

Then for the ſecond worke ſay, If 100 ℔ yelde 2 ℔, what yeldeth 430 ℔? Multiply and diuide, and you ſhall finde 8 ℔—12 ſ, and ſo much comes the loane of 430 ℔ to for 3 moneths

neths, after the rate of 8 ℔ in the 100 ℔ for 12 moneths.

Otherwise wrought thus by the rule of 3 at twice also.

If 100 ℔ giue 8 ℔, what giueth 430 ℔? Multiplie and diuide, and you shall finde 34 ℔ $\frac{2}{5}$.

Then againe for the second worke say: If 12 moneths giue 34 ℔ $\frac{2}{5}$, what giueth 3 moneths? Worke and finde 8 ℔—12 s̄ as before.

Otherwise yet at one working. By the first part of the rule of 5 numbers forward, in saying, if 100 ℔ in 12 moneths gaine 8 ℔, what shall 430 ℔ gaine in 3 moneths? Multiplie the first by the second for your diuisor, and the other 3 the one into the other for the diuident, and you also find 8 ℔—12 s̄ as aforesaid.

Proofe.

Item a friend of mine receiued of me 8 ℔ 12 s̄ for the Interest and vse of 430 ℔ for 3 moneths terme: the question is nowe, what he take in the 100 ℔ for 12 moneths after that rate.

Answere. For most briefe say by the first part, or rule of 5 numbers forward: If 430 ℔ in 3 moneths did pay 8 ℔—12 s̄, what doeth 100 ℔ in 12 moneths take after that rate? Worke, and you shall finde 8 ℔, and so much he take vpon the 100 ℔ for 12 moneths.

A third queſtion and Proofe alſo wrought by the backer rule of 5 Numbers.

Item, lent my friende 430 ℔ to receiue for the Intereſt thereof, after the rate of 8 ℔ in the 100 ℔ for 12 moneths. The queſtion is nowe, howe long time my friende ought to haue the vſe thereof, that it may be returned with 8 ℔—12 ſ̃ gaines.

You may worke it if you pleaſe, by the rule of 3 direct at twice, in ſaying: If 100 ℔ yéeld 8 ℔, what yéeldeth 430 ℔? Multiplie and diuide, and finde 34 ℔ and $\frac{2}{3}$.

Then againe for the ſecond worke ſay, If $34\frac{2}{3}$ giue 12 moneths, what giueth $8\frac{2}{3}$ ℔? Multiplie and diuide, and you ſhall finde 3 moneths, and ſo long time ought my friend to vſe it, to returne it with 8 ℔—12 ſ̃ gaine.

Otherwiſe at one working by the backer rule of 5 numbers, in ſaying: If 100 ℔ in 12 moneths doe gaine 8 ℔, howe long time ſhall 430 ℔ be a gaining of 8 ℔—12 ſ̃? Multiplie the firſt and the ſecond into the laſt for your diuident, and the third & fourth multiply together for your diuiſor, & then diuide, & you ſhal finde 3 moneths the iuſt time that my friende ought to vſe it, to returne it with 8 ℔—12 ſ̃ gaine.

A fourth deriued question out of this Braunch which is a Proofe of this last, and also of the other two going before.

ITem, howe much money ought a merchant to deliuer after 8 ℔ in the 100 for 12 moneths that in 3 moneths he may gaine 8 ℔—12 s.

Ans. You may also if you please worke it by the Golden Rule of three at twice, first saying, if 3 moneths gaine 8 ℔, what 12 moneths gaine: you shall finde 34 ℔. Then say againe, if 8 ℔—be come of 100 ℔, what shall come of 34 ℔—8 s: Worke and you shal find the answere to the question, which is 430 ℔, and so much ought the merchant to deliuer.

But most briefely it is aunswered by the Backer Rule of 5 numbers, where I argue thus, saying: If 100 ℔ be 12 moneths a gaining of 8 ℔, then but for 3 moneths terme, onely to take 8 ℔—12 s must nedes be a good round summe: to worke it, set your number thus, 100—12—8—3—8$\frac{1}{4}$: multiplying the first into the second, and also by 43 the product of the fift for your diuident, and the third and fourth together with 5 the Denominator of your fraction, for your Diuisor: then diuide, and you shall finde as before 430 ℔. The true solution to your question.

The

The twelfth Chapter treateth of the making of Factors which is taken in two sortes.

The first is, when the estimation of the factor, is taken vpon the sending of the Merchant, as if the estimation of his person be $\frac{1}{4}$, it is vnderstoode, that he shall haue $\frac{1}{4}$ of the gaine, and the merchant the other $\frac{3}{4}$.

The other sort is when the estimation of his making is out of the sending of the merchant, as if the order and agréement betwéne them were such, that the merchant shall put in 800 l, and the Factor for his making shall haue $\frac{1}{4}$, neuertheleſſe he shall haue but $\frac{1}{5}$ of the gaine or profite, for the $\frac{1}{4}$ of 800 is 200 (for the estimation of his making) which with the 800 l make 1000 l, whereof the 200 l is $\frac{1}{5}$.

A Merchant doeth put in 800 l into the handes of his Factor: vnder such condition, that the saide Factor shall haue $\frac{1}{4}$. And after certaine time, they finde in profite 124 l—6 s 8 d, I demaund how much the merchant shal haue hereof, and howe much ought the Factor to haue?

Answere. When the estimation of the factor is out of the sending of the merchant, it maketh

l	s	d	
99	9	4	for the Merchant
24	17	4	for the Factor

But if that his estimation be at the sending of the merchaunt, then it maketh but

l	s	d	
93	5	0	for the Merchant
31	1	8	for the Factor.

For the Merchaunt is then to haue $\frac{3}{4}$, and the Factor $\frac{1}{4}$.

A merchant doth put into the hands of his Factor 800 l, and the Factor 400 l to haue the $\frac{1}{5}$ of the profite, I demaund nowe for howe much his person is esteemed, when the same is counted vpon the sending of the merchant.

Answere. According to the tenor and order before prescribed in the first rule, that is, if his estimate be $\frac{1}{4}$, he shall haue the $\frac{1}{4}$ of the gaine. Therefore say by the rule of 3 direct, If $\frac{1}{4}$ taken put in 400 l, what is the estimate, or putting in of $\frac{1}{5}$ taking? Multiplie and diuide, and you shall finde 320 l, and so much is the person of the Factor estimated.

Otherwise.

To finde the estimation of the person of the

the factor, you shal consider, that seeing it was agreede betweene them, that the factor should take the $\frac{1}{5}$, then the merchant shall haue the residue, which are $\frac{4}{5}$: wherefore the gaine of the merchant vnto that of the factor is in such proportion as 5 vnto 4. Then if you wil know the estimation of the person of the factor. Say if 5 giue 4, what will 400 giue? Multiplie and diuide, and you shall finde 320 l. And so much is the person of the Factor esteemed to be worth.

Other conditions than these aforesaid, may also be betweene Merchaunts and Factors without respect, either of sending or not sending of the merchant, where most commonly the estimation of the bodie of the Factor is in such proportion to the stocke which the Merchant layeth in, as the gaine of the saide factor is vnto the gaine of the merchant. As thus, If a merchant do deliuer into the hands of his factor 400 l, and he to haue halfe the profite. The person of the saide factor shal be esteemed to be worth 400 l, and if the factor doe take but $\frac{1}{5}$ of the gaine, he should haue but $\frac{1}{5}$ so much of the gaine as the merchant taketh, which must haue $\frac{2}{5}$, wherefore the person of the factor is esteemed but the $\frac{1}{5}$ of that which the merchant layeth in. That is to say 200 l.

And if the factor did take the $\frac{2}{5}$ of the gaine, then

then the merchant shal take the residue which are $\frac{2}{3}$, wherefore the gaine of the merchant vnto the factor is then in such proportion as 3 vnto 2: whereupon if you will then knowe the estimation of the person of the factor. Say if 3 giue 2, what shall 400 giue? Worke and you shall finde 266 $\frac{2}{3}$ pounds. And so much is the person of the factor esteemed to be worth.

And if the merchant should deliuer vnto his factor 400 pound and the factor would lay in 80, and his person to the ende he might haue the $\frac{1}{2}$ of the gaine, I demaund how much shall his person be esteemed.

Answere. Abate 80 from 400, and there will remaine 320. And at so much shall his person be esteemed.

A merchant hath deliuered vnto his factor 900 l. to gouerne in the trade of merchandise, vpon condition that he shall haue the $\frac{1}{3}$ of the gain, if any thing be gained, and also to beare the $\frac{1}{3}$ of the losse if any thing be lost. Nowe I demaund howe much his person was esteemed at.

Answere. Seeing that the factor taketh the $\frac{1}{3}$ of the gaine, his person ought to be esteemed as much as $\frac{1}{2}$ of the stock, which the merchant layeth in. That is to say the $\frac{1}{2}$ of 900 pound which is 450. The reason is, because $\frac{1}{3}$ of the gaine that the factor taketh, is the $\frac{1}{2}$ of the $\frac{2}{3}$ of the gaine that the merchant taketh, and so the

the factor his person is estéemed to be worth 450 pound.

A merchant hath deliuered vnto his factor 600 pound. And the factor layeth in 250 poũd and his person. Now because he layeth in 250 pound & his person, it is agréed betwéene them, that he shall take the $\frac{2}{3}$ of the gaine, I demand for howe much his person was estéemed?

Answere. For as much as the factor taketh $\frac{2}{3}$ of the gaine, he taketh $\frac{2}{3}$ of that which the merchant taketh: for $\frac{2}{3}$ are the $\frac{2}{3}$ of $\frac{3}{3}$. And therefore the factors laying in, ought to be 400 pound, which is $\frac{2}{3}$ of 600 pound that the merchant laid in. Then subtract 250, which the factor did laye in, from 400 pound which should haue béene his whole stocke, and there remaineth 150 pound for the estimation of his person.

More a merchant hath deliuered vnto his factor 800 l, vpon condition that the factor shall haue the gaines of 160 l, as though he laide in so much readie money, I demaunde what portion of the gaine the saide factor shall take?

Answere. Sée what part the 160 l (which the factor laide in) is of 960 which is the whole stocke of their companie, and you shall finde $\frac{1}{6}$. And such part of the gaine shall the factor take.

But in case, that in making their couenants,

nants, it were so agréed betwéene them that the factor should haue the gaine of 160 ℔ of the whole stocke which the merchant layeth in. That is to say of the 800 ℔, then should the factor take $\frac{1}{5}$ of the gaine: for 160 is $\frac{1}{5}$ of 800 pound

The thirteenth Chapter entreateth of Rules of Barter, and exchanging Merchandise, which is distinct into 7. Rules, with diuerse other necessarie Questions incident thereunto.

1. Rule.

TWo Merchants willing to change their merchandise the one with the other. The one hath 24 broade clothes at 10 ℔—10 s̄ the péece. The other hath Mace, at 12 s̄ the pound. The question is howe many pound of Mace he ought to giue him for his clothes, to saue himselfe harmelesse and be no loser.

Answere. Séeke first by the rule of thrée, what the 24 clothes cost at 10 ℔—10 s̄ the péece, and you shall finde 252 ℔. Then to finde the quantitie of Mace, say againe by the rule of 3, if 12 s̄ buy one pound, what shall 252 ℔ buye me? Worke, and you shall finde 420 pound of Mace. And so many pound ought he to

to giue for his clothes.

The Proofe.

Two barter, the one hath 420 pound of Mace at 12 s the pound, to barter or change for broad clothes at 10 l—10 s the péece. The question is how many broad clothes he ought to giue for all his mace.

Answere. First say if 1 cost 12 s, what 420? You shall finde 5040 s. Then say againe if 10 ½ l giue 1 cloth, what shal 5040 s giue? Worke, and you shall finde 24 clothes your desire.

2. Rule.

Two change merchandise for merchandise. The one hath Pepper at 2 s—4 d the pound, to sell for readie money. But in barter he wil haue no lesse than 3 s the pound. And the other hath Holland at 5 s—6 d the Elle ready money. The question is nowe at what price he ought to deliuer the elle in barter to saue himselfe harmelesse.

Answere. Say by the rule of three direct: if 2 ⅓ s readie money giue 3 s in barter, what shall 5 ½ s giue in barter? you shall finde 7 1/14 s, and at that price ought the second Merchant to sell his Holland in barter.

The

The Proofe.

Two barter. The one hath Holland at 5 s̄—6 d̄ in the elle to sell for readie money. And in barter he will haue 7 $\frac{1}{14}$ s̄. The other hath Pepper at 2 s̄—4 d̄ the pound to sell for readie money. The question is nowe howe he ought to sell it in barter.

Answere. Say by the rule of three direct. If 5 $\frac{1}{2}$ readie mony giue 7 $\frac{1}{14}$ s̄ in barter, what ought 2 $\frac{1}{3}$ s̄ to take in barter? Multiplie and diuide, & you shall finde 3 s̄ your desire.

3. Rule.

Two barter. The one hath cloth of Arras at 30 s̄ the Elle readie money, but in barter he will haue 35 $\frac{1}{2}$ s̄. And the other hath white wines, which he deliuered in barter for 16 l̄ for a Tun. The question is nowe what his wines cost the Tunne in readie money.

Answere. Say by the rule of 3 direct. If 35 $\frac{1}{2}$ s̄ in barter giue but 30 readie money, what did 16 l̄ in barter cost? Worke and you shall finde 13 l̄—10 s̄ $\frac{30}{71}$. And so much cost his wines for a Tunne readie money.

The Proofe.

Two barter merchandise for merchandise. The one hath wines white at 13 l̄—10 s̄ $\frac{30}{71}$ the

the Tunne to sell for readie money, But in barter he deliuered it for 16 ℔. The other to make his match good & saue himselfe harmelesse, deliuereth Arras at 35 ½ s̃ the elle. The question is nowe, what an Elle of his Arras cost in readie money.

Answere. Say by the rule of three direct. If 16 ℔ in barter giue but 13—10 $\frac{30}{71}$ s̃ in readie money, what shall 35 ½ s̃ yelde in barter? Worke and you shall finde 30 s̃ your desire.

4. Rule.

Two barter. The one hath Kerseyes at 14 ℔ the peece readie money. But in barter he will haue 18 ℔. And yet he will haue the ⅓ part of his ouerprice in readie money. And the other hath Ginger at 8 groats the pound to sell for readie money. The question is how he ought to deliuer the Ginger by the ℔ in barter to saue himselfe harmelesse, and make the barter equall.

Answere. Item for the working of this question, and such other the like, you must vnderstand, if the partie ouerselling his wares, require to haue also some portion in ready money, as ½ : ⅔ : ¾ &c. Then shall you first rebate the same demaunded part whatsoeuer it be from the ouer price, and also from the iust price. And those two numbers that shall remaine after the subtraction is made, shall be the

the two first numbers in the rule of three. And the iust price of the same merchandise shall be ỳ third number, which by the operation of the rule of three direct shall yeeld you a true solution howe and at what price you shall oversell that your merchandise to saue your selfe harmeleſſe, and make the barter equall.

Example.

Take the $\frac{1}{3}$ (of eighténe) which is the ouer price of his cloth, which $\frac{1}{3}$ of eighténe is sixe, which as appeareth here in the margent, you must subtract from 18, there resteth 12. And also abate it from 14, which is the iust price of the cloth, and there remaineth 8, which 8 and 12 are the two first numbers in the rule of three. Then take 8 groates or $2\frac{2}{3}$ s for the third number. Then say by the rule of 3 direct, If 8 l giue 12 l, what shall $2\frac{2}{3}$ s giue? Multiplie and diuide, and you shall finde 4 s. And for so much shall the second Merchant sell his Ginger, or his commoditie in barter, to ballaunce the same equall.

14	18
6	6
8	12

The Proofe.

Two barter. The one hath fine Kerseyes at 14 l the péece readie money. But in barter

ter he will haue 18 ℔, and yet he will haue the $\frac{1}{3}$ part of his ouerprice in readie money. And the other hath Ginger, which he hauing cunning ynough to make the barter equall, deliuered in barter for 4 s̄ the pound. The question is nowe what his Ginger cost him in readie money.

Anſ. After you haue made the ſubtraction, abating 6 the $\frac{1}{3}$ part of 18, both from 18 & 14, as before was taught you. Then will there remaine 8 and 12 for your two firſt numbers in the rule of three. Then ſay, if 12 giue but 8, what ſhall come of 4 the ouerprice of the pound of Ginger? Multiplie and diuide, and you ſhall finde 2 s̄—8 d̄ your deſire.

Two Merchants barter merchandiſe for merchandiſe. The one hath Denſhire whits at 7 ℔—13 s̄—4 d̄ the peece readie money, but in barter he doth them away for 8 ℔—3 s̄—4 d̄. And yet he will haue the $\frac{1}{3}$ part of his ouerprice in readie money. And the other hath Cottens at 3 ℔ the peece readie money. The queſtion is nowe at what price he ought to ſel or exchange his Cottens in barter to ſaue him ſelfe harmeleſſe, & make the barter equall.

7—13——4——	8——3——4——
2—14—5 $\frac{1}{3}$——	2—14——5 $\frac{1}{3}$——
4—18—10 $\frac{2}{3}$——	5——8—10 $\frac{2}{3}$——

Answere. First seeke $\frac{1}{3}$ part of 8 l—3 s—4 d, which is 2 l—14—5 $\frac{1}{3}$ d, which rebated from 8—3—4 d, there resteth as appeareth by the example aboue saide 5—8—10 $\frac{2}{3}$ d, which $\frac{1}{3}$ of 8—3—4 d also rebated from 7—13—4 d, there resteth 4—18—10 $\frac{2}{3}$ the two first numbers in the rule of 3. And the 3 l which is the neate price of the peece of Cotten is the third number. Then say by the rule of three direct, as was taught before: If 4—18 10 $\frac{2}{3}$ d giue 5 l—8 s—10 $\frac{2}{3}$ d, what shall 3 l giue? Multiplie and diuide, and you shal finde 3—6—0 $\frac{73}{89}$ d the iust price that he ought to deliuer his Cottens in barter.

5. Rule.

Two merchants will change merchandise for merchandise. The one hath Kerseyes at 40 s the peece to sell for readie money. And in barter he will sell them for 56 s—8 d, and he will gaine after 10 l vpon the 100 l. And yet he will haue the $\frac{1}{2}$ of his ouerprice in readie money. The other hath Flare at 3 d the poũd readie money. The question is nowe howe he shall sell the pound of his Flare in barter.

Answere. See first at 10 l vpon the 100 l what the 56 $\frac{2}{3}$ s commeth to, in saying by the rule of three direct. If 100 l giue 110 l, what 56 $\frac{2}{3}$? Multiplie and diuide, and you shal finde 3 l—2 s—4 d of which the $\frac{1}{2}$ that he demandeth

beth in readie money, is, 1 ℔—11 s̃—2 d̃, the same 31 s̃—2 d̃ abated from 40 s̃, and also from 56 s̃—8 d̃: there will remaine 8 s̃—10 pence, and 25 s̃—6 d̃, for the two first numbers in the rule of three. And 3 pence the price of the pound of flare for the third number. Then multiplie and diuide, and you shal finde 8 $\frac{33}{53}$ d̃. And for so much shall he sell the pound of flare in barter.

6. Rule.

Two are willing to exchange Merchandise: The one hath Norwich Grograines at 25 s̃ the peece readie money. And in barter he will haue 30 s̃, and he will haue the $\frac{1}{4}$ part of his ouerprice in readie money. The other hath Norwich stockings at 40 s̃ the dozen to sel for readie money. But in as much as the first merchaunts Grograines are no better, he would deliuer them so to ballance the barter, that he may gaine after 10 ℔ in the 100 ℔. The question is nowe howe he shall sell his hose the dozen in barter, according to his request.

Answere. Say, if 100 giue 110, what shall 40 s̃ giue, which is the iust price of the dozen of stockings? Multiplie and diuide, and you shall finde 44 s̃. Then take the $\frac{1}{4}$ of 30 s̃, which is 7 s̃—6 d̃. And subtract it from 25 s̃, and also from 30 s̃, and there will remaine

17 s̄—6— and 22 s̄—6 d̄, for the two first numbers in the rule of three, and 44 s̄ which is the iust price (with his gaine in the dozen of stockings) for the third number. Then multiplie and diuide, and you shall finde 56 s̄ $6\frac{6}{7}$: and for so much he is to sell his dozen of stockings in barter.

7. Rule.

Two merchants will change their merchandise one with the other. The one hath 720 Elles of Cambricke at 5 s̄ the Elle to sel for readie money, but in barter he requireth 6 s̄—8 d̄. And yet notwithstanding he loseth by it after 10 l̄ vpon the 100 l̄, whereupon he requireth the $\frac{1}{3}$ of his ouerprice in readie money, and the other merchant hauing skill ynough to make the barter equall, deliuereth English Saffrons at 30 s̄ the l̄. The question is nowe what his Saffron cost the pound in readie money.

Answere. You must first seeke what is lost vpon the 100 l̄, which to doe you may say if you please, If 100 l̄ lose 10, what shall $6\frac{2}{3}$ lose? Worke and you shall finde $\frac{2}{3}$ s̄ (or 8 d̄) which must be rebated from 6 s̄—8 d̄. So resteth 6 s̄ still: or you may say if 100 l̄ giue me but 90 l̄, what shall 6 s̄—8 d̄ giue? Worke this way either, and you shall finde also as before, directly in your quotient 6 s̄ your de-

sire.

sire. Then are you next to cast vp what the 720 Elles of Cambricke commeth to at 6 s 8 d the Elle, and you shall finde 240 pounds the ½ whereof the Cambricke Merchant will haue in readie money (which is 120 pound) nextly you must cast what the Cambricke commeth to after his losse in the 100 l, which as you found is but 6 s an elle, and you shall finde 216 l. Nowe must you subtract (his readie money (which is 120 l) out of 240 l, and also out of 216 l, and there will remaine 120 l and 96 l for your two first numbers in the rule of three, and 30 s the ouerprice of your Saffron for the third number. Then multiplie and diuide, and you shall finde 24 shillings. And so much did his Saffron cost in readie money.

Two Merchaunts barter. The one hath 50 Clothes to put away for readie money at 11 l the Cloth, & in barter putteth them away for 12 l, taking Hollande Cloth at 20 d the Flemmish Elle, which was worth no more but 18 d. The question is now, what Holland payeth for the Cloth, and what he winneth or loseth by the bargaine?

Answere. 50 Clothes at 11 l the cloth commeth to 550 l, & put away at 12 d the peece, maketh 600 l: Then to finde what Holland payeth for the Cloth, say by the rule of 3 direct, If 20 d buye 1 Elle, what 600 l? Worke,

and you shall finde 7200 Elles. Now to finde the estate of his gaine or losse, you must seeke what his 7200 Elles commeth to at 18 s the Elle. Worke by the rule of Proportion direct, and you shall finde 540 l, which is not so much as his Clothes were worth in readie money by 10 pound, and so much lost the first Merchant by his exchange.

A Venetian hath in London 100 peeces of silke, to put away for readie money at 3 l the peece. But in barter he deliuereth them for 4 l the peece, taking wools of a Felmonger at 7 l—10 s the C. weight, which was worth no more but 6 l the C. in readie money. The question is nowe, what wools payeth for the silkes, and which of them winneth or loseth by the barter.

Answere. 100 peeces of silkes at 3 l, is 300 l, and at 4 l is 400 l. Then to finde what woolles payeth for the silke, say by the rule of three direct, If 7½ l buy me 1 C. weight, what 400 l? Worke, and finde 53 ⅓ C. weight of woll. Nowe to finde the estate of their gaine or losse, cast vp his woll at 6 l the C. (for so much they were worth readie money) and you shall finde 320 l, which is 20 l more than the silkes were to be sold for in readie money, whereupon the Venetian gained 20 l by the barter.

A

A Merchant hath 53 ⅓ C. waight of woll at 6 l. the C. to sell for readie money, but in barter he will haue 7 l.—10 s: and an other doth barter with him for Silkes which are worth 3 pounds a peece the readie money. The question is nowe, how he ought to deliuer his silkes the peece in barter: and howe many payeth for the wolles.

Answere. Say by the rule of Proportion, or the rule of three direct, If 6 l. for a C. weight readie money yeeld me 7 l.—10 s, what will 3 l. yeelde, which is the iust price of a peece of silke in barter. To make the Trucke equall: Worke, and finde 3 l.—15 s the price of a peece of silke in barter: then say, if 3 l.—15 s require 1 peece of silke, howe many peeces of silke are bought with 400 l, which is the value of the 53 ⅓ C. weight of woll at 7 l.—10 s. Worke by the rule of three direct, and you shal finde 106 peeces of silke, and ⅔ of a peece, and so many peeces of silke payeth for the wolles, and neither partie hath aduantage of other.

Two men wil change merchandise the one with the other. The one of them hath beere, at 6 s—8 d the barrel to sel for readie money. But in barter he will sell the barrell for 8 s, and yet he will gaine moreouer after 10 l. vpon the 100 l. And the other hath white Spanish woll at 20 s the Roue to sell for readie money. The question is nowe, howe he shall

deliuer the Roue of woll in barter to saue himselfe harmeleſſe.

Answere. Say if $6\frac{2}{3}$ s̄ which is the iuſt price of the barrell of Beere, be ſolde in barter for 8 shillings, for howe much ſhall 20 s̄ (which is the iuſt price of the Roue of woll) be ſolde in barter? Worke by the rule of three direct, and you ſhall finde 24 s̄. Then for becauſe the firſt merchant will gain after 10 l̄ vpon the 100 l̄, he maketh of his 100 l̄—110 l̄. And therefore ſay by the rule of three, if the ſecond merchant of 110 l̄ doe make but 100 l̄, how much ſhall he make of 24 s̄? Multiplie and diuide, and you ſhall finde 21 s̄—9 d̄ $\frac{2}{11}$ of a penie. And for ſo much ſhall he ſell the Roue of woll to be deliuered in barter, to the end the firſt merchant may gaine 10 in the 100.

Two merchants wil change their commodities the one with the other. The one of them hath white paper at 4 s̄ the reſme to ſell for readie money. And in barter he will doe it away for 5 s̄, and yet he will gaine moreouer, after the rate of 10 l̄ vpon the 100 l̄, and the other hath Mace at 14 s̄—6 d̄ the pounde weight, to ſell in barter. Nowe I demaund what the pound did coſt in readie money?

Answere. Say, if 5 s̄ which is the ouer price of the paper in barter be come of 4 s̄ the iuſt price, of howe much ſhall come 14 s̄ $\frac{1}{2}$, which is the ſurprice of the pound of Mace in bar-

barter? Multiplie and diuide, and you shall finde 11 ß $\frac{1}{2}$. Then for because the first Merchant of Paper will gaine after 10 vpon the 100. Say, if 100 doe giue 110, what shall 11 $\frac{1}{2}$ ß giue? Worke, and you shal finde 12 ß — 9 d $\frac{12}{23}$, and so much did the pound of Mace cost in readie money.

The fourteenth Chapter entreateth of exchanging of money from one place to another.

EXchange is no other thing than to take or receiue money in one Citie, to render or pay the value thereof in an other Citie, or else to giue money in one place, and receiue the value thereof in an other, at terme of certain dayes, moneths, or fayres, according to the diuersitie of the place.

But this practise chiefely consisteth in the knowledge of the Money or Coines in diuers places, of which for thy benefite, (after a few examples giuen to the Introduction of this worke) I will set downe certaine notes of the diuersitie of the common and vsuall Coines in most places of Christendome for traffike.

And first I wil begin at Antwerpe, where they vse to make their accounts by Deniers de gros:

gros: that is to say by pence Flemmish, whereof 12 doe make 1 s̄ Flemmish, and 20 s̄ doe make 1 l̄ de gros.

Item a Merchant deliuered at Antwerpe 400 pounds Flemmish, to receiue in London 20 s̄ sterling for euerie 23 s̄—4 d̄ Flemmish. The question is nowe, howe much sterling money is to be receiued at London for the said 400 pounds Flemmish?

Answere. Say by the Rule of three, if 23 $\frac{1}{3}$ Flemmish giue 20 s̄ sterling, what 400 pounds Flemmish? Worke, and you shall finde 342 l̄—17 s̄—1 $\frac{5}{7}$ d̄, and so much sterling shall I receiue in London for the 400 l̄ Flemmish.

Otherwise also wrought by Rules of Practise in taking the $\frac{1}{7}$ of the Flemmish money deliuered, and abating the same from the principall, the rest is English money sterling, as before.

400	0	0	
57	2	10	$\frac{2}{7}$
342	17	1	$\frac{5}{7}$ sterling.

A Merchant at London deliuereth 200 poūd sterling for Antwerpe at 23 s̄ 5 d̄ Flemmish the pound sterling. The question is, howe much he must receiue at Antwerpe.

An-

Answere. Say by the Rule of three, if 1 ℔ sterling giue 23 s—5 d Flemmish, what 200 pound sterling? Worke, and you shall finde 234 ℔—3 s—4 d. So many poundes Flemmish shall he receiue at Antwerpe for the saide 200 ℔ sterling.

Otherwise also by Practise.

1	23	5	200
	3 s 4 d		33 6 8
	1 d		16 8
	maketh		234 ℔ 3 4 sterling.

In London 200 ℔ sterling is deliuered by exchange, for Antwerpe at 23 s—9 d Flemmish the ℔ sterling. The question is, what rate the Flemmish money ought to be returned to gaine 4 ℔ vpon the 100 ℔ sterling at London.

Answere. First say by the Rule of three direct. If 1 ℔ sterling giue 23 $\frac{3}{4}$ Flemmish, what 200 ℔ sterling? Multiplie and diuide, & you shall finde 237 ℔—10 s. The which to returne to gaine 8 ℔ sterling in London, say by the backer Rule, If 200 ℔ sterling require the exchange 23 s—9 d Flemmish, what the exchange to make 208 ℔ sterling? Worke by the Rule, and finde 22 s—10 d $\frac{1}{22}$ d Flemmish, the effect in the question required.

If

If I take vp money at Antwerpe after 19 s—4 d Flemmish, to pay for the same at London 20 s sterling, and when the day of payment is come, I am forced to returne the same money againe in London to pay my Bil of exchange, so that for 20 s which I take vp here at London, I must pay 19 s—6 d at Antwerpe, I demaund whether I doe win or lose, and howe much in or vpon the 100 l of money.

Answere. Say by the rule of three: If 19 1/3 giue 19 1/3, what will 100 l giue? Multiplie and diuide, and you shall finde 99 l—2 s 106/117 which being abated from 100 l, there will remaine 17 s 11/117, and so much doe I lose vpon the 100 l of money.

If I take vp at London 20 s sterling to pay at Antwerpe 22 s—4 d, and when the day of payment is come, my Factor is constrained to take vp money againe at Antwerpe, wherewith to pay the aforesaide summe, and there he doeth receiue 23 s——4 d Flemmish, for the which I must pay 20 s at London, the question is nowe whether I doe win or lose, and howe much vpon the 100 pound of money after that rate.

Answere. Say by the Rule of Proportion, If 22 1/3 s giue 23 1/3 s, what will 100 l giue? Mul-

Multiplie and diuide, and you shall finde 104 l 9 111/201 s, from the which abate 100 l, and there will remaine 4 l—9 111/201 s, & so much is there gained vpon the 100 l of money.

In Antwerpe is deliuered 200 l Flemmish by exchange for London, at 20 s sterling for euerie 23 s—4 d Flemmish. The question is at what rate the same is to be returned to gaine 10 l vpon the 100 l Flemmish in Antwerpe.

Answere. First say by the rule of three, if 23 1/3 Flemmish giue 20 s, what shall 200 l giue? Worke, and you shall finde 171 l—8 s 6 6/7 d. Then say againe by the rule of 3 direct, if 171 l 8 s 6 6/7 sterling giue me 210 l Flemish, what shal 20 s sterling giue? worke, & you shal finde 24 s—6 d Flemmish. And at the same rate ought the same to be returned at Antwerpe to gaine 10 l vpon the 100 Flemmish.

A Merchant of Antwerpe deliuereth 234 l—3 s—4 d Flemmish, to receiue at London 200 l sterling. The question is now, how the Exchange goeth after this rate.

Answere. Say by the Rule of 3 direct, If 200 giue 20, what giueth 234 1/6? Multiplie and diuide, and you shall finde 23 s—5 d? And for so much goeth the Exchange.

Item the Exchange from London into France,

France, is not like as it is in Flanders, but is deliuered by the French Crowne, which is worth 50 soulx Turnois the pece.

Whereupon also you must note, that in France they make their accounts by Francks Soulx and Deniers Tournois, whereof 12 Deniers maketh 1 soulx Tournois, and 20 soulx maketh 1 ℔ Tournois, which they call a Liure or Francke. But the Merchaunts to make their accounts doe vse French crowns, which is currant among them for 51 soulx Tournois. But by exchange it is otherwise, for it is deliuered but for 50 soulx Tournois the Crowne, or as the taker vp of the money can agree with the deliuerer. And note that this △ Caracter representeth the Crowne by exchange, and is euer 50 soulx Tournois or French money.

A merchant deliuereth in London 240 ℔ sterling after 5 s̄—6 d̄ the crowne, to receiue at Paris 50 soulx Tournois for euerie crown, I demaund howe much Tournois or French money payeth the bils for the saide 240 ℔ sterling.

Answere. Say by the rule of three, if 5 ½ s̄ sterling giue me 50 s̄ Tournois, what shall 240 ℔ sterling giue? Reduce the poundes into shillings, then multiplie and diuide, and you shal finde 2181 Liuers—16 Soulx,—4 Deniers

niers, and $\frac{4}{11}$ Tournois, and so much payeth the Bils at Paris for the saide 240 ℔ sterling.

A Merchant deliuereth in Roan, or elsewhere in France 1430 ℔, or Franckes, the which Francke or ℔ is 20 soulx or poũd Tournois, to receiue in London 6 s̃—4 d̃ sterling for euerie △ of 50 soulx Tournois. The question is howe much sterling money I ought to receiue at London for my 1430 pound Tournois.

Answere. Say, if $2\frac{1}{2}$ ℔ giue me $6\frac{1}{3}$ s̃, what will 1430 giue me? Worke, and you shall finde 3622 s̃ $\frac{2}{3}$ sterling, which maketh 181 ℔ 2 s̃—8 d̃, and so much money is to be receiued at London for the saide 1430 Liuers Tournois, after 6 s̃—4 d̃ for euerie Crowne of 50 soulx.

In London is deliuered 200 ℔ sterling by exchange for Paris, at 5 s̃—9 d̃ the △ of 50 soulx Tournois. The question is at what price the saide crowne is to be returned to gaine 6 ℔ vpon the 100 ℔ sterling at London.

Answere. First say by the rule of three direct, if $5\frac{3}{4}$ s̃ sterling giue 50 soulx Tournois, what shall 200 ℔ sterling giue? Worke, and you shall finde 1739 Franckes or Liuers, 2 soulx $\frac{14}{23}$. Then the which to returne & gaine 6 ℔ vpon the 100 ℔ in London, say by the rule of three direct, if 1739 Franckes 2 soulx $\frac{14}{23}$

yeld

yeld 212 ℔, what the △ of 50 soulr? worke, and finde 6 s̄—1 d̄ $\frac{7}{25}$ the effect required in the question.

A Merchant deliuered in London 160 ℔ sterling, to receiue in Biskay for euerie 5 s̄ 6 d̄ 1 Duckate of 374 Marueides. The question is, howe many Marueides I ought to receiue at Biskay.

Answere. Say, if 5 $\frac{1}{2}$ s̄ sterling giue 374 Marueides, what shall 100 ℔ sterling giue? Multiplie and diuide, and you shall finde 217600 Marueides, and so many I ought to receiue at Biskay for my 160 ℔ sterling.

A Merchant deliuereth in Bayon 20000 Marueides to receiue in London 5 s̄—6 d̄ sterling, for euerie Duckate of 374 Marueides. The question is nowe how much sterling money payeth the Bils of Exchange for the saide 20000 Marueides.

Answere. Say, if 374 Marueides make 1 Duckate, what 20000 Marueides? Multiplie and diuide, and finde 106 Duckates $\frac{178}{187}$.

Then say againe, if 1 Duckate giue 5 $\frac{1}{3}$ s̄, what giueth 106 $\frac{178}{187}$ Duckates? Worke, and finde 30 ℔—6 s̄ and $\frac{34}{561}$ s̄ which is worth $\frac{408}{561}$ partes of a penie.

Otherwise it is wrought more briefer at one working, as in the last question before, in

con-

considering that 5 s̄—8 d̄ containeth 1 Duckate, or 374 Marueides. Therefore say by the rule of 3, if 374 Marueides giue 5 $\frac{2}{3}$ s̄, what 40000 Marueides? Worke, and you shall also finde in your quotients 30 l̄—6 s̄ $\frac{34}{561}$. And so many pound sterling is to be receiued for the 40000 Duckates.

In London 200 l̄ deliuered by Exchange for Vigo 374 Marueides the Duckate, of 5 s̄ 10 d̄ sterling, maketh 25645 $\frac{5}{7}$ Marueides, the which to returne and gaine 10 l̄ vpon the 100 l̄ in London, say by the rule of three direct, if 220 l̄ require 25645 $\frac{5}{7}$ Marueides, what 5 s̄—10 d̄? worke, and finde 340 Marueides, prises of euerie Duckate in returne, which is the effect in the question required.

These may seeme sufficient for instructions.

NOtwithstanding for thy further aide and benefite hereafter followeth 6 special and most briefe Rules of Practise for English, French, and Flemmish money.

1	teacheth	how to turne Flemish to English sterling.
2		how to turn English sterling to Flemish.
3		how to turne Flemish to French.
4		how to turne French into Flemish.
5		how to turne sterling into French.
6		how to turne French into sterling.

The fifteenth Chapter entreateth of the saide 6 Rules of Breuitie, and of valuation of Englisſh, Flemiſh, and French money, and howe each of them may eaſilie be brought to o-thers value.

How briefely to reduce ℔, s̄, and d̄ Flemmiſh, into ℔, s̄, and d̄ Engliſh Sterling.

IT is to be noted, that 7 ℔ Flemiſh maketh but 6 ℔ ſterling, 7 s̄ Flemmiſh maketh 6 s̄ ſterling, and 7 d̄ Flemmiſh 6 d̄ ſterling, ſo that 7 yeldeth but 6. Wherein is euident, that there is loſt $\frac{1}{7}$, (if it may be ſo called) whē it is reduced into Engliſh money. Wherfore to knowe howe much 233 ℔, 13 s̄, 4 d̄, Flemmiſh maketh Engliſh, you muſt ſubtract from it $\frac{1}{7}$, beginning with the pounds, &c. and that which reſteth after this ſubtraction is the ſumme required: ſo that 233 ℔. 13 s̄, 4 d̄ Flemiſh maketh 200 ℔, 5 s̄, $8\frac{4}{7}$ d̄ ſterling.

Example. ℔	s̄	d̄	An other Example. ℔	s̄	d̄
233	13	4	311	0	0
$\frac{1}{7}$ 33	7	$7\frac{3}{7}$	$\frac{1}{7}$ 44	8	$6\frac{6}{7}$
200	5	$8\frac{4}{7}$ ſter.	264	11	$5\frac{1}{7}$

To reduce ℔, s̄, and d̄, Ster. into ℔, s̄, d̄, Flem.

Note that a ℔ Sterling, maketh 1 ℔, 3 s̄, 4 d̄ Flemmish, that is 1 ℔ ½ : 1 s̄ sterling maketh 1 s̄ ½ flem. and 1 ster. maketh 1 ½ flem. So that there is gained (if it may be so called) ½ of the summe being thus reduced to Flem. For of ½ is made ½, which is 1 whole, and ½. Then to knowe howe much 237 ℔, 7 s̄ 6 d̄ ster. maketh flem. subtract from your ster. the ½ of the whole summe, and adde it to the same summe, and it maketh 276 ℔ 18 s̄ 4 d̄, which is the summe required.

Example.			An other Example.		
℔	s̄	d̄	℔	s̄	d̄
237	7	6	337		
½ 39	11	3	½ 56	3	4
276	18	9 ster.	393	3	4

Ye shall note, that the equalitie of Flemmish and French money is this, that is to say the ℔ Flemmish maketh 7 ℔ ½ French or Turnois, 1 s̄ Flemish maketh 7 s̄ ½ French, and a groat Flemmish, maketh 7 d̄ ½ French.

Wherefore to knowe howe much 143 ℔ 4 s̄ 9 d̄ Flemmish maketh French, ye must multiplie the whole number twice by 6, begin-

ning at 8 ß, and so forward, and the product of your second multiplication diuide by 5, so that worke is finished. Or multiplie the saide sum by 7, and take out of it $\frac{1}{5}$, adding it to the product of your multiplication by 7, and that is your number required. So that as well by the one as by the other, 143 ℔ 4 ß 9 ß Flemmish, maketh 1031 ℔ 6 ß 2 ß $\frac{2}{5}$ French or Turnois.

Example.

℔	ß	ß	
143	4	9	flem.
		6	
859	8	6	
		6	
5156	—11	—0	
$\frac{1}{5}$ 1031	—6	—2 $\frac{2}{5}$	frē.

The same otherwise.

℔	ß	ß
143	4	9
		7
1002	13	3
$\frac{1}{5}$ 28	—12	—11 $\frac{2}{5}$
1031	—6	— 2 $\frac{2}{5}$ frē

An other Example.

143 ℔ flem.	
6	
858	
6	
5148	
$\frac{1}{5}$ 1029 ℔ 02 12 ß fren.	

Or thus.

143
7
1001
$\frac{1}{5}$ —28—12
1029 ℔ 12 french.

A briefe Reduction of ℔ s and d French, into ℔ s and d Flemish.

Multiplie 233 ℔, 8 s, 4 d French by 5, and diuide the product twice by 6, that is the laide number by 6, and the product againe by 6, and the quotient of this second Diuision is the thing required. So that 233 ℔, 8 s, 4 d fren. maketh 32 ℔, 8 s, 4 d, ⅔ flemish.

Example.				Another.			
℔	s	d		℔	s	d	
233	8	4	fre.	753			fren.
		5		5			
1167	1	8		2756			
⅙ 194	10	3⅔		⅙ 627	10		
⅙ 32	8	4⅔	fle	⅙ 104	11	8	fle.

To reduce ℔ s and d Sterling, into ℔ s and d French or Tournois.

The ℔ sterling maketh 8 ℔, 8 s french, that is to say 8 ℔ ⅓: the s maketh 8 s ⅓, and the peny 8 d ⅓ french. Wherefore to knowe what 231 ℔, 13 s, 4 d sterling, maketh french, ye must multiplie your whole summe by 42, that is by 7, and the product of it by 6, and diuide this

second product by 5, and that is the summe required.

Otherwise multiplie the summe sterling by 8, and adde twice to the product $\frac{1}{5}$, and it shall produce the summe required. So that both wayes 231 l, 13 s, 4 d sterling, maketh 1946 l French, as here vnder followeth.

Example.

l	s	d	
231	13	4	ster.
		6	
1390	— 0	— 0	
		7	
$\frac{1}{5}$ 9730	— 0	— 0	
$\frac{1}{5}$ 1946	— 0	— 0	fren.

The same otherwise.

l	s	d	
231	— 13	— 4	ster.
		8	
1853	6	— 8	
46	6	— 8	
46	6	— 8	
1946	0	— 0	fren.

An other Example.

753	ster.
6	
4518	
7	
31626	
$\frac{1}{5}$ 6325 — 4	fre.

The same.

753	ster.
8	
6024	
$\frac{1}{5}$ 150 — 12	
$\frac{1}{5}$ 150 — 12	
6325 — 4	fren.

To

To reduce ℔ ß and ð Fren. into ℔ ß ð Ster.

To know howe much 1256 ℔ 12 ß 6 ð fren. maketh in sterling money, multiply the sum by 5, and diuide the product by 7 & 6 at twice, and the last quotient shall be the thing required, that is to say, 1256 ℔ 12 ß 6 ð maketh 149 ℔ 11 ß 11 ð 5/7 sterling.

Example.				An other Example.			
℔	ß	ð		℔	ß	ð	
1256	12	6	fren.	2531			fren.
		5		5			
6283	2	6		12755			
1/2 1047	3	9		1/2 2109	3	4	
1/7 149	11	11 5/7	ster.	1/7 301	6	2 5/7	ste.

Note that when any money is giuen by exchange at London for Roan at 71 ð 1/2, or rather 71 1/7 for the crowne of 50 ß french, there is neither gaine nor losse, for it is one money for an other, accounting 8 ℔ 8 ß French for 1 ℔ sterling. So the giuer loseth the time of payment which is about 15 dayes, and he that taketh it, hath gaine of the same.

They of Roan that put forth or take money by exchange for London, ought to haue like consideration.

Item when any man giueth at London 64 d ½, or rather 64 d ⅔, to haue at one of the Faires of Lions a crowne de Marc, he that so giueth the money loseth the time, and he that taketh it, gaineth the same: for 62 d ⅔ is equal in value to 45 s French. He that putteth or taketh money at Lions for London, ought to consider the same.

Item, when any deliuer in Antwerpe 75 d, to receiue at Lions a crowne of Marcke, he that putteth it forth loseth the time, and hee that taketh it, gaineth the same. For 75 groats Flemmish is equall in value to 45 s French.

Thus for this time I make an ende of the practise of exchange and the instructions thereunto belonging, and according to my promise yet further to gratifie such as are desirous to knowe the common coynes vsed for traffike among Merchants in these Cities following. Here followeth a briefe declaration of their moneys and the reckenings and accoumts of them.

The

The sixteenth Chapter containeth a declaration of the valuation and diuersitie of Coines of most places of Christendome for traffike. And the maner of exchange in those places from one Citie or Towne to an other. which knowne is right necessarie for Merchants, by meanes whereof they doe finde the gaine or losse vpon the exchange.

ITem for as much as the greatest diuersitie of money of exchange is at Lions. Therfore I will begin duely of the money of that place.

At Lyons they vse Franckes, Soulz and Deniers Tournois: a Francke maketh 20 Soulz, and one Soulz 12 Deniers. But the Merchants to keepe their bookes of accounts doe vse French Crownes of the marke at 45 soulz the peece, and doe diuide it into 20 s, 1 s. and 12 d.

Item a Marke of Golde maketh 65 △ of the Marke, which serueth for exchange, and diuide it into 8 ounces. The ounce into 24 pence or deniers, the denier into 24 graines. And so the sum or whole by imagination or gesse.

Also at Lyons there are 4 faires in a yere, at the which they doe commonly exchange, which are from three moneths to 3 moneths.

At

At Geanes they vse the soulx: one Ducket maketh £ 3.

At Naples they vse Duckets, Tarie, and Graines. The Ducket maketh 5 Taris, and one Tarie 20 graines, but they take 6 Duckets (which maketh thirtie taries) for the ounce.

A Ducket maketh 10 Carlins, and a Carlin 10 graines, so that 2 Carlins make a Tarie, and 100 graines make a ducket.

At Rome they vse the duckets of the chamber: one ducket is worth 12 Guylis, and a Guylis 10 soulx.

At Venice they vse duckets Curraunts at 124 soulx a pece or 24 deniers, and 1 denier maketh 32 picolis.

At Palerme and Messine they write, after ounce, tarie, and graines, & 1 ounce is worth 6 duckets or 30 taris, & 1 tarie is 20 graines, and 1 graine 6 picolis, 1 ducket is also worth 24 Carlins.

At Millan they vse £, s, d, of Duckets Imperials, and △ of exchange is worth 4 £.

At Lucques, Florence and Ancone, they vse the △ of golde: in golde the french crowne is worth £ 7, but at Buloigne £ 3, 10 s.

At Barselone they vse the soulx, the ducket of exchange is worth 22 soulx.

At Valence and Saragosse they vse the Liuer, Soulx and Denier, the French crowne of exchange is worth 20 soulx, and 1 soulx is

12 deniers.

At the Fayres of Castil they vse the Marueides, the ducket is worth 375 Marueides.

At Lisbone they vse the Raies, one ducket of exchange is worth 400 rayes.

At Noremburge, Franckford, and August, in Germanie, they vse the Krentzérs, whereof 60 make a Florin.

At Antwerpe they vse ś and d de Gros, and they exchange into the denier de Gros, to wit our English penie.

At London they vse the 1 l. sterling and 1 d sterling, & they exchange in 1 d sterling.

The exchange of Lyons at sundrie places.

Item at Lyons there is exchange in three sorts, at the cities and townes following.

First they deliuer at Lyons one Marke to haue or receiue at Naples almost 41 ½ duckets, at Venice 70 duckets currant, at Rome 63 duckets of the Chamber, Luques and Florence 65 △ of Golde, at Millan 82 △.

And contrariwise at the saide Cities aforesaide they do giue so much of money to haue a marke at Lyons.

Secondly they giue at Lisbone one △ of Marke of 45 soulz Turnois a peece, to haue at Gennes almost 68 soulz. At Palerme and Messine almost 24 Carlins. At Barselone 22 soulz, at Valence or Saragosse 20 soulz. At the faire at Castil 350 Marueides, at Lisbone

bone 360 Rais, in Antwerpe 57 deniers de Gros, and at London 70 ß sterling.

And contrariwise they doe giue in the saide Cities almost as much of their money to haue a French crowne, of the marke at Lyons.

Thirdly they doe giue at Lyons a △ of the sunne to haue almost 93 Krentzers at Franckford, August, Norenberge, or other Cities in Almaine.

Also at Lyons only they doe pay the change the $\frac{2}{3}$ in golde, and $\frac{1}{3}$ in money, or else all in money, in giuing $1\frac{1}{2}$ for the hundreth.

Changes at Naples and other Townes.

Item at Naples they giue or deliuer almost 112 duckets to receiue at Rome 100 duckets of the chamber at the olde value.

Through Luques and Florence they deliuer 100 duckets Carlins to receiue there almost 86 △ of golde.

Through Palerme and Messine one ducket of 5 Tary to receiue there almost 164 grains.

Through Millan one ducket to receiue there almost 90 soulz.

Through Geanes one ducket to receiue there almost 65 soulz. The whole summe to be paide within 10 daies after the sight of the bill of exchange.

Also at Naples they deliuer one ducket to receiue in Antwerpe almost 67 ß or deniers de Gros, within 2 moneths. At London almost

most 60 ß sterling in 3 moneths. At Barselone almest 20 soulx within 2 moneths. At Valence almost 18 soulx within 2 moneths. At Lisbone 333 Raies, within 3 moneths, and at the fayze at Castil, almost 340 Marveides at the same faire.

Change of Venice to other places.

At Venice they deliuer 100 duckets currant to receiue in Almaine almost 140 Florenes at 60 Krentzers the pēce.

At Lucques and Florence almost 108 △ of golde in 10 dayes.

Likewise at Venice they deliuer a Ducket currant to receiue at Palerme and Messine almost 21 Carlins, at Milan almost 93 soulx. At Geanes almost 62 soulx, the whole at 10 dayes ende.

Of the Pair or Pari.

As touching the exchange, it is necessarie to vnderstand or knowe the Pair, which the Italians call Pari, which is no other thing than to make the money of the change of one Citie or Towne to or with the money of another, by meanes whereof they doe finde the gaines or losse vpon the exchange.

Example.

Item hauing receiued letters of credite of one of Antwerpe that the △ of the sunne is there worth 7 soulx. The question is what the same is worth at London when the Pair

or

or exchange goeth for 23 s.

Answere. Say if 23 giue but 20, what giueth 7? worke and find 6 s 1 $\frac{1}{23}$ d, and so much is the △ of the sunne worth at London.

The seuenteenth Chapter conteineth also a declaration of the diuersitie of the waights and measures of most places of Christendom for traffike. At the end of which discourse are two tables, the one for waight, and the other for measure, proportionate and reduced to an equality of our English measure and waight, by the aide whereof the ingenious may easily by the Rule of three, conuert the one into the other at their pleasure, &c.

AT London and so al England through are vsed two kindes of weights and measures, as the Troy waight & the Haberdepoise: from the Troy waight is derined the proportiõ and quantity of al kind of drie & liquide measures, as Pecks, Bushels, Quarters, &c. wherewith is bought and solde all kinde of graine and other commodities mette by the bushell. And in Liquid, Ale, Beere, Wine, Oyle, Butter, Honie, &c. vpon these grounds and statutes is bread made, and sold by the Troy waight. And so is gold, siluer, pearle, pretious stones and Jewels. The least quantitie of this Troy

weight

waight is a graine, 24 of these graines make a penie waight, 20 penie waights an ounce, and 12 ounces a pound: 2 ℔ or 2 pintes of this waight maketh a quart. And so ascending into bigger quantities is produced the measures whereby are solde our other naturall sustenance, viz. Ale, or Beere, with also other necessarie commodities, as Butter, Honie, Herrings, Eeles, Sope, &c. All which last before rehearsed, though their measures (wherein they are contained) be framed and derived from the Troy waight, yet are they in traffike, with diuerse commodities, as Lead, Tinne, Flaxe, Ware, with all other commodities both of this Realme, and of other forraine Countries whatsoeuer, bought and sold by the Haberdepoise waight, after 16 ounces to the pound, and 112 ℔ to the C. waight. And vnto euerie C. is allowed 12 ℔ waight at the common beame. From hence is also deriued the wey of Suffolke Cheese, which conteineth 32 Cloues, 8 ℔ to a Cloue, and weyeth in all 256 ℔. And also the barrell of Suffolke Butter is or should be of like weight with the wey of cheese, viz. 256 ℔. Wore 14 of these ℔ make a stone, and 26 stone containeth a sacke of English woll. Forraine woolles, to wit, French, Spanish and Estrich, is also solde by the pound or C. waight, but most commonly by the Roue, 25 ℔ to a Roue: other commodities of Male are bought and solde by the C.

fiue

fiue score to the C. Except headed ware, to wit cattel, nailes, and Fish, which are solde after fire score to the C. There is also two other sortes of measures, to wit, the Elle, and the Yarde. By the Elle is vsually mette Linnen Cloth, as Canuas, &c. And by the yarde, Silkes, woollen Clothes, &c.

Antwerpe.

At Antwerpe are also 2 sortes of weights, their golde and siluer waight, and their common waight. Gold and siluer is weyed by the Marke, the Marke is 8 ounces, the ounce 20 Esterlings, and the Esterling 32, as our graines. The Goldsmithes diuide that into smaller, but not the merchants, the proofe of golde is made by Caracts, whereof 24 maketh a Marke of fine golde: the Karact is 24 graines, the proofe of the money is made by Deniers, 12 deniers is 1 ℥ fine, that is a Mark of fine siluer, the Denier also is diuided into 24 graines, and the graine into 4 quarters.

Item, 100 Markes in Antwerpe, Troy waight, maketh at Lions 103 Markes, 2 ½ ounces, and 20 graines 23 ℔. At Noremburg 103 markes, 2 ½ ounces, 2 quints, 3 Deniers, at Frankford 105 Markes, at Ausburge 104 markes, 3 ounces, 1 quint. At Venice 103 markes, 1 ounce, 7 deniers, 18 graines. At London 66 ℔.

The marke of golde or siluer at Antwerpe Troy

Troy waight, which is 8 ounces, maketh 7 ½ ounces common waight, with which all other merchandise is weyed. So that the Troy weight is greater than the common waight by 6 ½ in the C. By this waight of Troy they also wey Muske, Amber, Perle, &c.

All silkes are bought at Antwerpe, by the Bruges Elle, which is greater than the common measure, by which they retaile by 2 in the C. Their common Elle is ¾ of our yarde, and ⅔ of our Elle.

Lyons.

At Lyons is vsed 3 sorts of waights, whereof the first is the cõmon Towne waight, with which they wey all kinde of Spicerie, and diuerse other merchandise. The second is called Geneua waight, which is 8 in the 100 greater than the common waight, with which they weigh silkes, &c. The third is French waight, called commonly the Marke waight, and 100 ℔ thereof maketh 106 ½ ℔ Geneua, & 114 ¼ of their common waight, with which French waight is weyed all things that payeth custome or toll.

At Lyons is also vsed two sorts of Elles or Aulnes. The one wherewith they measure grosse Clothes, as Canuas, and such like. The other is called the French Elle or Aulne, with which they measure all other kinde of Merchandize, whereof 7 common Towne Elles

maketh 11 ordinarie French Elles.

Roan.

At Roan 6 ½ Muydes of salt, being the measure of the place, make an hundred at Armuiden in Zeland, and the C. of Bronage measure of Armuidē maketh at Roan 11 Muides, 30 Mines make a Last of Corne, and 16 a last of Dates, 100 ℔ waight there, maketh at London 114 ½, and 109 ⅝ at Antwerpe. And 100 Elles make at London 115 ⅜.

Noremburge.

A 100 ℔ waight of Noremburge maketh at London 111 ½ at Antwerpe 107 ½. And 100 Elles of Noremburge make at London 75 ⅝, at Antwerpe 95 ⅜, &c.

Lisbone.

The C. waight at Lisbone maketh foure Roues, euerie Roue 32 ℔, so that their C. waight is 128 pound, and their pound containeth 14 ounces, and 100 ℔ of their waight maketh at London 113 ½.

Their silke, cloth of Golde and Woollen is measured with a measure which they call a Cubite, containing about ¾ of a Varre of Castile. Howbeit, their common measure is called a Varre which maketh 5 Palmes, & containeth 1 ½ of a Varre of Castile, our Elle of London is equall with the Varre of Lisbone.

All

All kinde of merchandise brought from Flaunders, Roane, or Brittaine, payeth at Lisbone, as a duetie or custome to the king, 20 in the C. which they call the tenth in merchandise, and the other tenth in money.

Note also that all kinde of merchandise comming to Lisbone by land, payeth lesse in custome than that that commeth by water.

Ciuill.

The Roue of Ciuill is 30 ℔, 4 Roues make their C. waight which is 120 ℔. The 100 ℔ of Ciuill maketh at London 102 ℔. Their other common measure is a Varre, whereof 100 maketh at London 74 Elles, & at Rome 40 Canes, &c.

Venice.

At Venice be 2 sorts of waights, the one called La Grosse, the other La Suttle, with the grosse is weyed all kinde of great wares, and with the small, all kinde of Spicerie, and such like: 96 ℔ of grosse waight there maketh at London 100 ℔, and 100 ℔ of spicerie there without any tare or allowance, make at London 64, and with tare 56.

Their other common measure are Braces, whereof 100 make at London 55 ½ Elles, at Antwerpe 92 ½, &c.

Florence.

At Florence the 100 ℔ waight maketh at

Aquila, for Saffron 110, and 145 ℔ of Floréce make at Roan but 100 ℔, the waight of Florence, and that of Luke is all one.

Their other measures are braces, wherof 100 maketh at Antwerpe Burges measure 81 ⅓ Elles, 100 Braces there make at London 49 Elles, &c.

Lucque.

The Lucque Sattens are commonly solde at Lions by waight and 133 ½ ℔, maketh at Lions 100 ℔, so that 1 ℔ ⅓ maketh at Lions but 1 ℔.

Their other measures are Braces, wherof 100 of them make at London 50 Elles, at Antwerpe 83 ⅓ Elles, &c.

Aquila.

At Aquila their 100 ℔ maketh at London 71 ¼, their 136 ⅔ ℔ of Saffron maketh at Geneua but 100, and 11 pound of Geneua maketh 15 pound at Aquila.

Valentia.

At Valentia be 2 sorts of waights, a great and a small. The C. waight of great waight containeth 4 Roues, the Roue 36 ℔, so the C. great waight is 144 ℔, & the C. waight small containeth but 120 ℔, and is also parted into 4 Roues, which is 30 ℔ to a Roue. By the small is solde the Skarlet graine, with al other kinde of spicerie, and by the great is solde woll with all such like grosse wares. The 1 ⅓ ℔ of

silke

silke at Valentia maketh at Lions 1 ℔ Geneua waight. The charge of great marchandise at Valencia containeth 432 ℔, and in smal wares 360 ℔. The waight here and at Barsellone is all one. Their 100 ℔ waight maketh at London 78 ℔, and at Antwerpe 75.

Dansicke.

At Dansicke or Spruce land the rule is, that whosoeuer buyeth any merchaundise there, buyeth it by the Shippound, which is 320 ℔, 20 Lispounds make a Shippound, and the Lispound containeth 16 pound, which Shippoūd of Dansicke maketh at Antwerpe 266 $\frac{2}{3}$ ℔. Their 100 ℔ waight maketh at London 86 $\frac{1}{3}$, &c.

Their other common measures are Elles, whereof 100 make at London 72 $\frac{1}{4}$, and at Antwerpe 120 $\frac{1}{2}$ Elles.

Toulouse.

At Toulouse 6 Cabes of Woad, maketh a Charge, 2 Cesternes of Corne, and all kinde of graine maketh a Charge, the Cesser waieth 160 ℔ waight of that place. Their 100 ℔ in waight maketh at London but 91 $\frac{1}{4}$ ℔.

Geanes.

At Genua or Geanes, a 100 ℔ of their waight maketh at London 71 $\frac{1}{4}$, and at Antwerpe 68 $\frac{3}{4}$, 100 ℔ waight at Genua maketh at Venice, to wit, Suttle 106 ℔.

Their other cõmon measures are Palmes, whereof 100 make at London 20½ Elles, and at Antwerpe 34⅓.

The eighteenth Chapter treateth of Sportes and Pastimes, doone by Number.

IF you would knowe the number that any man doth thinke or imagine in his mind, as though you could diuine, bid him triple it, or put twice so much more to it, as it is, which doone, aske him whether it be euen or odde: if he say odde, bid him take 1 to it, to make it euen, and for that 1 keepe 1 in your minde. Nowe after he hath taken 1 to it to make it euen, bid him giue away halfe, and keepe the other halfe for himselfe, which when he hath doone, bid him triple that halfe, and againe after he hath tripled it, aske him whether it be euen or odde: if he say odde: then bid him take 1 to make it euen again, and for that last 1 keepe 2 in your minde, nowe after he hath made his number euen, bid him cast away the one halfe, and keepe the other still, from which halfe that he keepeth, cause him subtilly to put away or giue you 9 out of his number, and for each 9 that he giueth you keepe 4 in your minde, and thereunto ioyne the 3 which I bade you keepe, and you shall haue your desire.

Ex-

Example.

Imagine he thought 7, the triple whereof is 21, and because it is odde, he is to take 1 to make it euen, which first 1 giuen is 1 for you to keepe in minde. Then the halfe of his 22 being cast away, he reserueth stil 11, which after you haue bid him triple, it maketh 33, then in giuing of him 1 againe to make it euen: vpon that last 1 reserue 2 in your mind, then his halfe of 34 maketh 17, from whence he can giue you 9 but once, Therfore that yelding to you 4, and the 3 that you keepe, make 7 your desire.

An other kinde of Diuination, to tell your friende howe many pence or single peeces, reckening them one with an other, he hath in his purse, or should thinke in his minde.

Which to doe, first bid him double the peces he hath in his purse, or the number he thinketh, if he participate his number or secrecie vnto some one friende that sitteth by him, that can but multiplie, and adde neuer so litle: if their number be great, then shall they worke as you bid thē so much the surer.

Now after he hath doubled his number, bid him adde thereunto 5 more, which done, bid him multiplie that his number by 5 also: which done, bid him tell you the iust summe

of his last Multiplication, which summe the giuer thinking it nothing auaileable, because it is so great aboue his pretended imagination: yet thereby shall you presently with the helpe of Subtraction tel his proposed number.

The Rule is this.

Imagine he thought 17: double 17, and it maketh 34, whereunto if you adde 5, it maketh 39: which multiplied by 5, as here is practised in the margent, it yeldeth 195, which 195 is the summe deliuered you in the worke: then for a generall rule, you shall euermore cut off the last figure towards your right hand, with a dash of your pen, as here is performed, as a figure nothing auaileable vnto your worke, and then rebate 2 from your first figure, after 5 is cut off, and the rest shall euermore be your desire, as by this example doth appeare.

17
2
34
5
39
5
195
2|
17

If in any companie you are disposed to make them merie by maner of Diuining, in deliuering a Ring vnto any one of them, which after you haue deliuered it vnto them, that you wil absent your selfe frō them & they to deuise after you are gone which of them shall haue the keeping thereof, and that you at your returne wil tel them what person hath it, vpon what hand, vpon what finger, and what ioint

Which

Which to doe, cause the persons to sit downe all on a rowe, and to keepe likewise an order of their fingers: nowe, after ye are gone out from them to some other place, say vnto one of the lookers on, that he double the number of him that hath the Ring, and vnto the double bid him adde 5, and then cause him to multiplie that Addition by 5, and vnto the product bid him adde the number of the finger, of the person that hath the Ring. And lastly, to end the worke beyond that number towardes his right hand, let him set downe a figure, signifying vpon which of the ioynts he hath the ring, as if it be vpon the second ioint, let him put downe 2. Then demaund of him what number he keepeth, from the which you shal abate 250. And you shall haue three figures remaining at the least. The first towards your left hand shall signifie the number of the person which hath the Ring, the second or midle number shall declare the number of the finger, & the last figure towards your right hand shall betoken the number of the ioynt.

Example.

Imagine the seuenth person is determined to keepe the Ring vpon the fifth finger, and the third ioint: first double 7, it maketh 14, therto adde 5, it maketh 19, which multiplyed by 5, yeldeth 95: vnto which 95, adde the number of the finger, and it maketh 100: and beyond

yond 100 toward the right hand, I set downe 3 the number of the ioynt, all maketh 1003, which is the number that is to be deliuered you, from which abating 250, there resteth 753, which prefigureth vnto you the seuenth person, the fifth finger, and the third ioynt.

But note, that when you haue made your subtraction, if there doe remaine a 0: in the place of tennes, that is to say, in the second place, you must then abate 1, from that figure which is in the place of hundreds, that is to wit from the figure which is next your left hand, and that shall be worth 10 tenths, signifying the tenth figure, as if there should remaine 803, you must say, that the seuenth person vpon his tenth finger, and vpon his third ioynt, hath the Ring.

And after the same manner, if a man doe cast 3 Dice, you may knowe the points of euerie one of them. For if you cause him to double the points of one Dye, and to the double to adde 5, and the same summe to multiply by 5, and vnto the product adde the points of one of the other Dice. And behinde the number towards the right hand to put the figure which signifieth the points of the last Die, and then to aske what number he keepeth, from which abate 250, and there will remaine 3 figures, which doe note vnto you the points of euerie Die.

An

Another.

If three diuerse things are to be hidden of three diuerse persons, and you to diuine which of the three persons hath the three diuerse things doe thus: imagine the three things to be represented by A. B. C. Then secondly keep well in your minde which of the persons you meane to be the first, second and third. Then take 24 counters or stones, and your three things, and giue A vnto the partie whom you imagine to be your first man, and therwithal giue him one of your 24 counters in his hand. And B. vnto your second man, and therewithall 2 counters. And C. vnto your third man, and therewithall 3 counters. And leaue the rest which are 18 still among them, which done, separate your selfe from them, and afterwards bid them change the things among them as they shall thinke good, which done after they are agreed, bid him that hath such a thing, as before you haue represented by A. for euerie counter that he hath in his hand to take vp as many more. And for him that hath B, for euerie one in his hand to take vp two. And for him that hath C. for euerie one in his hand to take vp 4, and the rest of them to leaue still vpon the bord. These three things and the three persons being fully printed in your minde, come to the Table, and you shall euermore finde one of these fiue numbers, 1. 2. 3, 5. 6, or 7. If therefore one remaine still vp-

on

on the boꝛd, then haue they made no erchãge, but kẽpe them ſtil as they were deliuered vnto them. So that the firſt man hath A, the ſecond B, and the third C. But if 2 remaine, then the firſt man hath B, your ſecond man A, and your third man C. The reſt of the woꝛke and the oꝛder thereof are here apparant by the Table following.

	1	A			1	B	
1	2	B		5	2	C	
	3	C			3	A	
	1	B			1	C	
2	2	A		6	2	A	
	3	C			3	B	
	1	A			1	C	
3	2	C		7	2	B	
	3	B			3	A	

Another Diuination of a number vpon the casting of two Dice.

First let the caster cast both the Dice, and marke well the number: then let him take vp one of them, it maketh no matter which, and loke what number it hath in the bottom, and adde al together: then cast the die againe, and kæpe in his minde what altogether maketh: then let the Dice stande: bring 7 with you and thereunto adde the rest of the pits that you see vpon the vpper side of the Dice, and so many did the caster cast in all.

FINIS.

I. D. to the earneſt Arithmetician.

MY louing friend to Science bent,
Something thou haſt by this booke woon:
But if thou wilt be excellent,
Another race thou muſt yet runne.

2

Supplyes there to but (fewe doe) neede,
And none but ſuch as in our phraſe,
(By Records pen) thou mayſt well reade.
Proceede therefore. Be not ſtunt dwaſe.

3

The ground moſt ſure, whereon this race
With ſpeedefull courage muſt be paſt,
Of late hath turnd his Greekiſh face,
By Engliſh tilth, which aye will laſt.

4

The famous Greeke of Platos lore,
Euclide I meane Geometer:
So true, ſo plaine, ſo fraught with ſtore,
(As in our ſpeach) is yet no where.

A treaſure ſtrange, that booke will prooue,
With numbers skill, matcht in due ſort,
This I thee warne of ſyncere loue,
And to proceede doe thee exhort.

Plus oultre.

Reade I ſee
number of th
termes.

285. 29. The $\frac{23}{80}$ ſhould be $\frac{24}{80}$.
304. 11. For proar. Reade you may pr
318. 5. For $\frac{7}{12}$ of a yere, make it $\frac{7}{13}$.
319. 6. For prooued, make proponed.
330. In the table of perches in the firſt co
reckening doward in the 9. proportion. For 14 fo
and $\frac{1}{7}$, make $\frac{2}{3}$. Then again in the 12 proportion, fo
10 foote and $\frac{5}{14}$, make 10 $\frac{3}{14}$.
Againe in the 16 proportion, for 16 foote and $\frac{5}{9}$, make
16 $\frac{3}{9}$.
Then in the ſeconde columne the 8 proportion, for 9
foote make 9 $\frac{3}{8}$.
There alſo in the 12 proportion, for $\frac{2}{13}$, make 1 $\frac{2}{13}$.
In the 13 proportion for 15 foote and $\frac{6}{42}$, make $\frac{15}{42}$.
Then in the laſt proportion of the ſecond columne,
for 9 foote and $\frac{1}{8}$ make 9 $\frac{1}{8}$.
341. 1. For 6 ouer the totall ſumme, make it 6 o-
uer, of the totall ſumme.
343. 5. For the ſonne muſt haue $\frac{1}{48}$, make $\frac{1}{8}$.
346. 7. For $\frac{13}{25}$, make $\frac{13}{12}$.
348. 3. For medicines, make mixtures.

...th ...
...hildren o...
...rte of Arith...
...hose method is such ...
...d to friend) he will bring them (if the ...
...y thing) to their desire therin in a short time.
...learne them to write any maner of hande vsuall ...
...is Realme of Englande.
... also after reasonable vnderstanding of Arithme-*..., If any be minded to haue them taught the famous account of Debitor and Creditor, they shall finde him readie to accomplish their desire. More also to further such as are desirous that way into the principles of* Algebar *or* Cossicke *numbers. Lastly to learne them to drawe any maner of Demonstration, Deuise, or Proportions. Or to learne them to drawe eyther white or blacke Capitall Letters. Or to drawe or reduce any Mappe or Carde in true proportion, from a great quantitie to a smaller, or to bring a smaller to a greater: Of all or any these things rehearsed, you shall finde the Author (according to his small talent) readie to accomplish the same for ... reasonable reward. Whose dwelling & hath beene t... ...re within the Mayes gate in short ...outhwarke nigh ...ridge.*

FINIS.

Faults escaped in the Printing, which I desire thee good Reader to correct with thy pen for thine owne ease and benefite.

The first number signifieth the page, the second the line of the page.

76 In the first line for where I finde 3 make it 6.
177. 4. For I set. Reade I set forth.
179. 24. For the number of the tens. Reade number of the termes.
285. 29. The $\frac{23}{80}$ should be $\frac{24}{80}$.
304. 11. For proar. Reade you may proue.
318. 5. For $\frac{7}{12}$ of a yere, make it $\frac{7}{15}$.
319. 6. For prooued, make proponed.
330. In the table of perches in the first columne reckening doward in the 9. proportion. For 14 foote and $\frac{1}{2}$, make $\frac{2}{3}$. Then again in the 12 proportion, for 10 foote and $\frac{5}{14}$, make 10 $\frac{3}{14}$.
Againe in the 16 proportion, for 16 foote and $\frac{5}{5}$, make 16 $\frac{3}{5}$.
Then in the seconde columne the 8 proportion, for 9 foote make 9 $\frac{3}{7}$.
There also in the 12 proportion, for $\frac{9}{15}$, make 1 $\frac{9}{15}$.
In the 13 proportion for 15 foote and $\frac{6}{4}$, make $\frac{15}{41}$.
Then in the last proportion of the second columne, for 9 foote and $\frac{1}{8}$ make 9 $\frac{1}{8}$.
341. 1. For 6 ouer the totall summe, make it 60 uer, of the totall summe.
343. 5. For the sonne must haue $\frac{5}{89}$, make $\frac{5}{8}$.
346. 7. For $\frac{13}{22}$. make $\frac{15}{14}$.
348. 3. For medicines, make mixtures.

383. 11. (For further tillage) make furthering of tillage.

385. 19. For 180, make 1800.

391. 10. For 148 make 48, & for 238 make 138.

Finally the Author giueth intelligence. That if any be minded to haue their Children or Seruants instructed or taught in this noble Arte of Arithmetike, *or any briefe exercise thereof. Whose method is such by long custome of teaching, that (God to friend) he will bring them (if their capacitie be any thing) to their desire therin in a short time. As also to learne them to write any maner of hande vsuall within this Realme of Englande.*

Item also after reasonable vnderstanding of Arithmetike, *If any be minded to haue them taught the famous account of Debitor and Creditor, they shall finde him readie to accomplish their desire. More also to further such as are desirous that way into the principles of* Algebar *or* Cossicke *numbers. Lastly to learne them to drawe any maner of Demonstration, Deuise, or Proportions. Or to learne them to drawe eyther white or blacke Capitall Letters. Or to drawe or reduce any Mappe or Carde in true proportion, from a great quantitie to a smaller, or to bring a smaller to a greater: Of all or any these things rehearsed, you shall finde the Author (according to his small talent) readie to accomplish the same for a reasonable reward. Whose dwelling is & hath beene these 16 yere within the Mayes gate in short Southwarke nigh Battle bridge.*

FINIS.